International Journal of Statistics and Applied Mathematics 2018; 3(1): 377-383

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(1): 377-383

www.mathsjournal.com
Received: 18-11-2017
Accepted: 19-12-2017

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Performance evaluation of stochastic model on automatic teller machines considering major/minor faults

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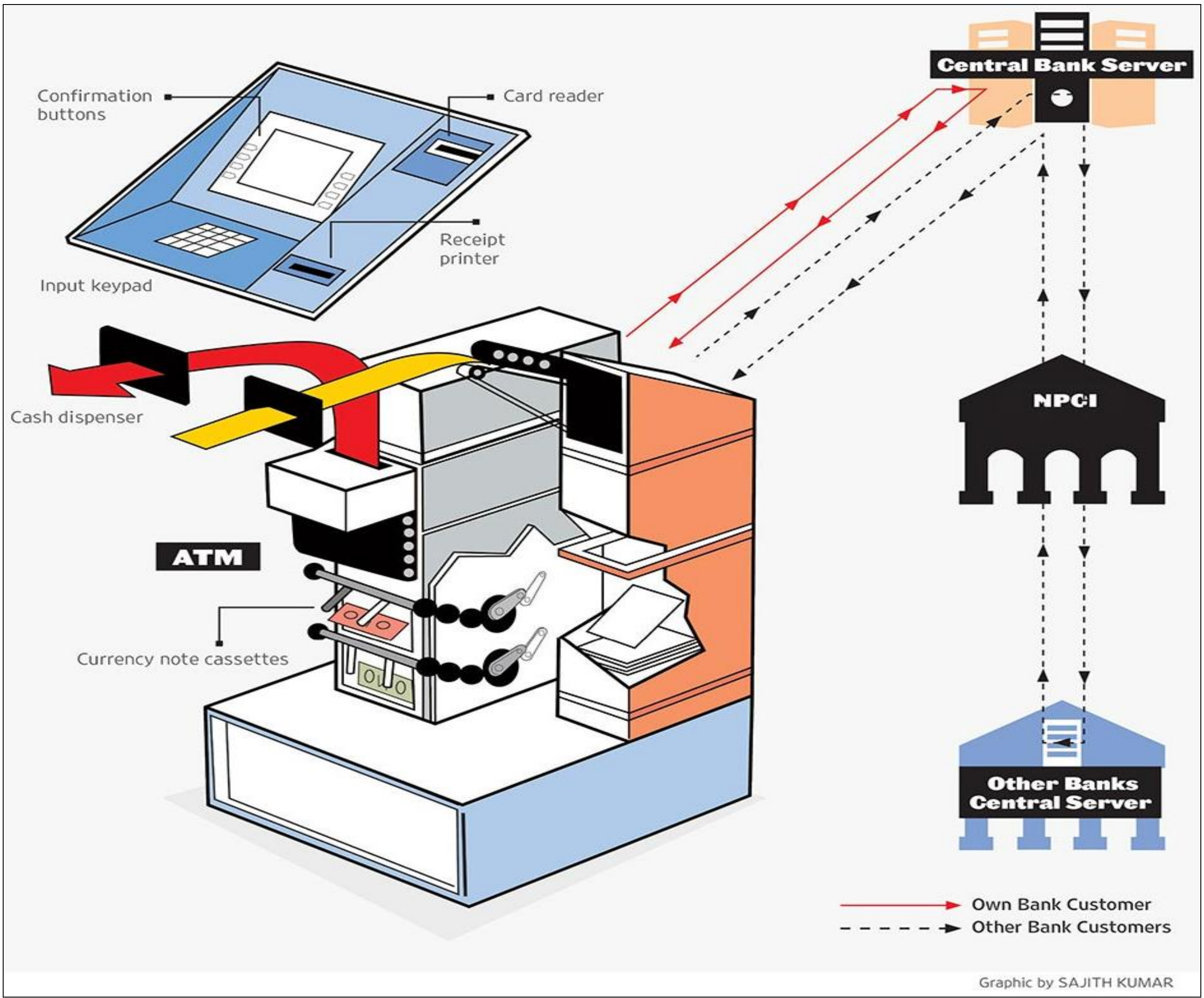
Abstract
The paper considers a reliability model for Automated Teller Machines (ATM) that may have faults in hardware/software components. In the ATM, hardware and software components may have minor or major fault(s) and the occurrence of a minor fault leads to degradation state whereas a major fault leads to failure of the system. Whenever the ATM fails or goes under degradation state then the service engineer first inspect whether there is fault in the hardware or software component then recovery of the relevant component is done. Using Markov processes and regenerative point technique various measures of system performance are obtained. Using these measures the profit analysis of the system is carried out. Various conclusions about reliability and cost of the system are made on the basis of the graphical studies

