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# Stochastic analysis of two unit system with increasing failure and repair rates

**Sunita and Dr. Pardeep Goel**

**Abstract**
The paper analyzes the sensitivity of two unit system for system parameters using Regenerative Point Graphical Technique (RPGT). Taking failure and repair rates constant. A state diagram of the system depicting the transition rates is drawn. Expressions for path probabilities mean sojourn times, mean time to system failure, availability of the system, busy period of the server, expected number of server's visits are derived using RPGT. Tables are prepared to compare and draw the conclusion.

**Keywords:** Regenerative Point Graphical Technique (RPGT), Sensitivity Analysis, System Parameters

## Introduction

A system may consist of a number of units and individual units have great importance in a system for its proper functioning. Parametric values of a system largely depend on the failure and repair rate of individual units. In this paper a stochastic analysis and sensitivity analysis of a two units (one having subunits in parallel and other having subunits in series) system is carried out. Kumar, J. & Malik, S. C. [1] have discussed the concept of preventive maintenance for a single unit system. Liu., R. [2], Malik, S. C. [3], Nakagawa, T. and Osaki, S. [4] have discussed reliability analysis of a one unit system with un-repairable spare units and its applications. Goel, P. & Singh J. [5], Gupta, P., Singh, J. & Singh, I.P. [6], Kumar, S. & Goel, P. [7], Gupta, V. K. [8], Chaudhary, Goel & Kumar [9] Sharma & Goel [10], Ritikesh & Goel [11] and Goyal & Goel [12] have discussed behavior with perfect and imperfect switch-over of systems using various techniques. Most of researchers have assumed that failure rates of units are constant, whereas it is not practically, so, whereas a unit fails its failure rate increases generally on expert on each repair of a unit practically. Here in this paper both situations have been considered to model a tow unit system.

Here for the sensitivity analysis we have considered two units system 'A' & 'B' in which unit 'A' having sub units in parallel hence if one or more sub units fail then the system works in reduced capacity and if the number of sub units failure is greater than a predefined number or else, then system is considered to be in the failed state. Unit 'B' have sub units connecting in series hence if any of its sub unit fail than the unit fail causing the whole system in failed state. Fuzzy concept is used to determine failure/working state of a unit. Taking failure rates exponential (constant), repair rates general & independent and taking into consideration various probabilities, a transition diagram of the system is developed to find Primary circuits, Secondary circuits & Tertiary circuits and Base state. Problem is solved using RPGT to determine system parameters. System behavior and sensitivity analysis is carried out and ell illustrated with the help of tables. Particular cases are taken for different repair and failure rates for sensitivity analysis of the system.

**Assumptions and Notations:** The following assumptions and notations are taken: -

1. A single repair facility is always available.
2. The distributions of failure times and repair times are exponential and general respectively and also different for different units.
3. Failures and repairs are statistically independent.
4. Repair is perfect and repaired system is as good as new one.

5. Nothing can fail when the system is in failed state.
6. The system is discussed for steady-state conditions.
7. Replacement of Un-repairable unit and repair facility is immediate.

$\left(i \xrightarrow{sr} j\right)$ : *r*-th directed simple path from *i*-state to j-state; r takes positive integral values for different paths from *i*-state to j-state.

$\left(\xi \xrightarrow{sff} i\right)$ : A directed simple failure free path from $\xi$ -state to i-state.

$V_{m,m}$ : Probability factor of the state m reachable from the terminal state m of the M-cycle.

$V_{\overline{m,m}}$ : Probability factor of the state m reachable from the terminal state m of the $m - \overline{cycle}$.

$R_i$ (t) : Reliability of the system at time t, given that the system entered the un-failed Regenerative state 'i' at t = 0.

$V_i$ (t) : The expected no. of server visits for doing a job in (o,t] given that the system Entered regenerative state 'i' at t = 0.

$\mu_i$ : Mean sojourn time spent in state i, before visiting any other states;

$\mu_i = \int_0^\infty R_i(t)dt$

$\mu_i^1$ : The total un-conditional time spent before transiting to any other regenerative states, given that the system entered regenerative state 'i' at t=0.

$n_i$ : Expected waiting time spent while doing a given job, given that the system entered regenerative state 'i' at t=0; $\eta_i = W_i^*(0)$.

$\xi$ : Base state of the system.

$f_j$ : Fuzziness measure of the j-state.

$\alpha_1/n$: Constant failure rate of server A from full working state to reduce state.

