International Journal of Statistics and Applied Mathematics 2018; 3(1): 481-487

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(1): 481-487

www.mathsjournal.com
Received: 22-11-2017
Accepted: 24-12-2017
Impact Factor: RJIF 5.34

Mamta Kumari
Research Scholar, Maharshi Dayanand University, Rohtak, Haryana, India

Correspondence
Mamta Kumari
Research Scholar, Maharshi Dayanand University, Rohtak, Haryana, India

Anisotropic Bianchi Type-V Cosmological Models in General Relativity with Fluid Cosmology

Mamta Kumari

Abstract
Einstein's field equations with cosmological term Λ varying with time are considered in the context of general homogeneous, anisotropic universes in a way which conserves the energy–momentum tensor of matter content. It is shown that the field equations are solvable for any arbitrary cosmic scale functions. We obtain anisotropic Bianchi type V bulk viscous fluid cosmological models in general relativity with fluid cosmology. We also discuss the physical features of the cosmological models.

Keywords: Cosmological Models, General Relativity, Fluid Cosmology

Introduction
The adequacy is isotropic Friedman-Robertson- Walker (FRW) cosmological models for describing the present-state of the universe has been no basis for expecting that these are equally suitable for describing the early stages of evolution of the universe. At the early stages of evolution of universe when radiation in the form of photons as well as neutrion decoupled, the behavior of the matter has been observed like that of a viscous fluid. Since the viscosity counteracts the gravitational collapse, a different picture of the initial stage of the universe may appear due to dissipative processes caused by viscosity. Misner (1967, 1968) [9, 10] studied the effect of viscosity on the evolution of the universe and has suggested that the strong dissipation due to the neutron viscosity may considerably reduce the anisotropy of the black-body radiation. Murphy (1973) [12] investigated an exact cosmological model of zero-curvature FRW type in the presence of bulk viscous fluid alone which exhibits the interesting feature that the big-bang singularity appears in the infinite past. Szydlowski and Heller (1983) presented models of the universe filled with interacting matter and radiation including bulk viscosity dissipation. Mohanty and Pradhan (1983) [11] obtained a class of non –static exact solutions in the closed elliptic Robertson-Walker space-time filled with bluk viscous fluid in the presence of attractive scalar fields. The viscosity mechanism in cosmology can explain the anomalously high entropy per baryon in the present universe. Exact solutions of the isotropic and spatially homogeneous cosmology for open, closed, and flat models have been investigated by Santos *et al*. (1985) [165].

Spatially homogeneous and anisotropic Bianchi type I-IX cosmological models play a significant role in the description of the large scale behavior of universe and such models have been widely studied within the framework of general relativity in search of a realistic picture of the universe in its early stages. The nature of cosmological solutions for spatially homogenous and anisotropic Bianchi type- I model was investigated by Belinsky and Khalatniko (1976) [3] by taking into account the dissipative process due to viscosity. They have shown that viscosity can not remove the cosmological singularity but results in a qualitatively new behavior of the solutions near singularity. Saha and Rikhvitsky (2006) [15] investigated the nature of the cosmological solutions for a spatially homogenous Bianchi type-I models in the presence of a cosmological constant by taking into account dissipative process due to viscosity. Huang (1988) [7] presented exact solution for a Bianchi type I cosmological model with bulk viscosity without shear viscosity. Banerjee and Sanyal (1988) [1] obtained an irrotational anisotropic Bianchi type V model under the influence of both shear and bulk viscosity together with heat flow. Coley (1990) [4], Coley and Hoogan (1994) [5] studied diagonal Bianchi type- V imperfect fluid with both viscosity and heat conduction with and without cosmological term. Singh and Chaubey (2007) investigated the evolution of Bianchi type V model with viscous fluid and cosmological constant. Verma and Shri Ram (2010) [19] presented some hypersurface homogenous bulk viscous fluid universe model with time-dependent cosmological term. Further, Shri Ram and Verma (2010) [19] obtained bulk viscous fluid hyper surface homogenous cosmological model with time-varying cosmological and gravitational constant. Pradhan and Kumar (2009) [13] studied LRS Bianchi type-II viscous fluid models with time-varying cosmological constant. Verma and Shri Ram (2010) [19] investigated anisotropic Bianchi type III cosmological models with time dependent cosmological and gravitational constants. Shri Ram *et al*. (2009) [20, 21] investigated a Bianchi type V viscous fluid cosmological model with heat in Saez-Ballester theory of gravitation. In such a study, the space-time continuum requires Riemannian geometry for their description.

