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Estimation of parameters of a multivariate time series var model

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Abstract
Multivariate Time Series model building involves five important steps namely, Identification Specification, Estimation and testing the hypotheses, Diagnostic checking and Forecasting. Estimation of parameters of multivariate Vector Autoregressive (VAR) model is more complicated than that of univariate autoregressive models. Under normality of the errors, Maximum likelihood estimation as well as Likelihood Ratio test can be performed in the context of multivariate VAR models.
In the present research article, the parameters of multivariate VAR model have been estimated by using the method of maximum likelihood estimation based on ordinary least squares regression. The dispersion matrix of errors in multivariate VAR model has been estimated by using Internally Studenti zed residuals. A test procedure has been developed is testing number of lags of variable for multivariate VAR model by using the Likelihood Ratio test.

Keywords: parameters, multivariate time series, likelihood estimation

1. Introduction

Univariate Time series models have the advantage of being able to predict a variable only on the basis of current, past and future values. However, the explanatory power of univariate models can be improved by incorporating the political economic information contained in interacting variable.
A Multivariate Time Series Model provides an adequate unrestricted approximation to the reduced form of an unknown structural specification of a simultaneous equations model. Zellner and palm (1974) [1] and Zellner (1979) [2] have shown that any structural model can be written in the form of a Multivariate time series model.
Methods of time series analysis may also be divided into linear and nonlinear; univariate & multivariate methods. In context of statistics, econometrics, quantitative finance, seismology, meteorology and geophysics, the primary goal of time series analysis is "forecasting" while in the context of Datamining, pattern recognition and machine learning, time series analysis can be used for clustering classification, query by content, anomaly detection as well as forecasting.

2. Vector Autoression (Var) Models

Sims (1980) [3] suggested Vector Auto regression (VAR) models for forecasting macro time series. VAR assumes that all the variables are endogenous for instance, consider the following three macro series; money supply, interest rate and output. Vector of three independent variables as a (AR) function of its lagged values. If the number of lags (x) and number of equations (g) increase, then the degrees of freedom problem becomes more difficult. Generally, the number of parameters to be estimated becomes
$g + xg^2$
For small samples, individual parameters may not be estimated. So, only simple VAR model can be considered for a small sample. The system of equations has the same set of variables in each equation SUR on the system is equivalent to OLS on each equation.
Under Normality of the disturbances, MLE as well as likelihood ratio test can be performed.

One important application of LR tests in the context of VAR is in determining the choice of lags to be used. In this case, the Log – likelihood for restricted model with m lags and the unrestricted model with q > m lags.

LR test is asymptotically distributed as $\chi^2_{(q-m)g^2}$. In case of sample size T (large) estimate the large number of parameters (qg^2+g) for the unrestricted model. VAR model have been used to test the hypothesis that some variables do not Granger cause some other variables.

2.1 A Simple Vector Auto Regression (VAR)

Consider the simple form as

$$y_{1t} = m_1 + a_{11} y_{1,t-1} + a_{12} y_{2,t-1} + \in_{1t}$$

$$y_{2t} = m_2 + a_{21} y_{1,t-1} + a_{22} y_{2,t-1} + \in_{2t}$$

$$y_t = m + A y_{t-1} + \in_t \qquad \dots (2.1)$$

Each variable is expressed as a linear combination of lagged values of itself. The VAR equations may be expanded to consider deterministic time trends and other exogeneous variable.

Where as in univariate case, the behaviour of y's will depend on the properties of the A matrix.

Let the eigen values and eigen vectors of a matrix A be

$$\wedge = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad c = \begin{bmatrix} \vdots & \vdots \\ c_1 & c_2 \\ \vdots & \vdots \end{bmatrix}$$

Provided that the eigen values are distinct. The eigen vectors are linearly independent and C will be non-singular.

Then

$$C^{-1} A C = \wedge \quad \text{And } A = C \wedge C^{-1} \qquad \dots (2.2)$$

Let us consider a new vector of variables z_r as

$$Z_t = c^{-1} y_t \text{ Or } y_t = c Z_t \qquad \dots (2.3)$$

By pre multiplying equation (2.1) by c^{-1} gives,

$$c^{-1} y_t = c^{-1} m + c^{-1} A y_{t-1} + c^{-1} \in_t \backslash$$

$$\Rightarrow Z_t = m^* + \wedge Z_{t-1} + \eta_t \qquad \dots (2.4)$$

Where $c^{-1} y_t = Z_t$; $c^{-1} m = m^*$ and $c^{-1} \in_t = \eta_t$ which is a white noise vector. Thus.

$$Z_{1t} = m_1^* + \lambda_1 Z_{1,t-1} + \eta_{1t}$$

$$Z_{2t} = m_2^* + \lambda_2 Z_{2,t-1} + \eta_{2t}$$

Each Z variable follows a separate AR (1) process and is stationary.