$\alpha_2/n$: Constant failure rate of local server from reduce state to complete failure.

$\alpha/n$ : Constant failure rate of remote server from full working to failed state.

$\beta_1/m$: Constant repair rate of unit A from full reduced state to full state.

$\beta_2/m$: Constant repair rate of unit 'A' from failed state to reduced state.

$\beta/m$ : Constant repair rate of unit 'B' from failed state to good state.

m : Decreasing repairman's expert coefficient $0 < m < 1$, so on subsequent repairs m decrease, so that repair rate will increase.

n : Increasing repairman's expert coefficient $0 < n < 1$, so on subsequent failures n increase, so that failure rate will increase.

Full Capacity Working State

Reduced State

Failed State

A/$\bar{A}$/a : Unit in full capacity working / reduced state / failed state.

B/b : Unit 'B' in full capacity working / failed state.

Taking into consideration the above assumptions and notations the Transition Diagram of the system is given in Figure 1.

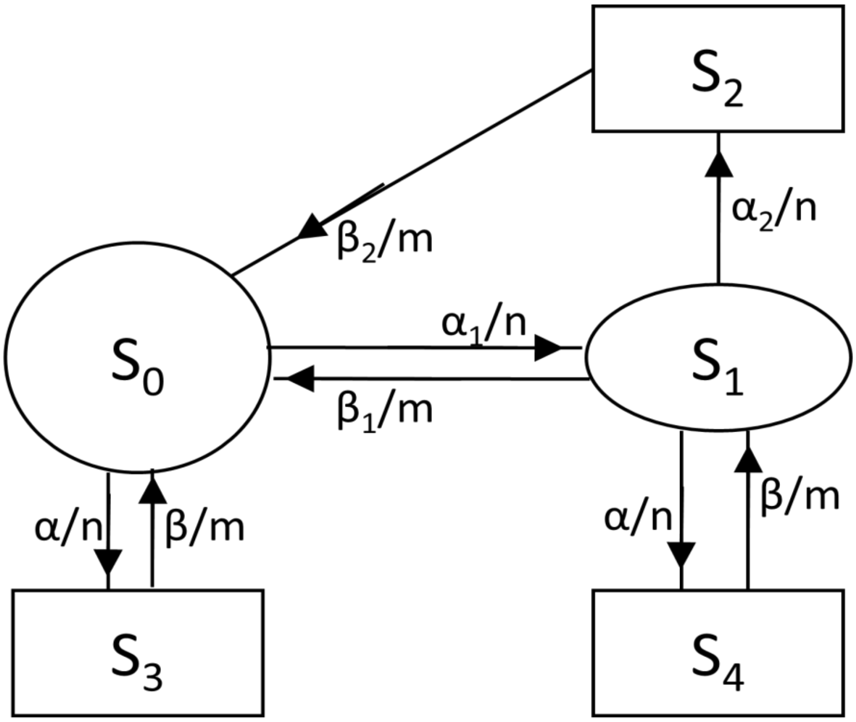


**Fig 1:** $S_0$ = AB, $S_1$ = $\bar{A}$B, $S_2$ = aB, $S_3$ = Ab, $S_4$ = $\bar{A}$b

**Various Paths from vertices**

**Table 1:**

| Vertex | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| 0 | (0,1,0)<br>(0,1,2,0)<br>(0,3,0) | (0,1) | (0,1,2) | (0,3) | (0,1,4) |
| 1 | (1,0)<br>(1,2,0) | (1,0,1)<br>(1,4,1)<br>(1,2,0,1) | (1,2) | (1,2,0,3)<br>(1,0,3) | (1,4) |
| 2 | (2,0) | (2,0,1) | (2,0,1,2) | (2,0,3) | (2,0,1,4) |
| 3 | (3,0) | (3,0,1) | (3,0,1,2) | (3,0,3) | (3,0,1,4) |
| 4 | (4,1,0)<br>(4,1,2,0) | (4,1) | (4,1,2) | (4,1,0,3)<br>(4,1,2,0,3) | (4,1,4) |

**Primary, Secondary & Tertiary Circuits at various vertices**

**Table 2:**

| Vertex i | Primary Circuits (CL1) | Secondary Circuits (CL2) | Tertiary Circuits (CL3) |
|---|---|---|---|
| 0 | (0,1,0)<br>(0,3,0)<br>(0,1,2,0) | (1,4,1)<br>-<br>(1,4,1) | -<br>-<br>- |
| 1 | (1,0,1)<br>(1,4,1)<br>(1,2,0,1) | (0,3,0)<br>-<br>(0,3,0) | -<br>-<br>- |
| 2 | (2,0,1,2) | (0,3,0), (1,4,1)<br>(0,1,0) | -<br>(1,4,1) |
| 3 | (3,0,3) | (0,1,0) | (1,4,1) |
| 4 | (4,1,4) | (1,0,1) | (0,3,0) |

From the table 2, we see that at working state '0', there are maximum number of primary circuits, hence state '0' is the base state.