The propose of this paper is to study again Bianchi type-V cosmological models in two fluid cosmology. The paper is organized as follows. In Sect. 2, we present the metric and Einstein's field equations in two-fluid cosmology. In Sect. 3, we deal with the exact solutions of field equations in two types of cosmologies, one with power-law expansion and other one with exponential expansion. We also discuss the physical and kinematical behaviors of the anisotropic cosmological models. Some concluding remarks are given in Sect. 4.

The Metric and Field Equations

We consider the metric of anisotropic Bianchi type-V in the form

$$ds^2 = dt^2 - A^2 dx^2 - e^{mx}(B^2 dy^2 + C^2 dz^2) \tag{1}$$

where A(t), B(t), C(t) are cosmic scale functions and m is a constant.

The Einstein's field equations in two-fluid cosmology in proper units $8\pi G = 1, c = 1$ are [19].

$$R_{ij} - \frac{1}{2} R g_{ij} = -T_{ij} \tag{2}$$

where T_{ij} is the energy-momentum tensor for a two-fluid source is given by

$$T_{ij} = T_{ij}^{(m)} + T_{ij}^{(r)} \tag{3}$$

In (3), $T_{ij}^{(m)}$ is the energy-momentum tensor for the matter field given by

$$T_{ij}^{(m)} = (\rho_m + p_m) u_i^{(m)} u_j^{(m)} - p_m g_{ij} \tag{4}$$

and $T_{ij}^{(r)}$ is the energy-momentum tensor for radiation field given by

$$T_{ij}^{(r)} = \frac{4}{3} \rho_r u_i^{(r)} u_j^{(r)} - \frac{1}{3} \rho_r g_{ij} \tag{5}$$

Here ρ_m, p_m and ρ_r are matter energy density, matter pressure and radiation density respectively. In comoving coordinate system the four velocity vectors of matter and radiation are given as

$$u_i^{(m)} = (0,0,0,1), \quad u_i^{(r)} = (0,0,0,1), \tag{6}$$

With the use of (3), (4), (5) and (6), Einstein's field equations (2) for the metric (1) yield the following system of equations

$$\frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} + \frac{\dot{B}\dot{C}}{BC} - \frac{m^2}{A^2} = -p_m - \frac{\rho_r}{3}, \tag{7}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{C}}{C} + \frac{\dot{A}\dot{C}}{AC} - \frac{m^2}{A^2} = -p_m - \frac{\rho_r}{3}, \tag{8}$$

$$\frac{\ddot{A}}{A} + \frac{\ddot{B}}{B} + \frac{\dot{A}\dot{B}}{AB} - \frac{m^2}{A^2} = -p_m - \frac{\rho_r}{3}, \tag{9}$$

$$\frac{\dot{A}\dot{B}}{AB} + \frac{\dot{B}\dot{C}}{BC} + \frac{\dot{C}\dot{A}}{CA} - \frac{3m^2}{A^2} = p_m + \rho_r, \tag{10}$$

$$\frac{2\dot{A}}{A} - \frac{\dot{B}}{B} - \frac{\dot{C}}{C} = 0 \tag{11}$$

where an overdot denotes ordinary derivative with respect to cosmic time t
The average scale factor a(t), spatial volume V for the metric (1) are given by

$$V = a^3 = ABC \tag{12}$$

The kinematical quantities of observational interest in cosmology are the expansion scalar (θ), shear scalar (σ), Hubble parameter (H) and anisotropic parameter (A_m) given by

$$\theta = 3H = \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}, \tag{13}$$

$$\sigma^2 = \frac{1}{2}\left[\left(\frac{\dot{A}}{A}\right)^2 + \left(\frac{\dot{B}}{B}\right)^2 + \left(\frac{\dot{C}}{C}\right)^2\right] - \frac{1}{6}\theta^2, \tag{14}$$

$$A_m = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{15}$$

An observational quantity of cosmological interest is the deceleration parameter (q) defined as

$$q = -\frac{a\ddot{a}}{\dot{a}^2} \tag{16}$$

The sign of q denotes whether the model inflates or not. The positive sign of q corresponds to a standard cosmological model where as the negative sign indicates inflation.

