International Journal of Statistics and Applied Mathematics 2018; 3(1): 488-490

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(1): 488-490

www.mathsjournal.com
Received: 13-11-2017
Accepted: 15-12-2017

Himanshu Sikka
Department of Mathematics
Maharaja Agrasen Institute of Technology, Rohini, Delhi, India

Correspondence
Himanshu Sikka
Department of Mathematics
Maharaja Agrasen Institute of Technology, Rohini, Delhi, India

Differential equations and its areas of applications

Himanshu Sikka

Abstract
A mathematical equation that relates function and its derivatives is known as a differential equation. In this paper, different types of differential equations are discussed thoroughly. The method of finding the solution of a differential equation is also described. The relation between the differential equation and the difference equation is also mentioned here. There is a focus on various areas of applications of differential equations.

Keywords: Differential equations, applications

Introduction

A mathematical equation that relates function and its derivatives is known as a differential equation. Differential equation is a sub-field of calculus. The functions represent physical quantities in general and their derivatives represent the rates of change, and the differential equation describes a relationship between the function and its derivatives. It is used to determine the function over its entire domain. These relations are very common, hence differential equations play a major role in many areas including engineering, physics, economics, chemistry and biology.

In pure mathematics, we study differential equations from a number of different perspectives, concerned with their solutions. The solution is the set of functions that satisfy the differential equation. Differential equations can be solved by explicit formulas. We can also determine some properties of solutions of a differential equation without finding their exact form. The solution of the differential equation can be numerically approximated with the help of computers. A number of numerical methods are developed to find out the solution of Differential equation with higher degree of accuracy. Differential equations can be expressed as:

$$\frac{dy}{dx} = f(x) \text{ or } \frac{dy}{dx} = f(x,y) \text{ or } x1\frac{\partial y}{\partial x1} + x2\frac{\partial y}{\partial x2} = y$$

The ordinary differential equation is of the form: $y' + P(x)\, y = Q(x)\, y^n$.

Types: Differential equations are of different types. The classes of differential equations help us to reach to a solution. Some commonly used differential equations are: Ordinary/Partial, Linear/Non-linear, and Homogeneous/Inhomogeneous. There are many other properties and subclasses of these differential equations which may be very useful in particular contexts.

Ordinary Differential Equation: It is an equation containing an unknown function of one real or complex variable x, its derivatives, and some given functions of x. The unknown function is generally represented by a variable 'y', which depends on x. Thus x is known as the independent variable and y is known as the dependent variable of the equation. We use the term "ordinary" in contrast with the term "partial" differential equation. In partial differential equation there is more than one independent variable.

Linear differential equations are the differential equations which are linear in nature in the unknown function and its derivatives. We can express their solutions in terms of integrals in most of the cases.

Generally, the solutions of a differential equation cannot be expressed by a closed-form expressions, although we can use numerical methods for solving differential equations on a computer and get precise solution.

Partial Differential Equation: It is the differential equation in which unknown multivariable (more than one independent variable) functions and their partial derivatives are involved. PDEs are used to formulate problems having more than one independent variable functions. These are either solved in closed form, or with the help of computers to create a relevant computer model. PDEs model multidimensional systems. PDEs find their generalisation in stochastic form.

Non-linear differential Equation: These equations are formed by the products of the unknown function and its derivatives. Their degree is greater than 1. There are very few methods to solve such non-linear differential equations exactly. These depend on the equation having particular symmetries. These differential equations have complicated behaviour over extended time intervals, which is a characteristic of chaos. The fundamental problems of existence, uniqueness, and extendibility of solutions for nonlinear differential equations, and initial and boundary value problems for nonlinear PDEs are very difficult problems and their solution in special cases is considered to be a significant advance in the mathematical theory. These have unique solution.

Linear differential equations seem to be the approximations to nonlinear equations. These approximations are valid only under restricted conditions. For example, the harmonic oscillator equation is an approximation to the nonlinear pendulum equation that is valid only for small amplitude oscillations.

Equation order: Differential equations are expressed in terms of their order. The order is determined by the term with the highest derivatives in the differential equation. An equation containing only first derivatives is known as a first-order differential equation; an equation containing the second derivative is known as a second-order differential equation, and so on. Most of the differential equations that describe natural phenomena have only first and second order derivatives in them, but in exceptional cases higher order derivatives are used to express the function. For example, the thin film equation is a fourth order partial differential equation.

