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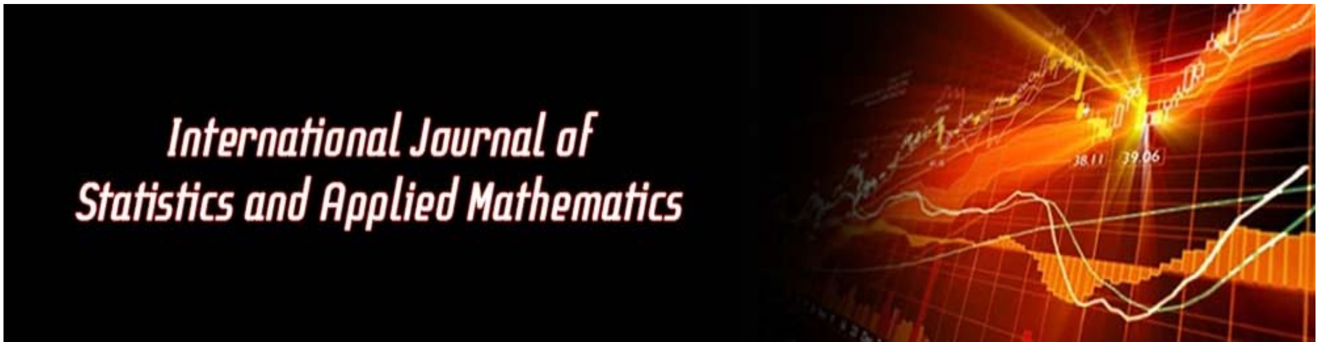

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Construction of supersaturated design using triangular type PBIBD

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Abstract

In this paper, a new method for the construction of supersaturated design using Triangular type partially balanced incomplete block designs is proposed. The $E(s^2)$ value of the design are also derived. The construction is also illustrated with suitable example. An attempt is also made to compare these design and some remarks are made.

Keywords: Triangular type PBIBD, Super-Saturated Design, $E(s^2)$-optimality

1. Introduction

Supersaturated design is a factorial design having the more number of factors when compared with number of design points. These designs can be used to identify active factor main effects when experimentation is expensive and the number of potential factors is large. These designs are more economical and flexible because of their run size.

Satterthwaite (1959) initially made an attempt to construct saturated designs randomly and suggested the random balance designs. Booth and Cox (1962) proposed a systematic method for the construction of super-saturated designs. After Booth and Cox (1962), no attempts were made till Lin (1993). Later several researchers made attempts on the construction of super-saturated designs.

Definition 1.1: A factorial design X is said to be a supersaturated design, if the number of factors "p" is more than the number of design points 'n' i.e. $p > n$.

Whenever, it is not possible to conduct the experiment with orthogonal design, we seek designs that are near orthogonal. To measure the lack of orthogonality, the degree of non-orthogonality is used. It depends on the all pairs of factors. Let s_{ij} be the sum of cross products between two different factors i and j of a design. If the expected value of s^2 i.e. the mean of s_{ij}^2 of all pairs (i, j) for $(i \neq j)$ is minimum, then the design is said to be $E(s^2)$-optimal design. This concept was proposed by Booth & Cox (1962) for selecting supersaturated designs.

In this paper, an attempt is made to construct supersaturated design using triangular type PBIBD.

2. Construction of Supersaturated design using Triangular PBIBD

Consider a triangular PBIBD with parameters v, b, r, k, λ_1, λ_2, n, n_1 and n_2 (where 'n' is the order of triangular matrix). For each factor, identify the first associate and second associate treatments. Construct a design of order v x v with elements as ±1 corresponding to each pair of treatments, put +1 if the pair of treatments belong to first associates and put –1 if the pair of treatment belongs to second associates. Obtain a super-saturated design X in (v+1) factors with v design points by augmenting a factor with level as +1 to each design point.

