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Elementary functions

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Abstract
Elementary function is an introduction to the college mathematics. It aims to provide a working knowledge of basic functions (exponential, logarithmic and trignometric) fast and efficient implementation of elementary function such as sin0, cos0, log0 are of ample importance in a large class of applications. The state of the art method for function evaluation involve either expensive calculations such as multiplication, large number of iteration or large looks up tables. Our proposed method evaluates trignometric, hyperbolic, exponential, logarithmic.

Keywords: Power series, logarithmic function, exponential function, trigonometric function

Introduction

Elementary functions are e^x, lon^x, a^x, sinx, cosx. We all familiar with these functions, but this acquaintances is based on a treatment which was essentially based on intuitive less rigorous geometrical considerations.

We shall base the study of these functions on the set of real numbers as a complete ordered field, the nation of limit and convergence of series.

As the definitions of function will be based on power series, we start with a brief study of power series.

Power Series

The series of the form $a_o + a_i x + a_2 x^2$ +....... + $a_n x^h$ + …..... = $\sum_{n=0}^{8} \quad a_n x^h$ Are called power series in x and the numbers are their co-efficient.

Some simple examples of the power series are.

1. The geometric series is convergent for $\sum_x h$ Divergent for /x/< /, divergent for /x/≥ I
2. $\sum \frac{x^h}{n!}$ Is convergent for every real x; likewise the series $\sum (-1)^h \frac{x^{2h}}{2h!}$
3. $\sum h^h \ x^h$ Converges for x = o, but diverges for x ≠ o For x = o, obviously every poser series $\sum a_n x^h$ Is convergent, whatever the value of coefficient a_n for a power series $\sum a_n x^h$, which does not merely converge everywhere or nowhere, a definite positive number R exists such that the series converges for every /x/x LR, but diverges for every/x/7R. The number R is called Radius of Convergence and the interval]-R, R [the interval of convergence.

For the power series $\sum a_n x^h$
Lim sup series ………………..
By couch's root test, it follows if
Lim sup $\sqrt[n]{19nl}$ = and R =

1. R = 0, ═ the series is nowhere convergent.
2. R = ∞, the series is everywhere convergent.
3. O<R<∞, the series converges absolutely.

For /x/< R, and diverges for /x/< R
i.e. R is radius of convergence.

Logarithmic Functions

It is convenient at this stage to introduce two of the most important functions is mathematics. It a huge range of applications, form the discharge of a capacitor to the population growth of acapacitor to the population growth of bacteria, the exponential function plays a crucial role and "log – log" graphs are a crucial part of methodology over a wide area of experimental science.

But first let us remind of what we mean by a logarithm. If a^x= b

⇒ x = log_ab, the logarithm of a to a base a. In words, log_ab is the power to which a must be raised to obtain b. there are difficulties about this definition.

For example we know that a^x means only if x is rational and a related difficulty it is not clear that log_ab is defined for every b.

However this difficulty will vanish very soon. We begin with the simple observation that the formula.

$\int x^h$ dx = $\frac{x^{h+1}}{n+1}$ is not valid when n= -1 on the other hand the function x → $\frac{1}{x}$ is .

Continuous in any interval [a, b] not containing 'o'

Let us define a new function L by

L (x) = $\int_i^x \frac{dt}{t}$ i (x 70)

Certain properties of L are immediate. By the fundamental theorem, L is differential and

L^i (x) = $\frac{i}{x}$

Thus L is increasing function for all x in ($o_i\infty$).

Notice also that L(i) = 0.

The crucial property of the function L is given in the following theorem.

Theorem: - For all x_i y in ($o_i\infty$).

L (xy) = L(x) + L(y)

L ($\frac{i}{x}$) = L (x)

Exponential Function

The power series

$\frac{1+x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots + \frac{x^n}{n!} + \ldots\ldots\ldots\ldots\ldots\ldots$ 1

Is everywhere convergent for real x.