**Primary, Secondary, Tertiary Circuits w. r. t. the Simple Paths (Base-State '0')**

**Table 3:**

| Vertex j | $\left(0 \xrightarrow{S_r} j\right)$: (P0) | (P1) | (P2) |
|---|---|---|---|
| 0 | $\left(0 \xrightarrow{S_1} 0\right)$: (0,1,0)<br>(0,3,0)<br>(0,1,2,0) | (1,4,1)<br>-<br>(1,4,1) | -<br>-<br>- |
| 1 | $\left(0 \xrightarrow{S_1} 1\right)$: (0,1) | (1,4,1) | - |
| 2 | $\left(0 \xrightarrow{S_1} 2\right)$: (0,1,2) | (1,4,1) | - |
| 3 | $\left(0 \xrightarrow{S_1} 3\right)$: (0,3) | - | - |
| 4 | $\left(0 \xrightarrow{S_1} 4\right)$: (0,1,4) | (1,4,1) | - |

**Transition Probability and the Mean Sojourn Times.**

$q_{i,j}(t)$ **:** Probability density function (p.d.f.) of the first passage time from a regenerative state 'i' to a regenerative state 'j' or to a failed state 'j' without visiting any other regenerative state in (0,t].

$p_{i,j}$ **:** Steady state transition probability from a regenerative state 'i' to a regenerative state 'j' without visiting any other regenerative state. $p_{i,j} = q^*_{i,j}(0)$; where * denotes Laplace transformation.

**Transition Probabilities**

**Table 4:**

| $q_{i,j}(t)$ | $P_{ij} = q^*_{i,j}(0)$ |
|---|---|
| $q_{0,1}(t) = (\alpha_1/n)e^{-\left[\frac{(\alpha_1+\alpha)}{n}\right]t}$<br>$q_{0,3}(t) = (\alpha/n)e^{-\left[\frac{(\alpha_1+\alpha)}{n}\right]t}$ | $p_{0,1} = \alpha_1/(\alpha_1+\alpha)$<br>$p_{0,3} = \alpha/(\alpha_1+\alpha)$ |
| $q_{1,0}(t) = (\beta_1/m)e^{-\left[\frac{(\beta_1)}{m}+\frac{(\alpha+\alpha_2)}{n}\right]t}$<br>$q_{1,2}(t) = (\alpha_2/n)e^{-\left[\frac{(\beta_1)}{m}+\frac{(\alpha+\alpha_2)}{n}\right]t}$<br>$q_{1,4}(t) = (\alpha/n)e^{-\left[\frac{(\beta_1)}{m}+\frac{(\alpha+\alpha_2)}{n}\right]t}$ | $p_{1,0} = n\beta_1/(n\beta_1+m\alpha+m\alpha_2)$<br>$p_{1,2} = m\alpha_2/(n\beta_1+m\alpha+m\alpha_2)$<br>$p_{1,4} = m\alpha/(n\beta_1+m\alpha+m\alpha_2)$ |

| | |
|---|---|
| $q_{2,0}(t) = (\beta_2/m)e^{-\left[\frac{(\beta_2)}{m}\right]t}$ | $p_{2,0} = 1$ |
| $q_{3,0}(t) = (\beta/m)e^{-\left[\frac{(\beta)}{m}\right]t}$ | $p_{3,0} = 1$ |
| $q_{4,1}(t) = (\beta/m)e^{-\left[\frac{(\beta)}{m}\right]t}$ | $p_{4,1} = 1$ |