Keywords: Automated teller machines (ATM), mean time to system failure (MTSF), expected uptime, expected degradation time, profit, Markov process, regenerative point technique

Introduction

Automated-Teller Machine (ATM) has been one of the best options for self service terminal in catering retail banking services. Each ATM is connected to a host computer, which is fitted inside the machine itself. This computer is connected to the central bank server also known as the ATM Switch, which resides at the bank level. The central server is the main brain behind the ATM and gives it instructions to perform various functions, including dispensation of cash. All the screen instructions in an ATM are sent from the central server. When a customer puts in a card, the card reader reads it and asks for the PIN and once the PIN is keyed in, it goes to the central server, which authenticates it and asks what the customer wants to perform through ATM. Once the withdrawal option is selected, the machine asks for the amount to be withdrawn. After the amount is entered, the server picks it up and sends it to the bank host, which could either be the same bank to which the ATM belongs or an even an outside bank, where the customer's account details are held. The central server then checks the balance, debits the account for that amount and then sends instructions to the ATM to dispense the cash. If the card is from the same bank, it is known as an "on-us" transaction and, when the card has been issued by a different bank, it is known as an "off-us" transaction. In an off-us transaction, the ATM switch recognizes the card as being from another bank and sends the information and request to the National Payments Corporation of India (NPCI), which - redirects the request to the respective bank (the outside bank host), which authenticates the request and issues instructions which are sent back to the ATM.

An ATM is the most important part of modern banking system consisting of both hardware and software components. The system can have different kinds of faults that may leads to complete failure/ degradation of the system. When an ATM encounter failures, either due to hardware or software, the machine may not provide its service continuously to its subscribers. In case of occurrence of major fault, there is complete failure of system whereas in case of minor fault system performance and capacity may decrease. ATM supports the customer interface connection with banks so the reliability and cost of ATM plays a very significant role in banking systems and hence need to be analyzed.



Many researchers in the field of reliability modeling including Tuteja *et al* (1991) [15], Welke *et al* (1995) [16], Rizwan and Taneja (2000) [10], Taneja *et al* (2004) [14], Kumar and Bhatia (2011) [6], Kumar and Batra (2013) [7], Kumar and Kapoor (2014) [9], analyzed a large number of systems considering various aspects of Reliability.

Batra (2010) [1] studied operational readiness of a computerized banking system. Sagar *et al* (2011) [2] presents design concept, security analysis for customer satisfaction, performance ability, and software reliability of ATM system. Bing *et al* (2011) [3] studied hardware error likelihood induced by the operation of software. Gupta *et al* (2015) [13] presents an overview of results related to the operational behavior of automatic teller system with respect to reliability analysis. Saini (2015) [12] performs cost-benefit analysis of two identical cold standby systems, subject to failure of airlines reservations system and failure of banking services caused by failure of internet connectivity. Iberahim *et al* (2016) [5] investigates the relationship between the reliability and responsiveness of ATM services with customer satisfaction and verify the determinants for service enhancement.

However, none of the researcher has carried out the profit analysis of ATM considering major/ minor hardware and software faults. While analyzing the working of ATM, it was observed that there are two types of faults one is major and another is minor.

So, keeping this practical situation in view, in the paper, one unit ATM is analyzed by considering the aspect of two types of faults. It is assumed that the occurrence of a minor fault leads to degradation state whereas the occurrence of a major fault leads to failure of the system. Whenever major or minor fault is detected, the service engineer inspects whether there is fault in the hardware or software component then he will repair the relevant component.