I (0), if the Eigen values has modulus < 1 is a random walk with drift

I (1), if the Eigen value is 1 and is explosive if the Eigen value exceeds 1 in numerical value.

2.2 A Three variable Vector Auto Regression (VAR):

By expanding the system of a first order VAR to three variables. Suppose the Eigen values of the A matrix are $\lambda_1 = 1$, $|\lambda_2| < 1$ and $|\lambda_3| < 1$. Thus there exists a (3x3) Nonsingular matrix C of Eigen vectors A. By defining a three element Z vector as in equation (2.3) follows that

Z_{1t} is I (1); Z_{2t} and Z_{3t} are each I (0). If all 'Y' variables are I (1) then y vector may be expressed as

$$y_t = \begin{bmatrix} \vdots \\ c_1 \\ \vdots \end{bmatrix} Z_{1t} + \begin{bmatrix} \vdots \\ c_2 \\ \vdots \end{bmatrix} Z_{2t} + \begin{bmatrix} \vdots \\ c_3 \\ \vdots \end{bmatrix} Z_{3t}$$

Consider a linear combination of the y variables i.e. I (0), we need to eliminate Z_{1t} element. Let $c^{(2)}$ and $c^{(3)}$ denote the second and third rows of c^{-1}, two co-integrating relations are available in

$$Z_{2t} = c^{(2)} y_t \text{ And } Z_{3t} = c^{(3)} y_t \qquad \dots (2.5)$$

A linear combination of I (0) variables is itself I (0) Thus, any linear combination of the variables in equation (2.5) is also a co-integrating relation with an co-integrating vector. When two or more cointegrating vectors are found there is an infinity of cointegrating vectors.

We consider π matrix then the Eigen values are

$$\pi = C(1 - \wedge)C^{-1}$$

$$= \begin{bmatrix} \vdots & \vdots & \vdots \\ c_1 & c_2 & c_3 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix} \begin{bmatrix} \cdots & c^{(1)} & \cdots \\ \cdots & c^{(2)} & \cdots \\ \cdots & c^{(3)} & \cdots \end{bmatrix} \qquad \dots (2.6)$$

$$= \begin{bmatrix} \vdots & \vdots \\ \mu_2 c_2 & \mu_3 c_3 \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \cdots & c^{(2)} & \cdots \\ \cdots & c^{(3)} & \cdots \end{bmatrix}$$

Thus π splits into the product of a (3x2) matrix of rank two. The matrix contains the two cointegrating vectors with which both cointegrating vectors enter into the Error Correction formulation for each Δy_i the

$$\Delta y_{it} = m_1 - (\mu_2 c_{12}) Z_{2,t-1} - (\mu_3 c_{13}) Z_{3,t-1} + \in_{1t}$$

$$\Delta y_{2t} = m_2 - (\mu_2 c_{22}) Z_{2,t-1} - (\mu_3 c_{23}) Z_{3,t-1} + \in_{2t}$$

$$\Delta y_{3t} = m_3 - (\mu_3 c_{32}) Z_{2,t-1} - (\mu_3 c_{33}) Z_{3,t-1} + \in_{3t}$$

The factorization of π is written

$$\pi = \alpha \beta^1$$

Where α and β are (3x2) matrices of rank two i.e., the rank of π is two and there are two cointegrating vectors. By substitution $\alpha \beta^1$ in.