**Mean Sojourn Times**

**Table 5:** $p_{0,1} + p_{0,3} = 1$, $p_{1,0} + p_{1,2} + p_{1,4} = 1$, $p_{2,0} = 1$, $p_{3,0} = 1$, $p_{4,1} = 1$

| $R_i(t)$ | $\mu_i = R_i^*(0)$ |
|---|---|
| $R_0(t) = e^{-\left[\frac{(\alpha_1+\alpha)}{n}\right]t}$ | $\mu_0 = n/(\alpha_1+\alpha)$ |
| $R_1(t) = e^{-\left[\frac{(\beta_1)}{m}+\frac{(\alpha+\alpha_2)}{n}\right]t}$ | $\mu_1 = mn/(n\beta_1+m\alpha+m\alpha_2)$ |
| $R_2(t) = e^{-(\beta_2/m)t}$ | $\mu_2 = m/\beta_2$ |
| $R_3(t) = e^{-(\frac{\beta}{m})t}$ | $\mu_3 = m/\beta$ |
| $R_4(t) = e^{-(\frac{\beta}{m})t}$ | $\mu_4 = m/\beta$ |

**Evaluation of Parameters:** The Mean time to system failure and all the key parameters of the system under steady state conditions are evaluated, applying Regenerative Point Graphical Technique (RPGT) and using '0' as the base-state of the system as under:

The transition probability factors of all the reachable states from the base state 'ξ' = '0' are:

Probabilities from state '0' to different vertices are given as

$V_{0,0} = 1$ (Verified)

$V_{0,1} = (0,1)/\{1-(1,4,1)\}, = p_{0,1}/(1-p_{1,4}p_{4,1}), = \{\alpha_1/(\alpha+\alpha_1)\}\{(n\beta_1+m\alpha+m\alpha_2)/(n\beta_1+m\alpha_2)\}$

$V_{0,2} = (0,1,2)/\{1-(1,4,1)\}, = p_{0,1}p_{1,2}/(1-p_{1,4}p_{4,1}), = \{m\alpha_1\alpha_2/(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}$

$V_{0,3} = (0,3), = p_{0,3} = \alpha/(\alpha+\alpha_1)$

$V_{0,4} = (0,1,4)/\{1-(1,4,1)\}, = p_{0,1}p_{1,4}/(1-p_{1,4}p_{4,1}), = \{m\alpha\alpha_1/(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}$

**MTSF ($T_0$):** The regenerative un-failed states to which the system can transit (initial state '0'), before entering any failed state are: 'i' = 1 taking 'ξ' = '0'.

$$\text{MTSF }(T_0) = \left[\sum_{i,sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}i\right)\right\}\mu i}{\Pi_{m_1\neq\xi}\{1-V_{\overline{m_1m_1}}\}}\right\}\right] \div \left[1-\sum_{sr}\left\{\frac{\left\{pr\left(\xi\xrightarrow{sr(sff)}\xi\right)\right\}}{\Pi_{m_2\neq\xi}\{1-V_{\overline{m_2m_2}}\}}\right\}\right]$$

$T_0 = (V_{0,0}\mu_0+V_{0,1}\mu_1)/\{1-(0,1,0)\}$

$= [1\{n/(\alpha_1+\alpha)\}+\{\alpha_1/(\alpha+\alpha_1)\}\{(n\beta_1+m\alpha+m\alpha_2)/(n\beta_1+m\alpha_2)\}\{mn/(n\beta_1+m\alpha+m\alpha_2)\}] /(1-p_{0,1}p_{1,0})$

$= [\{n/(\alpha_1+\alpha)\}+\{mn\alpha_1/(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}]/[1-\{\alpha_1/(\alpha+\alpha_1)\}\{n\beta_1/(n\beta_1+m\alpha+m\alpha_2)\}]$

$= [(n\beta_1+m\alpha_2)n+mn\alpha_1]/[(\alpha+\alpha_1)(n\beta_1+m\alpha_2)(\alpha+\alpha_1)(n\beta_1+m\alpha+m\alpha_2)-n\beta_1\alpha_1/(\alpha+\alpha_1)\ (n\beta_1+m\alpha+m\alpha_2)$

$= [\{(n\beta_1+m\alpha_2)n+mn\alpha_1\}\{(\alpha+\alpha_1)(n\beta_1+m\alpha+m\alpha_2)\}]/[(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\{(\alpha+\alpha_1)\ (n\beta_1+m\alpha+m\alpha_2)-\ n\beta_1\alpha_1\}]$

$= [(n^2\beta_1+mn\alpha_2+mn\alpha_1)(n\beta_1+m\alpha+m\alpha_2)]/[(n\beta_1+m\alpha_2)(n\alpha\beta_1+m\alpha^2+m\alpha\alpha_2+n\beta_1\alpha_1+m\alpha\alpha_1+m\alpha_1\alpha_2-n\beta)\}]$