Using Markov processes and regenerative point technique various measures of system performance such as MTSF/Expected uptime and Expected degradation time of the system are obtained. Using these measures profit analysis of the system is carried out. Various conclusions about reliability and cost of the system is made on the basis of graphical studies.

Other Assumptions

1. A minor or major fault may be in some hardware or software components. Hardware and Software faults are dealt separately by a team of service engineers.
2. The service engineers immediately handle all types of faults at ATM site.
3. Rate of occurrence of major and minor faults are constant whereas repair and inspection time distributions are arbitrary.
4. After each repair the system is as good as new.
5. Switching is perfect and instantaneous.
6. All random variables are mutually independent.
7. The service engineer takes negligible time to reach the site.

Notations

O	Operative state
λ_1	Rate of occurrence of major faults
λ_2	Rate of occurrence of minor faults
p_1/ q_1	Probability that the major/minor hardware fault occurs in the system
p_2/ q_2	Probability that the major/minor software fault occurs in the system
$q_{ij}(t)/ Q_{ij}(t)$	Probability of transitions from state 'i' to state 'j'
$g_{h1}(t)/ g_{h2}(t)$	P. d. f. of repair time of major/minor hardware fault
$g_{s1}(t)/ g_{s2}(t)$	P. d. f. of repair time of major/minor software fault
$i_1(t)/ i_2(t)$	P. d. f. of inspection time of major/minor fault
$I_1(t)/ I_2(t)$	C. d. f. of inspection time of major/minor fault
$G_{h1}(t)/ G_{h2}(t)$	C. d. f. of repair time of major/minor hardware fault
$G_{s1}(t)/ G_{s2}(t)$	C. d. f. of repair time of major/minor software fault

Transition Probabilities and Mean Sojourn Times

A transition diagram showing the various states of transition is shown as Fig. 1. The epochs of entry in to state 0,1,2,3,4,5,6 are regenerative point, i.e. all the states are regenerative states.

The transition probabilities are

$q_{01}(t) = \lambda_1 e^{-(\lambda_1+\lambda_2)t}$ $\quad$ $q_{02}(t) = \lambda_2 e^{-(\lambda_1+\lambda_2)t}$

$q_{13}(t) = p_1 i_1(t)$ $\quad$ $q_{14}(t) = q_1 i_1(t)$

$q_{25}(t) = p_2 i_2(t)$ $\quad$ $q_{26}(t) = q_2 i_2(t)$

$q_{30}(t) = g_{h_1}(t)$ $\quad$ $q_{40}(t) = g_{s_1}(t)$

$q_{50}(t) = g_{h_2}(t)$ $\quad$ $q_{60}(t) = g_{s_2}(t)$

The non-zero elements $p_{ij} = \lim_{s\to 0} q^*_{ij}(s)$

$p_{01} = \dfrac{\lambda_1}{\lambda_1+\lambda_2}$ $\quad$ $p_{02} = \dfrac{\lambda_2}{\lambda_1+\lambda_2}$