$$\Delta y_t = m - \pi y_{t-1} + \in_t$$

i.e. $\Delta y_t = m - \alpha \beta^1 y_{t-1} + \in_t$

$$= \Delta y_t = m - \alpha Z_{t-1} + \in_t \qquad \dots (2.7)$$

Where $Z_{t-1} = \beta^1 y_{t-1}$ contains two cointegrating variables. Suppose that the Eigen values are $\lambda_1 = \lambda_2 = 1$ and (λ_3) <1. Then it is possible to find a non-singular matrix p such that $P^{-1}AP = j$ where j is a Jordan matrix

$$J = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

By considering a three element vector $Z_t = P^{-1} y_t$ it follows that Z_1 is I(2), Z_2 is I(1) and Z_3 is I (0) generally all three variables are I(2), then

$$y_t = \begin{bmatrix} \vdots \\ p_1 \\ \vdots \end{bmatrix} Z_{1t} + \begin{bmatrix} \vdots \\ p_2 \\ \vdots \end{bmatrix} Z_{2t} + \begin{bmatrix} \vdots \\ p_3 \\ \vdots \end{bmatrix} Z_{3t}$$

Premultiplying by the second row of p^{-1} namely $P^{(2)}$ gives $P^{(2)} y_t = Z_{2t}$

Similarly $P^{(3)}$ gives $P^{(3)} y_t = Z_{3t}$. Which is I (0)

Thus there are two cointegrating vectors, but only one produces a stationary linear combination of y's. The eigen values are $\lambda_1 = 1$, $\lambda_2 = 1$ and $\lambda_3 = a$. Where the last Eigen value is assumed to have modulus less than one. The first two Y variables are random walks with drift, and thus I (1) and the last equation in the VAR connects all three variables. So, that $y^{(3)}$ is I(1).
Then π matrix is

$$\pi = I - A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -1 & 1-a \end{bmatrix}$$

The rank of π is one, and it may be factorized as

$$\pi = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & a-1 \end{bmatrix}$$

When the row vector is the cointegrating vector. This result may be seen from

$$Z_t = y_{1t} + y_{2t} + (a-1) y_{3t}$$

$$= y_{1t} + y_{2t} + (a-1)\left(y_{1,t-1} + y_{2,t-1} + a y_{3,t-1} + m_3 + \in_{3t} \right)$$

$$= \Delta y_{1t} + \Delta y_{2t} + a Z_{t-1} + (a-1) m_3 + (a-1) \in_{3t}$$

$$= \text{constant } + a Z_{t-1} + v_t$$

Where $v_t = \in_{1t} + \in_{2t} + (a-1) \in_{3t}$

Thus Z_t follows a stable AR (1) process and I (0).

2.3 VAR Higher Order Systems
Consider Second Order System

$$y_t = m + A_1 y_{t-1} + A_2 y_{t-2} + \in_t \qquad \ldots (2.8)$$

By subtracting y_{t-1} from both sides, we get

$$\Delta y_t = m + (A_1 - I)\, y_{t-1} + A_2 y_{t-2} + \in_t$$

The process of adding or subtracting (A_1 -I) y_{t-2} on the right side, we get

$$\Delta y_t = m + (A_1 - I)\, \Delta y_{t-1} - \pi\, y_{t-2} + \in_t \qquad \ldots (2.9)$$

Where $\pi = I - A_1 - A_2$

Similarly an alternative change in parameter is

$$\Delta y_t = m - A_2 \Delta y_{t-1} - \pi\, y_{t-1} + \in_t \qquad \ldots (2.10)$$

In second order system, there will be one lagged first difference term on the right hand side.
By continuing the procedure upto Var (p) system defined in equation (2.5) may be change in parameters. i.e,

$$\Delta y_t = m + B_1 \Delta y_{t-1} + B_{p-1} \Delta y_{t-p+1} - \pi\, y_{t-1} + \in_t \qquad \ldots (2.11)$$

Where the B's are the functions of the A's and $\pi = 1 - A_1 - \ldots\ldots - A_p$. The behaviour of the Y vector depends on the values of λ that solve

$$\left| \lambda^p I - \lambda^{p-1} A_1 - \ldots\ldots - \lambda A_{p-1} - A_p \right| = 0$$

For explosive roots, consider three possibilities

(i) Rank (π) = k If each root has modulus less than one, π will have full
Rank and be non-singular. All the 'y' variables in
Equation (2.5) will be I (0) and unrestricted OLS
Estimates of equation (2.5) and equation (2.11)

(ii) Will give same inferences about the parameters.
Rank (π) = r < k this situation will occur if there is a unit root with
Multiplicity (k-r) and the remaining 'r' roots are
Numerically less than one.

(iii) Rank (π) =0. This is a special case. It will only occur if A_1+ ……+
A_p= I, in this case π =0 and equation (2.11) shows the VAR should be specified in terms of first differences of the variables.