$= [(n^2\beta_1+mn\alpha_2+mn\alpha_1)(n\beta_1+m\alpha+m\alpha_2)]/[(n\beta_1+m\alpha_2)\{\lambda(n\beta_1+m\alpha+m\alpha_2)+m\alpha_1(\alpha+\alpha_2)]$

$= [n(n\beta_1+m\alpha_2+m\alpha_1)(n\beta_1+m\alpha+m\alpha_2)]/[(n\beta_1+m\alpha_2)\{\alpha(n\beta_1+m\alpha+m\alpha_2)+m\alpha_1(\alpha+\alpha_1)]$

**Availability of the System ($A_0$):** The regenerative states at which the system is available are 'j' = 0, 1 and the regenerative states are 'i' = 0 & 1 taking 'ξ' = '0' the total fraction of time for which the system is available is given by

$$A_0 = \left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\rightarrow}j)\}fj,\mu j}{\Pi_{m_1\neq\xi}\{1-V_{\overline{m_1m_1}}\}}\right\}\right] \div \left[\sum_{i,s_r}\left\{\frac{\{pr(\xi^{sr\rightarrow}i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\{1-V_{\overline{m_2m_2}}\}}\right\}\right] = \left[\sum_j V_{\xi,j}, f_j, \mu_j\right] \div \left[\sum_i V_{\xi,i}, f_j, \mu_i^1\right]$$

$A_0 = (V_{0,0}\mu_0+V_{0,1}\mu_1)/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,3}\mu_3+V_{0,4}\mu_4)$

Let $D = V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,3}\mu_3+V_{0,4}\mu_4$

$A_0 = n/(\alpha+\alpha_1)+\{\alpha_1/(\alpha+\alpha_1)\}\{(n\beta_1+m\alpha+m\alpha_2)/(n\beta_1+m\alpha_2)\}\{mn/(n\beta_1+m\alpha+m\alpha_2)\}+m\alpha_1\alpha_2/(\alpha+\alpha_1)$ $(n\beta_1+m\alpha+m\alpha_2)+n\alpha/(\alpha+\alpha_1)\beta+m^2\alpha\alpha_1/(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\beta$

$= \{n/(\alpha+\alpha_1)\}+\{mn\alpha_1/(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}+\{m^2\alpha_1\alpha_2/\beta^2(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}+\{m\alpha/\beta(\alpha+\alpha_1)\}+\{m^2\alpha\alpha_1/\beta(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}$

$= [n\{(n\beta_1+m\alpha_2)\beta\beta_2\}+\ mn\alpha_1\beta\beta_2+m^2\alpha_1\alpha_2\beta+\{m\alpha(n\beta_1+m\alpha_2)\beta_2\}+m^2\alpha\alpha_1\beta_2]/[\beta\beta_2(\alpha+\alpha_1)(n\beta_1+m\alpha_2)]$

$= [n\beta\beta_2(n\beta_1+m\alpha_2)+\ mn\alpha_1\beta\beta_2+m^2\alpha_1\alpha_2\beta+m\alpha\beta_2(n\beta_1+m\alpha_2)+m^2\alpha\alpha_1\beta_2]/[\beta\beta_2(\alpha+\alpha_1)(n\beta_1+m\alpha_2)]$

$= (n^2\beta\beta_1\beta_2+nm\beta\beta_2\alpha_2+mn\alpha_1\beta\beta_2+m^2\alpha_1\alpha_2\beta+mn\alpha\beta_1\beta_2+m^2\alpha\beta_2\alpha_2+m^2\alpha\alpha_1\beta_2)/[\beta\beta_2(\alpha+\alpha_1)(n\beta_1+m\alpha_2)]$

$D = [n\beta_1\beta_2(n\beta+m\alpha)+(\alpha_1+\alpha_2)(nm\beta\beta_2+m^2\alpha\beta_2)+m^2\alpha_1\alpha_2\beta]/[\beta\beta_2(\alpha+\alpha_1)(n\beta_1+m\alpha_2)]$

$= [\{n/(\alpha+\alpha_1)\}+\{\alpha_1/(\alpha+\alpha_1)\}\{(n\beta_1+m\alpha+m\alpha_2)/(n\beta_1+m\alpha_2)\}\{mn/(n\beta_1+m\alpha+m\alpha_2)\}]/D$

$= [\{n/(\alpha+\alpha_1)\}+\{mn\alpha_1/(\alpha_1+\alpha)(n\beta_1+m\alpha_2)\}]/D, = [(n^2\beta_1+nm\alpha_2+mn\alpha_1)/(\alpha+\alpha_1)(n\beta_1+m\alpha_2)]/D$