$p_{13} = p_1 i^*_1(0)$ $\quad$ $p_{14} = q_1 i^*_1(0)$

$p_{25} = p_2 i^*_2(0)$ $\quad$ $p_{26} = q_2 i^*_2(0)$

$p_{30} = g^*_{h_1}(0)$ $\quad$ $p_{40} = g^*_{s_1}(0)$

$p_{50} = g^*_{h_2}(0)$ $\quad$ $p_{60} = g^*_{s_2}(0)$

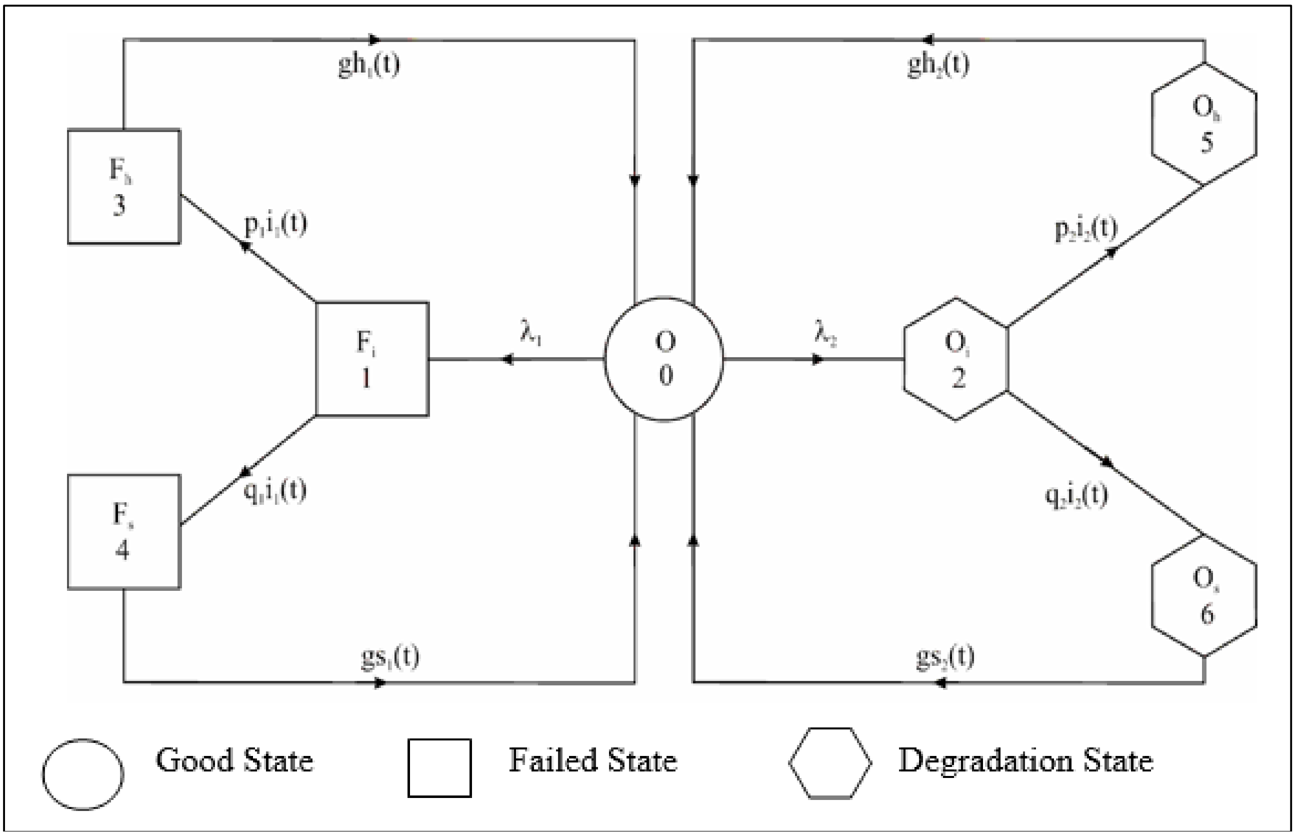


Fig 1: State Transition Diagram

By these transition probabilities, it can be verified that
$p_{01}+p_{02} = p_{13}+p_{14} = p_{25}+p_{26}= 1$
$p_{30} = p_{40}= p_{50}= p_{60}= 1$

The mean sojourn time (μi) in the regenerative state i is defined as the time of stay in that state before transition to any other state. If T denotes the sojourn time in regenerative state i, then

$\mu_0 = \frac{1}{\lambda_1+\lambda_2}$ $\mu_1 = -i_1^{*'}(0)$ $\mu_2 = -i_2^{*'}(0)$ $\mu_3 = -g_{h_1}^{*'}(0)$
$\mu_4 = -g_{s_1}^{*'}(0)$ $\mu_5 = -g_{h_2}^{*'}(0)$ $\mu_6 = -g_{s_2}^{*'}(0)$

The unconditional mean time taken by the system to transit for any regenerative state j, when it is counted from epoch of entrance into that state i, is mathematically stated as
$m_{ij} = \int t\, q_{ij}(t)dt$
Thus,

$m_{01} + m_{02} = \mu_0$ $m_{13} + m_{14} = \mu_1$ $m_{25} + m_{26} = \mu_2$
$m_{30} = \mu_3$ $m_{40} = \mu_4$ $m_{50} = \mu_5$
$m_{60} = \mu_6$