Definition

The function represented by the power series (1) is called Exponential function, denoted provisionally by E(x), thus.

E(x) = $\frac{1+x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots\ldots\ldots + \frac{x^n}{n!} + \ldots\ldots\ldots\ldots\ldots\ldots$ 2

E(1) =$\frac{1+1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \ldots\ldots\ldots\ldots + \frac{1}{n!} + \ldots\ldots\ldots\ldots\ldots\ldots$ 3

The series on the right hand side of (3) converges to a number which lies between 2 and 3.

This number is denoted by 'e', the exponential base and is the same number as represented by.

Lim (1 + $\frac{1}{n}$)^n

Since L is differentiable and increasing throughout its domain I there is an increasing differentiable inverse function L^1 : R → ($o_1\infty$), which for the moment we shall denote by E. Thus

E (L(x)) = x (x € ($o_1\infty$))

L (E (x)) = x (x € R)

Hence E^i (x) = (L^{-1}) (x) = $\frac{1}{L^i(L^i(x))}$ = $L^{-1}(x)$ = E (x)

The graph of E is given by

Theorem –

For all X_I Y in R,

E(x+y) = E (x) E(y)

E(-x) = $\frac{1}{E(x)}$

Relationship between exponential function and logarithm function

We can see the relationship between exponential function f(x) = e^x and the logarithm function f(x) - I_{n^x} by looking their graphs.

We can see that straight away that the logarithm function is a reflection of the exponential function in the line represented by f (x) = x. In other words, the axes have been swapped; x become f(x) and f(x) becomes x.

Key Point: The exponential function f(x) = e^x is the inverse of pgarithm f (x) = lgx

Remark

Before the advent of electronic, the students carried four – figure "log table" wherever they "log tables" wherever they went. To calculate (say) 325.7 x 48.43 one looked up the two logarithms [approximately 2.5128 and 1.6851 respectively), added the two logarithms, obtaining 4.1979 and then found 10^{44979} by looking up a table of "antilogarithms". The answer is not quite accurate but for most practical purposes an error of 0.02 %. is not significant.

Properties of exponential functions in terms of logarithms -

The logarithm function plucks form an expression. For this reason, the properties of exponent translate into proportions of logarithm.

4. For example, we know that when we multiply two terms with a common base, we add exponents.

(b^x) (b^y) = b^{x+y}

1. Product property of Logarithms
   log_b(MN) = $log_b N$ = $log_b N$
2. Quotient property of logarithms
   log_b($\frac{M}{N}$) = log_bM - log_b N
3. Exponent property of logarithms
   $log_b(M^c)$ = C log_bM

Each of these properties is merely a restatement of a property of exponents.

Trigonomeiric Functions

Trigonometric functions are mathematical relationships between the angles and sides of a right triangle. The three primary trigonometric functions are sinc, cosine, and tangent. Trigonometric functions can help you.

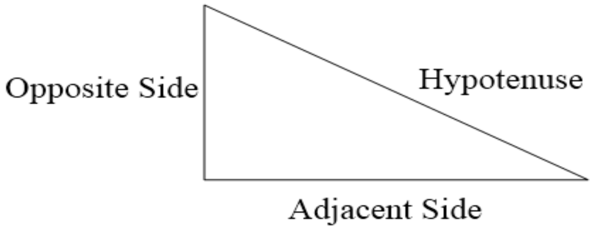
All you have to do is stand in your backyard, measure the distance between yourself and the tree, and use a protractor to gauge the angel of your line of sight to the top of tree. Using only these two measures. You can calculate the height of the free and know whether your house is safe.

Calculating the trigonometric functions

Let's look at each of the trigonometric function and see how it is calculated.

To calculate the sine of an angel in a right triangle you always divide the length of the side.