$= [\{nm(\alpha_1+\alpha_2)+n^2\beta_1\}\beta\beta_1]/[n\beta_1\beta_2(n\beta+m\alpha)+(\alpha_1+\alpha_2)\{(nm\beta\beta_2+m^2\alpha\beta_2)+m^2\alpha_1\alpha_2\beta)\}]$

**Busy Period of the Server:** The regenerative states where server is busy are j = 1, 2, 3, 4 and regenerative states are 'i' = 0 & 1, taking ξ = '0', the total fraction of time for which the server remains busy is

$$B_0=\left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\},nj}{\Pi_{m_1\neq\xi}\{1-V_{\overline{m_1m_1}}\}}\right\}\right]\div\left[\sum_{i,sr}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{m_2\neq\xi}\{1-V_{\overline{m_2m_2}}\}}\right\}\right]=\left[\sum_j V_{\xi,j}\,,n_j\right]\div\left[\sum_i V_{\xi,i}\,,\mu_i^1\right]$$

$B_0 = (V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,3}\mu_3+V_{0,4}\mu_4)/D$
$= (V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,3}\mu_3+V_{0,4}\mu_4-V_{0,0}\mu_0)/D$
$= 1-(V_{0,0}\mu_0/D)$, $= [1-\{n/(\alpha+\alpha_1)]/D$
$= 1-\{n/(\alpha+\alpha_1)\}[\{\beta\beta_2(\alpha+\alpha_1)(n\beta_1+m\alpha_2)/n\beta_1\beta_2(n\beta+m\alpha)+(\alpha_1+\alpha_2)(nm\beta\beta_2+m^2\alpha\beta_2)+m^2\alpha_1\alpha_2\beta\}]$
$= 1-\{n\beta\beta_2(n\beta_1+m\alpha_2)\}/\{n\beta_1\beta_2(n\beta+m\alpha)+(\alpha_1+\alpha_2)(nm\beta\beta_2+m^2\alpha\beta_2)+m^2\alpha_1\alpha_2\beta\}$
$= \{nm\beta_2(\beta_1\alpha+\alpha_2\beta)+m^2\alpha\beta_2(\alpha_1+\alpha_2)+m^2\alpha_1\alpha_2\beta\}/\{n\beta_1\beta_2(n\beta+m\alpha)+(\alpha_1+\alpha_2)(nm\beta\beta_2+m^2\alpha\beta_2)+m^2\alpha_1\alpha_2\beta\}$

**Expected Fraction of Inspections by the repair man:** The regenerative states where the repair man to do repair are j = 1, 3, 4 the regenerative states are i = 0 & 1, Taking 'ξ' = '0', the number of visit by the repair man is given by

$$V_0=\left[\sum_{j,sr}\left\{\frac{\{pr(\xi^{sr\to}j)\}}{\Pi_{k_1\neq\xi}\{1-V_{\overline{k_1k_1}}\}}\right\}\right]\div\left[\sum_{i,sr}\left\{\frac{\{pr(\xi^{sr\to}i)\}\mu_i^1}{\Pi_{k_2\neq\xi}\{1-V_{\overline{k_2k_2}}\}}\right\}\right]=\left[\sum_j V_{\xi,j}\right]\div\left[\sum_i V_{\xi,i}\,,\mu_i^1\right]$$

$V_0 = (V_{0,1}+V_{0,3})/ D$
$=[\{\alpha_1/(\alpha+\alpha_1)\}\{(n\beta_1+m\alpha+m\alpha_2)/(n\beta_1+m\alpha_2)\}+\{\alpha/(\alpha_1+\alpha)\}]/[\{n\beta_1\beta_2(n\beta+m\alpha)+(\alpha_1+\alpha_2)(nm\beta\beta_2+m^2\alpha\beta_2)+m^2\alpha_1\alpha_2\beta\}/\{\beta\beta_2(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}]$
$=[\{\alpha_1(n\beta_1+m\alpha+m\alpha_2)+\alpha(n\beta_1+m\alpha_2)\}/\{(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}]/[\{n\beta_1\beta_2(n\beta+m\alpha)+(\alpha_1+\alpha_2)(nm\beta\beta_2+m^2\alpha\beta_2)+m^2\alpha_1\alpha_2\beta\}/\{\beta\beta_2(\alpha+\alpha_1)(n\beta_1+m\alpha_2)\}]$
$= [\beta\beta_2\{\alpha_1(n\beta_1+m\alpha+m\alpha_2)+\alpha(n\beta_1+m\alpha_2)\}]/[n\beta_1\beta_2(n\beta+m\alpha)+(\alpha_1+\alpha_2)(nm\beta\beta_2+m^2\alpha\beta_2)+m^2\alpha_1\alpha_2\beta\}]$