Other Measures of System Performance
Using probabilistic arguments for regenerative processes, various recursive relations are obtained and are solved to derive following important measures of the system performance in steady state:

Mean Time to System Failure (T_0) = N/D
Expected Uptime of the system (UT_0) = N_1/ D_1
Expected Degradation Time of the System (DT_0) = N_2/ D_1
Busy Period of Repairman (inspection time only) (BI_0) = N_3/ D_1
Busy period of Repairman (repair time only) (BR_0) = N_4/ D_1

Where
$N= \mu_0 + p_{02}\, \mu_2 + p_{02}\, (p_{25}\, \mu_5 + p_{26}\, \mu_6)$
$D= p_{01}$
$N_1 = \mu_0$
$N_2 = p_{02}\mu_2+p_{02}\, p_{25}\mu_5+p_{02}\, p_{26}\mu_6,$
$N_3= p_{01}\mu_1+p_{02}\mu_2$
$N_4=p_{01}p_{13}\mu_3+p_{01}\, p_{14}\mu_4+ p_{02}p_{25}\mu_5+p_{02}\, p_{26}\mu_6$
$D_1 = \mu_0+ p_{01}\mu_1+ p_{02}\mu_2 + p_{01}p_{13}\mu_3 + p_{01}p_{14}\mu_4+ p_{02}p_{25}\mu_5+ p_{02}p_{26}\mu_6$

Profit Analysis
The expected profit of the system is

$P = C_0\, A_0 – C_1DT_0 – C_2\, BI_0 – C_3BR_0 – C_4$

Where
C_0 = revenue per unit uptime of the system
C_1= cost per unit degradation time of the system
C_2 = cost per unit time of inspection
C_3 = cost per unit time of repair
C_4 = cost of installation of the system

Graphical interpretation
For graphical analysis following particular cases are considered:

$g_{h_1}(t) = \beta_{h_1} e^{-\beta_{h_1}(t)}$; $g_{s_1}(t) = \beta_{s_1} e^{-\beta_{s_1}(t)}$; $i_1(t) = \alpha_1 e^{-\alpha_1(t)}$;
$i_2(t) = \alpha_2 e^{-\alpha_2(t)}$; $g_{h_2}(t) = \beta_{h_2} e^{-\beta_{h_2}(t)}$; $g_{s_2}(t) = \beta_{s_2} e^{-\beta_{s_2}(t)}$;

Various graphs for measures of system performance viz. MTSF, expected uptime, expected degradation time and profit are plotted for different values of rates of major/minor faults, probabilities of major/minor hardware and software faults (p_1, q_1, p_2, q_2), inspection rates (α_1,α_2), hardware/ software repair rates $(\beta_{h_1}, \beta_{h_2}, \beta_{s_1}, \beta_{s_2})$.

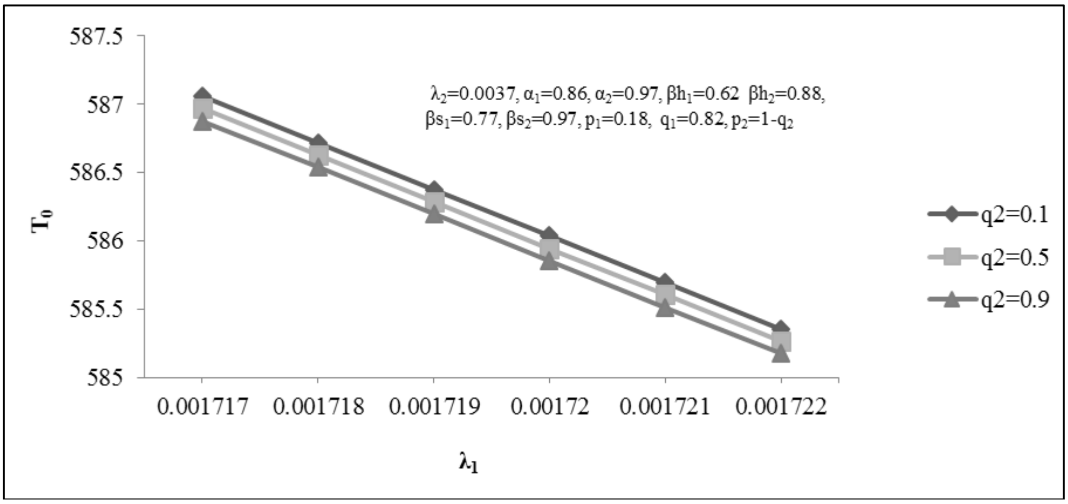


Fig 2: Mean Time To System Failure V/S Rate Of Major Faults For Different Values Of Probability Of Minor Software Faults