Theorem 2.1: A Supersaturated design X_{bxv} with two levels based on the association scheme of pairs of factors of triangular type PBIBD(2) with parameters v, b, r, k, λ_1, λ_2, $(n \geq 5)$, n_1 and n2 has E(s2) value given as

$$E(s^2) = \frac{(100n^2 - 250n - 192)}{4(v+1)}$$

Proof: Let N_{bxv} be the incidence matrix of a two associate class triangular PBIBD with parameters v, b, r, k, λ_1, λ_2, n_1 and n_2. Let T_i, T_j and T_k be any three factors. Let the pairs of factors (T_i, T_j) belong to first associate class and the pair of factors (T_i, T_k) belong to second associate class of the group divisible design. The elements of X'X matrix of saturated design will contain v(v-1)/2 distinct pairs of factors that are classified into three groups. The cross product terms corresponding to each pair of factor groups are

i) belong to the first associate class are (+1, +1), (+1, -1), (-1, +1) and (-1,-1) occur (n-2), (n-2), (n-2) and (n-3)(n-4)/2 times respectively. Therefore, the sum of cross products is [(n-4)(n-5) -4)] /2 and
ii) belong to the second associate class with the cross product terms being (+1, +1), (+1, -1), (-1, +1) and (-1,-1) occurs 4, (2n-8), (2n-8), [(n-4)(n-5)+4]/2 times respectively. Therefore, the sum of cross products is (n^2-17n+64) /2.
iii) cross product terms corresponding to each factor with augmented factor are (+1,+1), (-1, +1) occur (2n-4) and [(n-2)(n-3)/2 +1] times. Therefore, the sum of the cross product term is (-n^2+9n-16) /2.

The off diagonal elements in the last row / column of X'X are [-n^2+9n-16]/2 and in each other row/column the elements are [(n-4)(n-5)-4)]/2, [n^2-17n+64]/2 and (-n^2+9n-16)/2 occurring n_1, n_2 and one time respectively. Therefore, the E(s^2)- value is

$$E(s^2) = \frac{(100n^2 - 250n - 192)}{4(v+1)}$$

Example 2.1: Consider a Triangular PBIBD with parameters v=10, b=5, r =2, k=4, λ_1=1, λ_2=0, n_1 = 6 and n_2 = 3, n=5. Let A, B, C, D, E, F, G, H, I and J be the ten treatments.

$$D = \begin{bmatrix} * & A & B & C & D \\ A & * & E & F & G \\ B & E & * & H & I \\ C & F & H & * & J \\ D & G & I & J & * \end{bmatrix}$$

The first and second associates for each treatment are

Treatments →	A	B	C	D	E	F	G	H	I	J
First Associates Treatments	B	A	A	A	A	A	A	B	B	C
	C	C	B	B	B	C	D	C	D	D
	D	D	D	C	F	E	E	E	E	F
	E	E	F	G	G	G	F	F	G	G
	F	H	H	I	H	H	I	I	H	H
	G	I	J	J	I	J	J	J	J	I
Second Associates Treatments	H	F	E	E	C	B	B	A	A	A
	I	G	G	F	D	D	C	D	C	D
	J	J	I	H	J	I	H	G	F	E

Construct a supersaturated design $X_{v \times v+1}$ with levels ±1 by placing for each treatment put +1 if it is a first associate treatment and put -1 if it is a second associate treatment and augmenting a factor (column) with all +1's.

$$X_{v \times v+1} = \begin{bmatrix} -1 & +1 & +1 & +1 & +1 & +1 & +1 & -1 & -1 & -1 & +1 \\ +1 & -1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & -1 & +1 \\ +1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 \\ +1 & +1 & +1 & -1 & -1 & -1 & +1 & -1 & +1 & +1 & +1 \\ +1 & +1 & -1 & -1 & -1 & +1 & +1 & +1 & +1 & -1 & +1 \\ +1 & -1 & +1 & -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 \\ +1 & -1 & -1 & +1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 \\ -1 & +1 & +1 & -1 & +1 & +1 & -1 & -1 & +1 & +1 & +1 \\ -1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 & -1 & +1 & +1 \\ -1 & -1 & +1 & +1 & -1 & +1 & +1 & +1 & +1 & -1 & +1 \end{bmatrix}$$

with E(s^2) = 4.14.

3. Some Remarks On Proposed Supersaturated Design

1. The proposed supersaturated designs are with v design points in v+1 factors.
2. In case of the proposed methods, it is possible to estimate number of main effects be 'v' which maximum. It is more when compared with many of the existed designs.
3. It is possible to construct the super-saturated designs with 'n' design points if the series of group divisible and triangular type PBIBD's are exists with n+1 factors only.
4. Booth & Cox (1962) designs and the proposed supersaturated design both two levels and both not existing for all number of factors. If the super-saturated design exists then it is more efficient than the corresponding existed. Booth and Cox designs and the number of factors confounded is more in booth and Cox designs.
5. Lin(1993) designs are constructed based on Placket & Burman designs of order 'n' and may not exist for all possible values of n and these designs does not attain lower bound of E(s^2). Half the factors (n-1)/2 main effects are confounded.

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