Cosmological Models

In this section, we obtain the exact solutions of the equations (7)-(10), which are five equations in six unknowns. Singh *et al.* [20] obtained solutions of these equations by applying the special law of variation for Hubble parameter that yields a constant value of the deceleration parameter. Here we obtain solutions as (7)-(10) by a different approach proposed and studied by Tiwari *et al.* [21]

Subtracting (7) from (8), (8) from (9) and (9) from (7), and integrating the resulting equations, we obtain

$$\frac{\dot{A}}{A} - \frac{\dot{B}}{B} = \frac{k_1}{a^3}, \tag{17}$$

$$\frac{\dot{B}}{B} - \frac{\dot{C}}{C} = \frac{k_2}{a^3}, \tag{18}$$

$$\frac{\dot{A}}{A} - \frac{\dot{C}}{C} = \frac{k_3}{a^3}. \tag{19}$$

where k_1, k_2 and k_3 are arbitrary constant. Substituting (17), (18) and (19) in (14), we get

$$\sigma = \frac{k}{a^3} \tag{20}$$

where, $k^2 = k_1^2 + k_2^2 + k_3^2$. (21)

Now integrating (11), we get

$$A^2 = BC \tag{22}$$

taking the constant of integration unity. We set

$$B = AD, \; C = AD^{-1} \tag{23}$$

where D(t) is a function of t. Using (23) into (18) and simplifying, we obtain

$$\frac{\dot{D}}{D} = \frac{K}{a^3} \tag{24}$$

where $K\left(=\frac{1}{2}k_2\right)$ is a constant. From (24), we can determine the function D it the average scale factor a(t) is known as a explicit function of t.

Tiwari *et al.* [21] assumed that the anisotropy (σ/θ) in the model is inversely proportional to a^n, i.e.

$$\frac{\sigma}{\theta}=\frac{b}{a^n},\quad n.>0. \tag{25}$$

Substituting (13) and (20) in (25) and solving the resulting differential equation, we obtain

$$a(t)=(ct+d)^{\frac{1}{3-n}},\quad n\neq 3, \tag{26}$$

and $$a(t)=c_3\exp\left(\frac{kt}{3b}\right),\quad n=3, \tag{27}$$

where $c=\frac{K(3-n)}{3b}$, d and c_1 are constants. Without loss of geniality, we can take $c_1 = 1$. Thus, we obtain the power-law form and exponential form of the average scale factor a(t) in (26) and (27) respectively.

7.3.1 Model in power-Law Cosmology $(n\neq 3)$

From (12), (22) and (26), we obtain the solution of the scale factor

$$\mathbf{A}(t)=(c_1t+c_2)^{\frac{1}{3-n}} \tag{28}$$

Inserting Equation (26) in (24) and integrating, we get

$$D(t)=\exp\left\{\frac{K(n-3)}{nc}(ct+d)^{\frac{n}{n-3}}\right\} \tag{29}$$

From (23) and (29), we obtain the solutions for the scale factors B(t) and C(t) as follows

$$B(t)=(ct+d)^{\frac{1}{3-n}}\exp\left\{\frac{K(n-3)}{nc}(ct+d)^{\frac{n}{n-3}}\right\}, \tag{30}$$

$$C(t)=(ct+d)^{\frac{1}{3-n}}\exp\left\{-\frac{K(n-3)}{nc}(ct+d)^{\frac{n}{n-3}}\right\}. \tag{31}$$

Hence, the metric (1) with scale factors A(t), B(t) and C(t), as given by (28), (30) and (31), represents an expanding cosmological model with power-law expansion.

We now assume that the matter energy density and pressure satisfy the barotropic equation of state

$$p_m=(\gamma-1)\rho_m \tag{32}$$

Then, from (8)-(10), we obtain

$$\rho_m=\frac{3}{(4-3\gamma)}\left[\frac{4c^2}{(3-n)^2(ct+d)^2}-\frac{c}{(3-n)(ct+d)^2}+\frac{Kc}{(ct+d)^{\frac{3}{3-n}}}+\frac{2K^2}{3(ct+d)^{\frac{2n}{3-n}}}-\frac{2m^2}{(ct+d)^{\frac{2}{3-n}}}\right], \tag{33}$$