Fig. 2 gives the graph between MTSF (T_0) and rate of major faults (λ_1) for different values of probability of minor software faults (q_2). The graph reveals that MTSF decreases with increase in values of the rate of major faults. Further it can be observed that MTSF has lower values for higher values of probability of minor software faults.

Fig. 3 represents the graph between expected uptime (UT_0) and rate of major faults (λ_1) for different values of probability of major hardware faults (p_1). The graph shows that expected uptime decreases with increase in values of the rate of major faults. Further it can be observed that expected uptime has lower values for higher values of probability of major hardware faults.

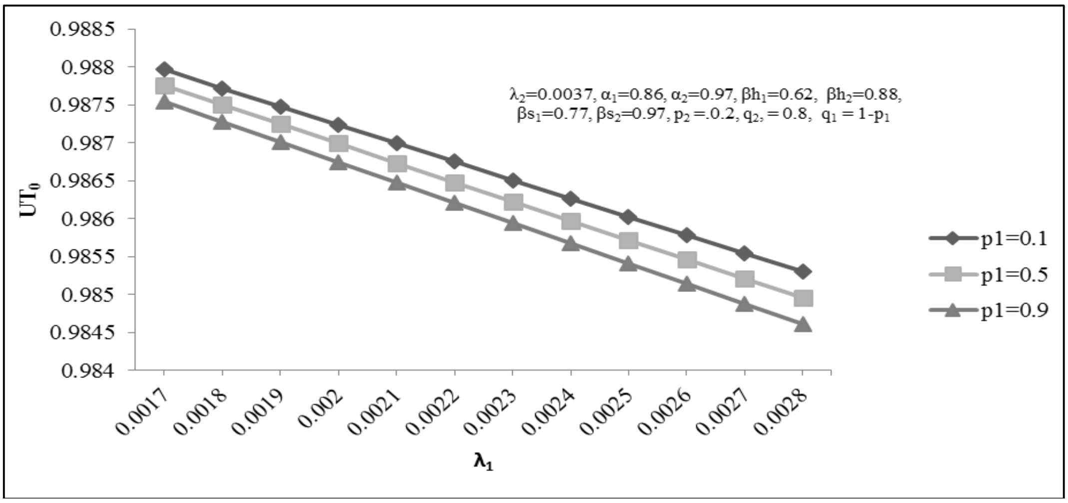


Fig 3: Expected Uptime V/S Rate of Major Faults for Different Values of Probability of Major Hardware Faults

Fig. 4 shows the graph between expected degradation time (DT_0) and rate of minor faults (λ_2) for different values of probability of minor hardware faults (p_2). The graph shows that expected degradation time increases with increase in values of the rate of minor faults. Further it can be observed that expected degradation time has higher values for higher values of probability of minor hardware faults.