**Particular Cases:** Particular results are carried out for 1st failure and repair rate for ease of calculations.

**MTSF ($T_0$):** Effect of failure rate of different units on MTSF for m = n = 1, $\beta_1 = \beta_2 = \beta = 0.8$

**MTSF ($T_0$) Table**

**Table 6:**

| **α** <br> **$\alpha_1 = \alpha_2 = 0.1$** | **$T_0$** |
|---|---|
| 0.10 | 9.259 |
| 0.15 | 6.392 |
| 0.20 | 4.888 |

**Table 7:**

| **$\alpha_1$** <br> **$\alpha = \alpha_2 = 0.1$** | $T_0$ |
|---|---|
| 0.10 | 9.259 |
| 0.15 | 8.974 |
| 0.20 | 8.730 |

**Table 8:**

| **$\alpha_2$** <br> **$\alpha = \alpha_1 = 0.1$** | **$T_0$** |
|---|---|
| 0.10 | 9.259 |
| 0.15 | 8.927 |
| 0.20 | 5.761 |

For optimum values of MTSF failure rates should be minimum one and for best values of MTSF failure rate of unit 'A' from initial state reduced state should be taken care of and should be kept minimum.
Effect of change of repair rate of units on MTSF
For $\alpha_1 = \alpha = \alpha_2 = 0.1$ $\beta = \beta_2 = 0.8$

**MTSF ($T_0$) Table**

**Table 9:**

| **$\beta_1$** | **$T_0$** |
|---|---|
| 0.80 | 9.259 |
| 0.85 | 5.397 |
| 0.90 | 5.090 |

As $S_1$ is the reduced state after the initial state $S_0$, thereafter there are failed states so here MTSF only depends upon failure rate $\alpha_1$ and repair rate $\beta_1$.

**Availability of the System ($A_0$): -** Effect of failure rate of different units on availability of the system for m = n = 1, $\beta = \beta_1 = \beta_2 = 0.8$

**Availability of the System ($A_0$) Table (Table 10, 11, 12)**

| **α** <br> **$\alpha_1 = \alpha_2 = 0.1$** | **$A_0$** |
|---|---|
| 0.10 | 0.879 |
| 0.15 | 0.869 |
| 0.20 | 0.792 |

| **$\alpha_1$** <br> **$\alpha = \alpha_2 = 0.1$** | $A_0$ |
|---|---|
| 0.10 | 0.879 |
| 0.15 | 0.875 |
| 0.20 | 0.871 |

| **$\alpha_2$** <br> **$\alpha = \alpha_1 = 0.1$** | **$A_0$** |
|---|---|
| 0.10 | 0.879 |
| 0.15 | 0.875 |
| 0.20 | 0.871 |

For assumed failure and repair rates optimum values is 0.879 and it is recommended for best values of availability failure rates of unit 'A' from full capacity working to reduced state should be kept minimum by the management.

**Availability of the System ($A_0$): -** Effect of change of repair rate of units on availability of the system for $\alpha = \alpha_1 = \alpha_2 = 0.1$

**Table 13:**

| **β** <br> **$\beta_1 = \beta_2 = 0.8$** | **$A_0$** |
|---|---|
| 0.80 | 0.879 |
| 0.85 | 0.884 |
| 0.90 | 0.889 |

**Table 14:**

| **$\beta_1$** <br> **$\beta = \beta_2 = 0.8$** | **$A_0$** |
|---|---|
| 0.80 | 0.879 |
| 0.85 | 0.879 |
| 0.90 | 0.880 |

**Table 15:**

| **$\beta_2$** <br> **$\beta = \beta_2 = 0.8$** | **$A_0$** |
|---|---|
| 0.80 | 0.879 |
| 0.85 | 0.879 |
| 0.90 | 0.880 |

From the above we see that increasing the repair rates of unit almost enhances the availability in equal proportions.
**Busy Period of the Server ($B_0$):** Effect of failure rate of different units on busy period of the server for n = m = 1, $\beta=\beta_1=\beta_2= 0.8$