3. Maximum Likeli Hood Estimates of Parameters of Var by Using Ols Regression

Consider the p^{th} order Gaussian Vector Auto regression Time series Model as

$$Y_t = \alpha + A_1 Y_{t-1} + A_2 Y_{t-2} + \ldots\ldots + A_p Y_{t-p} + \in_t \qquad \ldots. (3.1)$$

Where $\in_t \sim i.i.d.N(o, \Phi)$

Here, Y_t is an (nx1) vector containing the values that 'n' variables assume at time t;
α Is an (n x 1) vector of constants;
A_i is an (n x n) matrix of auto regressive coefficients for i =1, 2….p;
$\in_t$ Is an (n x 1) vector of error terms,
And Φ is an (n x n) symmetric positive definite matrix.
Suppose that one may be observed each of these 'n' variables for (T + p) time periods. The first p observations may be denoted by (Y_1, Y_2...Y_r).
Let X_t be a vector containing a constant term and p lags of each of the elements of Y as

$$X_t = \begin{bmatrix} 1 \\ y_{t-1} \\ y_{t-2} \\ . \\ . \\ y_{t-p} \end{bmatrix}$$ Which is an [(np+1) x 1] vector

Also, let Γ' be [n x (np+1)] matrix which is given by

$$\Gamma' = \left[\alpha \quad A_1 \quad A_2 \ldots\ldots\ldots A_p \right]$$

Now, the conditional mean Y_t is given by

$$E\left[Y_t\right] = \Gamma' X_t \qquad \ldots (3.2)$$

The j[th] row of Γ' contains the parameters of j[th] equation in the VAR model.
The conditional density function of Y_t is given by

$$L(\theta) = f_{y_t\, y_{t-1}\, y_{t-2} \ldots\ldots y_{-p+1}}\left(y_t / y_{t-1}, y_{t-2}, \ldots\ldots y_{-p+1}; \Theta\right)$$

$$= (2\pi)^{-\frac{n}{2}} \left|\Phi^{-1}\right|^{\frac{1}{2}} \exp\left[-(1/2)\left(y_t - \Gamma' x_t\right)^1 \Phi^{-1}\left(y_t - \Gamma' x_t\right)\right] \qquad \ldots (3.3)$$

Where Θ in a vector that contains the elements of α, A_1, A_2…. A_p and Φ.
The sample Log likelihood function is given by

$$\sum_{t=1}^{T} \log L(\theta) = -\left(\frac{nT}{2}\right)\log(2\pi) + \left(\frac{T}{2}\right)\log\left|\Phi^{-1}\right| - \frac{1}{2}\sum_{t=1}^{T}\left[\left(y_t - \Gamma' x_t\right)^1 \Phi^{-1}\left(y_t - \Gamma' x_t\right)\right] \qquad \ldots (3.4)$$

The maximum likelihood estimate of Γ is given by

$$\hat{\Gamma}^1_{\left[n\times(np+1)\right]} = \left[\sum_{t=1}^{T} y_t x_t^1\right]\left[\sum_{t=1}^{T} x_t x_t^1\right]^{-1} \qquad \ldots (3.5)$$

Which is the sample analog of the population linear projection of Y_t on a constant and X_t.
The j[th] row of $\hat{\Gamma}^1$ is given by

$$\hat{\Gamma}^1_{j\left[1\times(np+1)\right]} = \left[\sum_{t=1}^{T} y_{jt} x_t^1\right]\left[\sum_{t=1}^{T} x_t x_t^1\right]^{-1} \qquad \ldots (3.6)$$

Here, $\hat{\Gamma}^1{}_j$ is the estimated coefficient vector from an OLS regression of Y_{jt} on X_t.
From the equation (3.4), write the last term as

$$\sum_{t=1}^{T} (y_t - \Gamma^1 x_t)^1 \Phi^{-1}(y_t - \Gamma^1 x_t) = \sum_{t=1}^{T}\left[\left(y_t - \hat{\Gamma}^1 x_t + \hat{\Gamma}^1 x_t - \Gamma^1 x_t\right)^1\right.$$
$$\left.\Phi^{-1}\left(y_t - \hat{\Gamma}^1 x_t + \hat{\Gamma}^1 x_t - \Gamma^1 x_t\right)\right]$$

$$= \sum_{t=1}^{T} \left\{ \left[e_t + \left(\overset{\wedge}{\Gamma} - \Gamma \right)^1 x_t \right]^1 \Phi^{-1} \left[e_t + \left(\overset{\wedge}{\Gamma} - \Gamma \right)^1 x_t \right] \right\} \quad \ldots (3.7)$$

Where $e_t = \left(y_t - \hat{\Gamma}^1 x_t \right)$ is the $\left(n \times 1 \right)$ sample OLS residual vector for t^{th} observation.