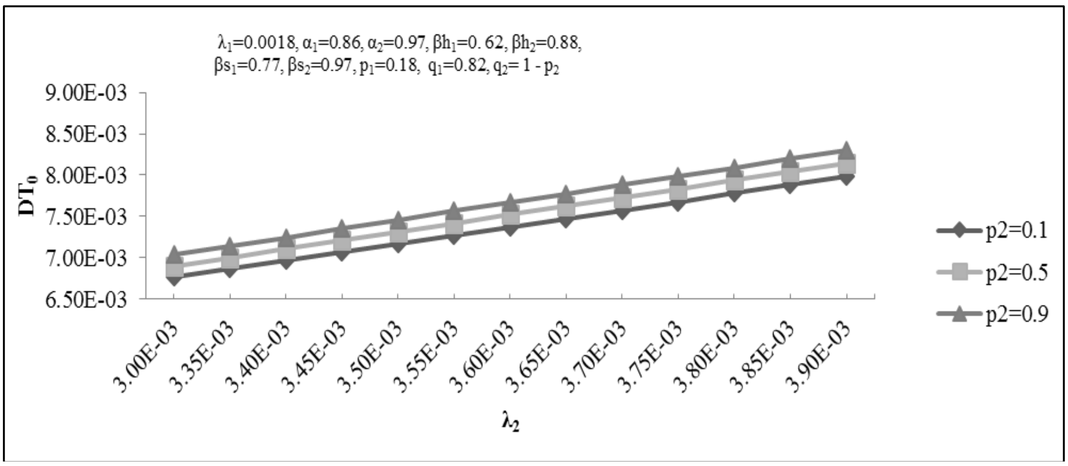


Fig 4: Expected Degradation Time V/S Rate of Minor Faults for Different Values of Robability of Minor Hardware Faults

The graph in fig. 5 shows the pattern of profit with respect to the rates of occurrence of minor faults (λ_2) for different values of rate of major faults (λ_1). The curve in the graph indicates that the profit of the system decreases with the increase in the values of the rates of occurrence of minor fault and has lower values for higher values of rate of major faults. Further from the graph it may also be noticed that for $\lambda_1 = 0.001$ the profit is > or = or < 0 according as λ_2 is < or = or > 0.581. Hence the system is profitable to the company whenever $\lambda_2 \leq 0.581$. Similarly, for $\lambda_1 = 0.301$ and $\lambda_1 = 0.601$, the profit is > or = or < 0 according as λ_2 is < or = or > 0.394 and 0.206 respectively. Hence in these cases the system is profitable to the company whenever $\lambda_2 \leq 0.394$ and 0.206 respectively.

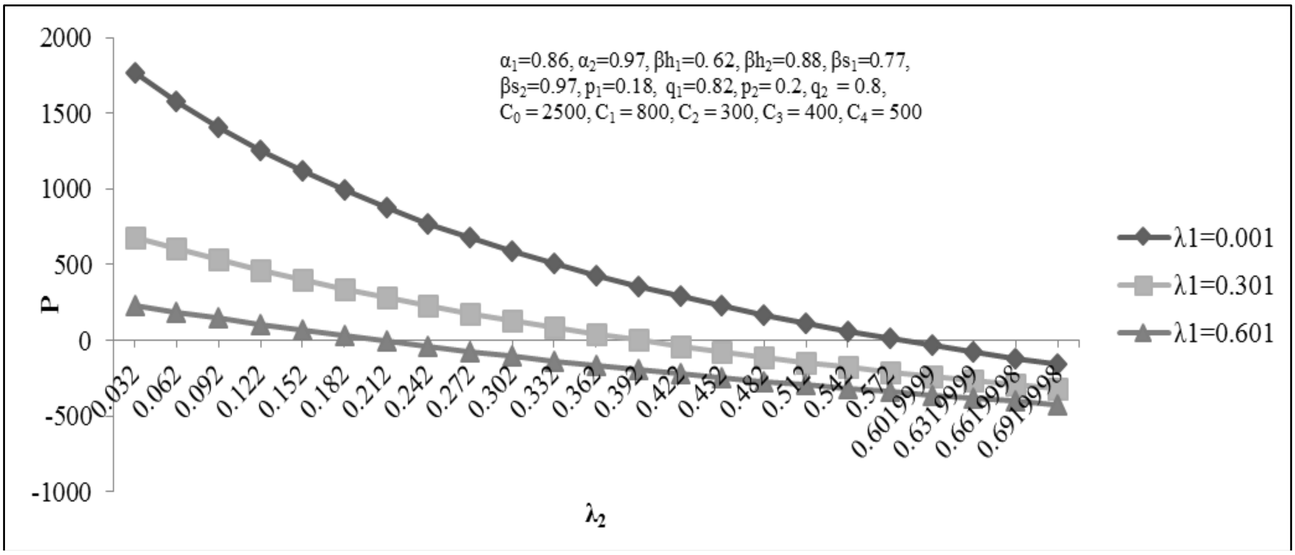


Fig 5: Profit V/S Rate of Minor Faults for Different Values of Rate of Major Faults