**Busy Period of the Server ($B_0$) Table (Table 16, 17)**

| **α** <br> **$\alpha_1 = \alpha_2 = 0.1$** | **$B_0$** |
|---|---|
| 0.10 | 0.154 |
| 0.15 | 0.194 |
| 0.20 | 0.234 |

| $\alpha_1$ <br> $\alpha = \alpha_2 = 0.1$ | $B_0$ |
|---|---|
| 0.10 | 0.154 |
| 0.15 | 0.195 |
| 0.20 | 0.235 |

**Table 18:** Busy Period of the Server ($B_0$)

| $\alpha_2$ <br> $\alpha = \alpha_1 = 0.1$ | $B_0$ |
|---|---|
| 0.10 | 0.154 |
| 0.15 | 0.163 |
| 0.20 | 0.171 |

For optimum of value of busy period failure rate of units in all units should be kept minimum one and to maintain its lower values rates $\alpha$ and $\alpha_1$ should be kept smaller in comparison to rate $\alpha_2$.

**Busy Period of the Server ($B_0$) Table:** Effect of change of repair rate of units on busy period of the server $\alpha = \alpha_1 = \alpha_2 = 0.1$

**Table 19:**

| $\beta$ <br> $\beta_1 = \beta_2 = 0.8$ | $B_0$ |
|---|---|
| 0.80 | 0.154 |
| 0.85 | 0.159 |
| 0.90 | 0.163 |

**Table 20:**

| $\beta_1$ <br> $\beta = \beta_2 = 0.8$ | $B_0$ |
|---|---|
| 0.80 | 0.154 |
| 0.85 | 0.158 |
| 0.90 | 0.162 |

**Table 21:**

| $\beta_2$ <br> $\beta = \beta_1 = 0.8$ | $B_0$ |
|---|---|
| 0.80 | 0.154 |
| 0.85 | 0.163 |
| 0.90 | 0.171 |

From above we see that there is no significance change in the values of busy period of server with the increase in repair of units, however to keep its lower value repair rate $\beta_2$ should be less than that of $\beta$ and $\beta_1$.

**Expected Fractional of Server's Visits ($V_0$):** Effect of failure rate of different units on expected fractional of server's visits for n= m = 1, $\beta = \beta_1 = \beta_2 = 0.8$

**Expected Fraction of Server's Visits ($V_0$) Table**

**Table 22:**

| $\alpha$ <br> $\alpha_1 = \alpha_2 = 0.1$ | $V_0$ |
|---|---|
| 0.10 | 0.167 |
| 0.15 | 0.208 |
| 0.20 | 0.249 |

**Table 23:**

| $\alpha_1$ <br> $\alpha = \alpha_2 = 0.1$ | $V_0$ |
|---|---|
| 0.10 | 0.167 |
| 0.15 | 0.200 |
| 0.20 | 0.229 |

**Table 24:**

| $\alpha_2$ <br> $\alpha = 0.1$ | $V_0$ |
|---|---|
| 0.10 | 0.167 |
| 0.15 | 0.166 |
| 0.20 | 0.166 |

For optimum value of $V_0$ failure rates of all units should be kept lowest and to have its value lower the failure rate of unit 'A' should be less than that of unit 'B'.
For $\alpha = \alpha_1 = \alpha_2 = 0.1$

**Expected Fraction of Server's Visits ($V_0$):** Effect of change of repair rate of units on expected fraction of server's visits **Table**

**Table 25:**

| $\beta$ <br> $\beta_1 = \beta_2 = 0.8$ | $V_0$ |
|---|---|
| 0.80 | 0.167 |
| 0.85 | 0.175 |
| 0.90 | 0.183 |

**Table 26:**

| $\beta_1$ <br> $\beta = \beta_2 = 0.8$ | $V_0$ |
|---|---|
| 0.80 | 0.167 |
| 0.85 | 0.167 |
| 0.90 | 0.168 |

**Table 27**

| $\beta_2$ <br> $\beta = \beta_1 = 0.8$ | $V_0$ |
|---|---|
| 0.80 | 0.167 |
| 0.85 | 0.167 |
| 0.90 | 0.167 |

There is no significant improvement in the value of $V_0$ on increasing the repair rates.

**Conclusion:** We have made analysis for $m = n = 1$, but for very poorly designed system results may be obtained when $m > 1$ and $n < 1$. For good system m should be less than one and n greater than one for optimum values of system parameters. From the above discussion and analysis managements may decide to maintain repair/ rates of units in view of the importance of a particular parameter in need of hour.

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