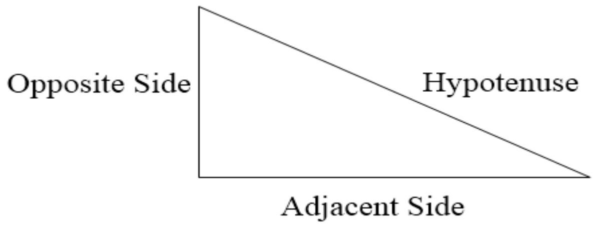
Opposite the angle by the length of hypotenuse of angle since is usually abbreviated as sin in mathematical statements.



$Sin\ B = \frac{opposite}{hypotenuse} = \frac{O}{H}$

Cosing

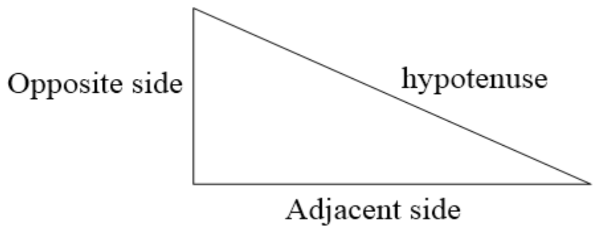
Typically cosine is abbreviated as cos, is calculated by dividing the length of the side adjacent to the angle by the length of the hypotenuse of triangle.



$cos\ B = \frac{opposite}{hypotenuse} = \frac{A}{H}$

Tangent

To calculate the tangent of an angle, divide the length of the side opposite the angle by the side adjacent to the angle.



$tan\ B = \frac{opposite}{hypotenuse} = \frac{O}{A}$

To remember how to calculate the three trigonometric functions, think about the acronym SHO CAH TOA.
Sine is equal to
Opposite over
Hypotenuse
Cosine is equal to
Adjacent over
Hypotenuse
Tangent is equal to
Opposite over
Adjacent.

Computation of values of the trigonometric functions

The values of sinx and cosx can be obtained from their Maclaurin series expansions. The expansion for sinx is

$$\text{Sinx} = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \ldots\ldots.$$

Derivation

The series can be shown to converge for all values of x. It can be used to compute the values of sinx for any value of x.

The maclaurin series expansion for cosx is

$$\text{Cosx} = 1 - \frac{x^2}{2!} + \frac{x^4}{4!}\ \frac{-x^6}{6!} + \ldots\ldots.$$

Once we have computed either sinx or cosx we can compute the other through the identity sin^2 x + cos^2x = 1.

The other trigonometric functions can then be computed through relations such as.

$$\text{tanx} = \frac{sinx}{cosx},\ \text{cotx} = \frac{1}{tanx},\ \text{secx} = \frac{1}{cosx}$$

$$\text{Cosecx} = \frac{1}{sinx}$$

Elementary functions when arguments is a complex number.

We will now treat the manner in which meaning have been assigned to the various elementary functions when the arguments are complex numbers.

The Exponential Functions Q^z, e^z

We define meaning of a^z when z is a complex number as follows.

Definition e^z

The maclavrin series expansion of e^x when x is a real number is

$$e^x = 1 + x + \frac{x^2}{2\,!} + \frac{x^3}{3\,!} + \ldots$$

When take this maclaurin series expansion of e^x, x is real, as our definition of e^z, when z is a complex number. Thus the definition of e^z for complex argument z is

$$e^z = 1 + z + \frac{z^2}{2\,!} + \frac{z^3}{3\,!} + \ldots$$

Def. a^z, a is real and positive. By definition for real and positive a

$$a^z = e^{zlna} \qquad 1$$

Where in a is the natural logarithm of a. Thus we evaluate a^z by substituting "zIna" into 2 in above the motivation for this definition comes from the fact.

References

1. Real Analysis by John M. Howie.
2. Real Analysis by H.L Royden and P.M. Fitzpartrick. (IVth Edition)
3. Elementary Functions by Dr. Ken W. Smith.
4. Mathematics, its content, Methods and Meaning.
5. James and james. Mathematics Dictionary Spiegel. Complex variables (Schaum)