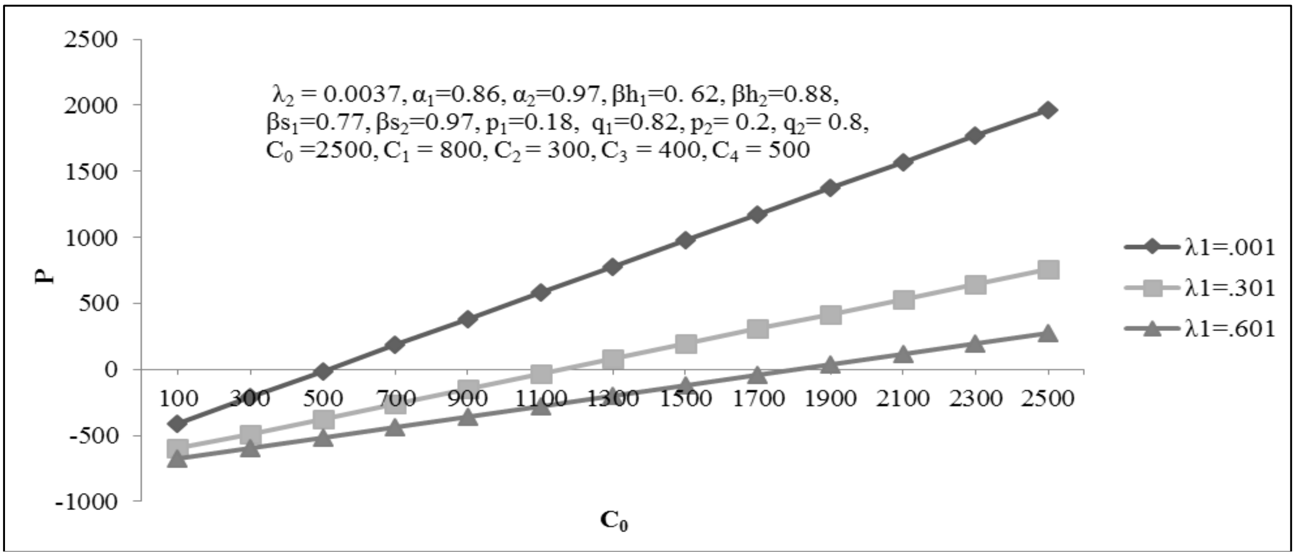


Fig 6: Profit V/S Revenue Per Unit Uptime of The System for Different Values of Rate of Major Faults

The graph in fig. 6 represents the pattern of profit with respect to the revenue per unit uptime of the system (C_0) for different values of rate of major faults (λ_1). The curve in the graph indicates that the profit of the system increases with the increase in the values of the revenue per unit up time of the system and has lower values for higher values of rate of major faults. Further from the graph it may also be noticed that for $\lambda_1 = 0.001$ the profit is < or = or > 0 according as C_0 is < or = or > 514.87. Hence the system is profitable to the company whenever $C_0 \geq 514.87$. Similarly, for $\lambda_1 = 0.301$ and $\lambda_1 = 0.601$, the profit is < or = or > 0 according as C_0 is < or = or > 1159.86 and 1804.85 respectively. Hence in these cases the system is profitable to the company whenever $C_0 \geq 1159.86$ and 1804.85 respectively.

Conclusion

From the graphs it is concluded that the mean time to system failure (MTSF) and expected uptime of the BTS deceases with the increase in the values of the rates of occurrence of major as well as minor faults. Further it is observed that the MTSF decreases with the increase in the probability of occurrence of minor software faults. The expected uptime of the BTS decreases with the increase in the values of probability of major hardware faults. Expected degradation time increases with increase in the values of rate of minor faults as well as with probability of minor hardware faults. It is also clear that the profit increases with increase in the values of revenue per unit uptime of the system but decreases with increase in the values of rates of occurrence of major and minor faults. For fixed values of rate of occurrence of major faults various cutoff points for revenue per unit uptime and rate of occurrence of minor faults are also obtained.

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