$$p_m=\frac{3(\gamma-1)}{(4-3\gamma)}\left[\frac{4c^2}{(3-n)^2(ct+d)^2}-\frac{c}{(3-n)(ct+d)^2}+\frac{Kc}{(ct+d)^{\frac{3}{3-n}}}+\frac{2}{3}\frac{K^2}{(ct+d)^{\frac{2n}{n-3}}}-\frac{2m^2}{(ct+d)^{\frac{2}{3-n}}}\right], \tag{34}$$

$$\rho_\gamma=\frac{-9\gamma c^2}{(4-3\gamma)(3-n)^2(ct+d)^2}+\frac{3K^2(\gamma-2)}{(4-3\gamma)(ct+d)^{\frac{2n}{3-n}}}+\frac{3m^2(3\gamma-2)}{(4-3\gamma)(ct+d)^{\frac{2}{3-n}}}+\frac{3c\gamma}{(4-3\gamma)(3-n)(ct+d)^2}-\frac{3Kc}{(4-3\gamma)(ct+d)^{\frac{3}{3-n}}} \tag{35}$$

For this model, expansion scalar shear scalar, Hubble parameter and anisotropic parameter are obtained as

$$\theta = 3H = \frac{3c}{(3-n)(ct+d)}, \quad (36)$$

$$\sigma = \frac{k}{(ct+d)^{\frac{3}{3-n}}}, \quad (37)$$

$$A_m = \frac{2(3-n)^2 K^2}{3(ct+d)^{\frac{2n}{3-n}}}. \quad (38)$$

The deceleration parameter has the value given as,

$$q = 2 - n \quad (39)$$

From, (39) it in clear that q = 0 when n = 2, q < o when n > 2 and q > 0 when m < 2. Thus, this model corresponds to an accelerated expanding model for n > 2 and a decelerating for n < 2.

We observe that the spatial volume is zero at $t = -d/c = t_1$ and becomes infinite when m < 3 as time t tends to infinity. The physical and kinematical parameters $\rho_m, p_m, \rho_r, \theta, \sigma$ and A_m are all infinite at $t = t_1$. Therefore the model starts evolving with a big-bang singularity at $t = t_1$. The physical and kinematical parameters are monotonically decreasing functions of time ultimately tend to zero for large time provided n < 3. The anisotropy parameter is initially very large and tends to zero for large time. Therefore the model isotropizes for large time. The model gives essentially an empty universe as t →. For an accelerating model, n is restricted to 2 < n <3 an for a decelerated model 0 < n < 2. Vishwkarma [22] shown that the decelerating models are also consistent with the recent cosmic background observations model by WMAP as well as the high-red shift supernova Ia data including SN 1997 ff at Z = 1.775.

Model in Inflationary Cosmology (n = 3)

We now obtain inflationary model by using the average scale factor a(t) obtained in (27). Inserting (27) in (24) and solving, we obtain

$$D(t) = \exp\left(-\frac{bK}{k} e^{-\frac{kt}{b}}\right) \quad (40)$$

taking the integration constant unity. Hence, the solutions for scale factors A(t), B(t) and C(t) are given by

$$A(t) = e^{\frac{kt}{3b}}, \quad (41)$$

$$B(t) = e^{\frac{kt}{3b}} \exp\left(\frac{-bK}{k} e^{-\frac{kt}{b}}\right), \quad (42)$$

$$C(t) = e^{\frac{kt}{3b}} \exp\left\{\frac{bK}{k} e^{-\frac{kt}{b}}\right\} \quad (43)$$

Hence, the metric of our solution is

$$ds^2 = dt^2 - e^{\frac{2kt}{3b}} dx^2 - e^{2mx}\left[\left\{e^{\frac{2kt}{3b}} e^{-\frac{2bK}{k}} e^{-\frac{kt}{b}}\right\} dy^2 + \left\{e^{\frac{2kt}{3b}} e^{+\frac{2Kb}{k}} e^{-\frac{kt}{b}}\right\} dz^2\right] \quad (44)$$