From equation (3.7), one may obtain,

$$\sum_{t=1}^{T} \left(y_t - \Gamma^1 x_t \right)^1 \Phi^{-1} \left(y_t - \Gamma^1 x_t \right) = \sum_{t=1}^{T} e_t^1 \Phi^{-1} e_t + 2 \sum_{t=1}^{T} e_t^1 \Phi^{-1} \left(\hat{\Gamma} - \Gamma \right)^1 x_t$$

$$+ \sum_{t=1}^{T} x_t^1 \left(\hat{\Gamma} - \Gamma \right) \Phi^{-1} \left(\hat{\Gamma} - \Gamma \right)^1 x_t \quad \ldots (3.8)$$

But, $\sum_{t=1}^{T} e_t^1 \Phi^{-1} \left(\hat{\Gamma} - \Gamma \right)^1 x_t = \text{trace} \sum_{t=1}^{T} e_t^1 \Phi^{-1} \left(\hat{\Gamma} - \Gamma \right)^1 x_t$

$$= \text{trace} \left[\Phi^{-1} \left(\hat{\Gamma} - \Gamma \right) \sum_{t=1}^{T} x_t e_t^1 \right] = 0 \qquad \left[\because \sum_{t=1}^{T} x_t e_t^1 = 0 \right]$$

$$\text{Thus,} \sum_{t=1}^{T} \left(y_t - \Gamma^1 x_t \right)^1 \Phi^{-1} \left(y_t - \Gamma^1 x_t \right) = \sum_{t=1}^{T} e_t^1 \Phi^{-1} e_t + \sum_{t=1}^{T} x_t^1 \left(\hat{\Gamma} - \Gamma \right) \Phi^{-1} \left(\hat{\Gamma} - \Gamma \right)^1 x_t \quad \ldots (3.9)$$

The expression (3.9) is to be minimized or the log likelihood function (3.4) is to be maximized for $\Gamma = \overset{\wedge}{\Gamma}$

It shows that the OLS regression of Y_{jt} on a constant term and p lags of all the variables X_{jt}'s in the system gives the maximum likelihood estimates of the coefficients for the j^{th} equation of a VAR model.

4. Estimating Φ by Using Studentized Residuals

Consider the log likelihood function for VAR model by using the maximum likelihood estimator $\hat{\Gamma}$ as

$$L^* \left(\Phi, \hat{\Gamma} \right) = \text{Log} L \left(\Phi, \hat{\Gamma} \right) = -\left[\frac{nT}{2} \right] \log \left(2\pi \right) + \left(\frac{T}{2} \right) \log \left| \Phi^{-1} \right| - \left(\frac{1}{2} \right) \sum_{t=1}^{T} e_t^{*1} \Phi^{-1} e_t^* \quad \ldots (4.1)$$

Where e_t^* is (nx1) sample Internally Studentized residual vector for t^{th} observation

Remark: Suppose that $e_t = \left[y_t - \hat{\Gamma} x_t \right]$ is the (nx1) sample OLS residual vector for t^{th} observation. The Internally Studentized residuals are given by

$$e_{tj}^* = \frac{e_{tj}}{\hat{\sigma}_{tj} \sqrt{1 - \nu_{tjj}}}, \; j = 1, 2, \ldots..n \quad \ldots. (4.2)$$

Where, $\hat{\sigma}_{tj} = \dfrac{\sum_{j=1}^{n} e_{tj}^2}{n - k}$

And $\left[\frac{e_{tj}^{*^2}}{n-k}\right]$ follows a Beta distribution with

Parameters 1/2 and $\left[\frac{n-k-1}{2}\right]$

Further, $E\left[e_{tj}^{*}\right] = 0, \forall\ j = 1, 2, \ldots, n$

$Var\left[e_{tj}^{*}\right] = 0, \forall\ j = 1, 2, \ldots, n$

$$Cov\left(e_{tj}^{*} e_{ts}^{*}\right) = \frac{-\nu_{tjs}}{\sqrt{\left(1-\nu_{tjj}\right)\left(1-\nu_{tss}\right)}}, \forall\ j \neq s = 1, 2, \ldots, n.$$