Examples

- Inhomogeneous first-order linear constant coefficient ordinary differential equation.

$$\frac{du}{dx} = cu + x^2$$

- Homogeneous second-order linear ordinary differential equation.

$$\frac{d^2u}{dx^2} - x\frac{du}{dx} + u = 0$$

- Homogeneous second-order linear constant coefficient ordinary differential equation.

$$\frac{d^2u}{dx^2} + w^2u = 0$$

- Inhomogeneous first-order nonlinear ordinary differential equation

$$\frac{du}{dx} = u^2 + 4$$

- Second-order nonlinear (due to sine function) ordinary differential equation describing the motion of a pendulum of length L

$$L\frac{d^2u}{dx^2} + g\sin u = 0$$

- Homogeneous first-order linear partial differential equation

$$\frac{\partial u}{\partial t} + t\,\frac{\partial u}{\partial x} = 0$$

- Homogeneous second-order linear constant coefficient partial differential equation of elliptic type, the Laplace equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

- Homogeneous third-order non-linear partial differential equation

$$\frac{\partial u}{\partial t} = 6u\frac{\partial u}{\partial x} - \frac{\partial^3 u}{\partial x^3}$$

Solution of differential equation: A solution of a differential equation is the relationship between its independent and dependent variables. It is free from any order of derivatives. It also satisfies the differential equation. A general solution of any differential equation of nth order is the one which contains ‘n’ number of arbitrary constants. We get particular solution of the differential equation by assigning particular values to the arbitrary constants in the general solution. We can calculate the values of arbitrary constants by using initial value problems and boundary conditions of the problem.

Singular solution is also known as the particular solution, but, singular solution is not obtained from the general solution by finding the values of arbitrary constants.

Order and degree of a differential equation: The order of a differential equation is the highest order derivative present in the given differential equation while the degree is the exponential power of the highest order derivative. Any order derivative present in the different equation must not contain fractional or negative exponents.

Connection to Difference Equation: The theory of differential equations and the theory of difference equations are closely inter-related. In difference equation, there are discrete values of the coordinates, and the relationship involves values of the unknown functions and values at nearby coordinates. Many numerical methods to find out the solutions and study the properties of differential equations involve the approximation of the solution of a differential equation by the solution of a corresponding difference equation.

Applications of Differential Equation: The differential equation is a wide concept used in the fields of pure and applied mathematics, physics, and engineering. Pure mathematics is used to compute the existence and uniqueness of solutions, while in applied mathematics differential equations are used for approximating solutions. Differential equations play an important role in almost every physical, technical, or biological process, from celestial motion, to bridge design, to interactions between neurons. Differential equations are also used to solve real-life problems. These problems can either be solved directly or solutions can be approximated using numerical methods.

Differential equation is a wide concept. It is used in Physics, Chemistry, Biology, Economics etc. along with Mathematics. For example: propagation of light and sound in the atmosphere and of waves on the surface of a pond are expressed by the second-order partial differential equation. The wave equation allows us to think that light and sound waves are much like familiar like water waves.

Physics: Different applications of differential equation in Physics are: Euler–Lagrange equation and Hamilton's equations in classical mechanics, Radioactive decay in nuclear physics, Newton's law of cooling and The Heat Equation in thermodynamics, The wave equation, Laplace's equation, Poisson's equation, The geodesic equation, The Navier–Stokes equations and The Convection–diffusion equation in fluid dynamics, The Diffusion equation in stochastic processes, The Cauchy–Riemann equations in complex analysis etc.

Classical mechanics: When we know about the force acting on a particle, then Newton's second law of motion is used to describe the motion of a particle. The ordinary differential equations used to express this force and motion is known as the equation of motion.

Electrodynamics

In classical electrodynamics, classical optics, and electric circuits, Maxwell's equations are a set of partial differential equations that are used along with the Lorentz force law. Maxwell's equations help us to understand the concept of how electric and magnetic fields are generated and altered by each other by using electric charges and currents.

General relativity: The Einstein field equations are a set of ten partial differential equations in Albert Einstein's general theory of relativity which are used to explain the concept of the fundamental interaction of gravitation as a result of spacetime being curved by matter and energy. The EFE equate local spacetime curvature with the local energy is expressed using the Einstein tensor and momentum within that spacetime is expressed using the stress–energy tensor.

Quantum mechanics: In quantum mechanics, the Schrödinger's equation is used as a substitute of Newton’s law. It is a linear partial differential equation, not an algebraic expression. It describes the time-evolution of the system's wave function.

Chemistry: in chemistry, differential equation is used to compute the rate law or rate equation for a chemical reaction that links the rate of reaction with the concentrations or pressures of reactants and constant parameters (known as rate coefficients and partial reaction orders). It is also helpful in determining the rate equation for a particular system which combines the reaction rate with a mass balance for the system. These are also used in the study of thermodynamics and quantum mechanics.

Biology: Different applications of differential equation in Biology are: Verhulst equation for biological population growth, von Bertalanffy model to compute biological individual growth, Replicator dynamics in theoretical biology, Hodgkin–Huxley model for neural action potentials.

Economics: Applications of differential equation in Economics are: the Solow–Swan model equation, Black–Scholes equation, Malthusian growth model, The Vidale–Wolfe advertising model etc.

Conclusion

A differential equation is a mathematical equation that relates function and its derivatives. Different types of differential equations are: linear and non-linear, ordinary and partial, homogenous and inhomogeneous are discussed thoroughly. In this paper, the ways to find out the solution of a differential equation are described. The relation between the differential equation and the difference equation is also explained. Different areas of applications of differential equations are mentioned in the field of Mathematics, physics, chemistry etc.

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