For the model (44) the expansion scalar, shear scalar, Hubble parameter and anisotropy parameter are obtained as

$$\theta = 3H = \frac{k}{b}, \quad (45)$$

$$\sigma = k e^{-\frac{kt}{b}}, \quad (46)$$

$$A_m = \frac{6K^2 b^2}{k^2} e^{-\frac{2kt}{b}}. \quad (47)$$

The deceleration parameter q has the value

$$q = -1 \tag{48}$$

which indicates that the model (44) is inflationary in nature

If we assume that the matter field satisfies the baroropic equation of state (32), the matter energy density, matter pressure and radiation energy density are calculated as

$$\rho_m = \frac{3}{(4-3r)}\left[\frac{4k^2}{9b^2} + \frac{2}{3}K^2 e^{-\frac{2kt}{b}} - 2m^2 e^{-\frac{2kt}{3b}}\right], \tag{49}$$

$$p_m = 3\frac{(r-1)}{(4-3r)}\left[\frac{4k^2}{9b^2} + \frac{2}{3}K^2 e^{-\frac{2kt}{b}} - 2m^2 e^{-\frac{2kt}{3b}}\right], \tag{50}$$

$$\rho_r = \frac{1}{4-3r}\left[-\frac{rk^2}{b^2} + 3(r-2)K^2 e^{-\frac{2kt}{b}} + 3(3r-2)m^2 e^{-\frac{2kt}{3b}}\right]. \tag{51}$$

For the model (44), we observe that the model has no finite singularty. Since ρ_m, p_m and ρ_r are all tend to zero as t $\rightarrow -\infty$, the model in infinitely old. The expansion scalar θ is constant throughout the evolution of the universe. The matter energy density matter pressure and radiation density are monotonically decreasing functions of time and attain constant values as t$\rightarrow \infty$. The shear scalar and the anisotropy parameter are decreasing function of time, which tend to zero as t$\rightarrow$. This means that this anisotropic model isotropizes at late-time. Since q = -1 and H is constant, the universe enters into the de-Sitter phase at late time.

Conclusions

We have presented exact solution of Einstein's field equations within the framework with time-dependent displacement field vector for totally anisotropy Bianchi type-V space-time filled with a bulk viscous fluid by assuming that the anisotropy (σ / θ) is inversely proportional to the n^{th} power of the average scalar factor of the model. We have obtained models of the universe in two types of cosmologies, one with power-law expansion with $n \neq 3$ and other one exponential-law expansion for n = 3. The universe with power-law expansion has a finite big bang singularity and approaches to an empty universe for large time. The universe with exponential expansion has a singularity in the infinite past. In this model the anisotropy parameter is a decreasing function of time which ultimately tends to zero t$\rightarrow \infty$. Therefore the universe isotropizes for large time which is in consistent with the observations on present-day universe. For sufficiently large time the universe into the de-Sitter phase.

References

1. Banerjee A, Sanyal A. Gen. Relativ. Gravit. 1988; 20:103.
2. Beesham A. Aust. J Phys. 1988; 41:833.
3. Belinsky VA, Khalatnikov IM. Sov. Phys. JETP. 1976; 42:205.
4. Coley AA. Gen. Relativ. Gravit. 1990; 22:3.
5. Coleg AA, Hoogan RJ. J Math Phys. 1994; 35:4117.
6. Halford WD. Aust. J Phys. 1970; 23:863.
7. Huang WH. Phys. Lett. A. 1988; 129:429.
8. Lyra G. Math. Z. 1951; 54:52.
9. Misner CW. Astro. Phys. J. 1968; 151:431.
10. Misner CW. Nature. 1967; 214:40.
11. Mohanty G, Pradhan BD. Int. J Theor Phys. 1983; 14:233.
12. Murphy L. Phys. Rev. 1973; 8:4231.
13. Pradhan A, Kumhar SS. Int. J Theor. Phys. 2009; 48:1466.
14. Pradhan A, Vishwakarma AK. J Geom. Phys. 2004; 49:332.
15. Saha B. Rikhvitsky V. Physica D. 2006; 219:168.
16. Santos NO, Dias RS, Banerjee A. J Math. Phys. 1985; 26:876.
17. Sen DK. Phys Z. 1957; 149:311.
18. Shri Ram, Chandal S, Verma MK. Chinese J Phys. 2016; 54:953.
19. Shri Ram, Verma MK. Astrophys. Space Sci. 2010; 330:151.
20. Shri Ram, Verma MK, Zeyauddin M, Fizika B. 2009; 18:207.
21. Shri Ram, Verma MK, Zeyauddin M. Modern Physic Lettrs A. 2009; 24:1847.
22. Shri Ram, Zeyauddin M, Singh CP. Int. J Mod. Phys. A. 2008; 23:1.