Here, ν_{tjs}'s are the elements of concerned Hat matrix and k is the number of parameters estimated per equation.
By applying the maximum likelihood estimation, one may obtain,

$$\frac{\partial L^{*}\left(\Phi, \hat{\Gamma}\right)}{\partial \Phi^{-1}} = \left(\frac{T}{2}\right)\Phi^{1} - \frac{1}{2}\sum_{t=1}^{T} e_{t}^{*} e_{t}^{*^{1}} \quad \ldots (4.3)$$

And hence, the maximum likelihood estimator of Φ is given by

$$\hat{\Phi} = \left[\frac{1}{T}\right]\sum_{t=1}^{T} e_{t}^{*} e_{t}^{*1} \quad \ldots (4.4)$$

The i^{th} diagonal element of $\hat{\Phi}$ is given by

$$\hat{\sigma}_{i}^{2} = \left[\frac{1}{T}\right]\sum_{t=1}^{T} e_{it}^{*^2}$$

It is the mean squared Internally Studentized residual from a regression of the i^{th} variable in the VAR on a constant term and p lags of all the variables. The $(i, j)^{th}$ element of $\hat{\Phi}$ is given by

$$\hat{\sigma}_{ij} = \left[\frac{1}{T}\right]\sum_{t=1}^{T} e_{it}^{*} e_{jt}^{*}$$

It is the average product of the Internally Studentized residuals for i^{th} and j^{th} variables

5. Testing Number of Lags of Variable for Var Model by Using the Likelihood Ratio Test.

Consider the log likelihood function for VAR model by substituting the estimators $\hat{\Phi}$ and $\hat{\Gamma}$ as

$$L^{*}\left(\hat{\Phi}, \hat{\Gamma}\right) = -\left(\frac{nT}{2}\right)Log\left(2\pi\right) + \left(\frac{T}{2}\right)\log\left|\hat{\Phi}^{-1}\right| - \left(\frac{1}{2}\right)\sum e_{t}^{*^{1}}\hat{\Phi}^{-1}e_{t}^{*} \quad \ldots. (5.1)$$

One may wish to test the null hypothesis that a set of variables was generated from a Gaussian VAR model with p_0 lags against the alternative hypothesis of $p_1 > p_0$ lags. Now, one may perform a set of n OLS regressions of each variable in the system on a constant term and on p_o and p_1 lags of all variables in the system under null and alternative hypotheses respectively. Let $\hat{\Phi}_{0}$ and $\hat{\Phi}_{1}$ be the estimators of variance-covariance matrices of the errors based on internally studentized residuals from these regressions under null and alternative hypotheses respectively.
To test the null hypothesis, the likelihood ratio test statistic is given by

$$Q = 2\left[L_1^* - L_0^*\right] = T\left\{\log\left|\hat{\Phi}_o\right| - \log\left|\hat{\Phi}_1\right|\right\} \quad \text{..... (5.2)}$$

Under the null hypothesis, Q has a χ^2 distribution with degrees of freedom equals to the number of restrictions imposed under the null hypothesis ie.

$$Q \overset{asy}{\sim} \chi^2_{n^2(p_1 - p_0)} \quad \text{.... (5.3)}$$

Remark: According to Sims (1980) [3], the test statistic Q may be sometimes defined as

$$Q = (T - k)\left\{\log\left|\hat{\Phi}_0\right| - \log\left|\hat{\Phi}_1\right|\right\} \quad \text{.... (5.4)}$$

Here, K=1+ np_1 is the number of parameters to be estimated per equation

6. Conclusions

Time series is a stretch of values on the same scale indexed by a time like parameter. The basic data and parameters are functions. Time series take on a dazzling variety of shapes and forms indeed there are as many time series as there are functions of real numbers. Concepts related to time series include: Longitudinal data, growth curves, repeated measures, economic models, multivariate analysis, signal processing and system analysis.

The parameters of VAR model have been estimated by using the method of maximum likelihood estimation based on ordinary least squares regression. The dispersion matrix of errors in VAR model has been estimated by using Internally studentized residuals. A test procedure has developed for testing number of lags of variables for VAR model by using Internally studentized residuals.

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