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Harpreet Kaur
Research scholar of IK Gujral Punjab Technical University, Kapurthala, Punjab, India

BP Garg
Research supervisor of IK Gujral Punjab Technical University, Kapurthala, Punjab, India

Correspondence
Harpreet Kaur
Research scholar of IK Gujral Punjab Technical University, Kapurthala, Punjab, India

Radiation effect on terminal velocity of a vertically falling non-spherical particle by diagonal Pade' [3/3] Approximants

Harpreet Kaur and BP Garg

Abstract
In this paper, the Radiation effect on velocity of a vertically falling non-spherical particle in incompressible Newtonian fluid is investigated. The use of radiation force for increasing the velocity of vertically falling non-spherical particles has the most important implication on increasing the efficiency of particulate control device such as gravity settling chambers. In this paper, the velocity of a vertically falling non-spherical single particle in the Newtonian fluid can be described by the force balance equation (Basset-Boussinesq-Ossen equation) in presence of Radiations. The main difficulty in the solution of this equation lies in the nonlinear term due to the nonlinearity nature of the drag coefficient. The terminal velocity was calculated by using the Diagonal Pade' Approximant method order [3/3] and Varinational Iteration method (VIM) 4^{th} iteration. The results were also compared with fourth order Runge-Kutta method(R-K 4^{th} order) to verify the accuracy of the proposed methods. The particle which is more effected by Radiation force have more velocity and reaches early at terminal velocity as compared to others. To obtain the results for all different methods, the symbolic calculus software MATLAB was used.

Keywords: Diagonal Pade' approximation method, non-spherical particle, Radiation effect Terminal velocity and Varinational Iteration method (VIM)

1. Introduction

The problem of acceleration motion of vertically falling spherical and non-spherical particle in Newtonian and Non-Newtonian fluids is relevant to many situations of practical interest. It is necessary to know the detailed trajectories of the accelerating particles for the purposes of design or improved operation. For example, the measurements of terminal velocity of raindrop in Newtonian fluids using the falling ball method. It is also necessary to know the time and distance required to reach the particle at terminal point to determine the reliable results for design models. In present, the non-spherical particle is considered in the presence of Radiation effect. It is clear from previous literature the motion of particle is affected by Radiation force. Considerable attentions have been devoted to the study of the acceleration motion of non-spherical particles in fluids and an excellent account of theoretical development in this area has been given by Clift et al. [17] for spherical bodies. Less information is available in the previous literature for the case of motion of non-spherical particle with Radiation force. Many correlation for the drag coefficient in terms of the Reynolds number for motion of non-spherical particles were given in the literature [1, 5]. From all these, one of the well-known analytical correlation between Reynolds numbers and drag coefficient for non-spherical particle is presented by Chien [18]

$$C_D = \frac{30}{Re} + 67.289e^{(-5.03\varphi)} \text{ Where Re}= \frac{\rho D u}{\mu} \qquad (1)$$

Where $C_D$drag coefficient with Sphericity φ and Re is is Reynolds numbers. These are based on the equal volume sphere diameter [13]. Eq. (1) was stated to be valid in the ranges of $0.2<\varphi<1$ and $0.001<Re<10000$ for the different shapes of partical [18]. The analysis derived by Pade' approximant and Varinational iteration method (VIM). The results of current methods are compared with the well-known R-K 4^{th} order method in order to verify the accuracy of the proposed methods.

Nomenclature	Greek symbols
Acc acceleration, m/s^2 C_D Drag coefficient D Particle diameter, m g acc. due to gravity, m/s^2 m particle mass,kg Re Reynolds number t time,s u Velocity,m/s H Radiation parameter	a,b,c,d constants φ Sphericity μ Dynamic viscosity,kg/ms ρ Fluid density,kg/m^3 ρ_s Spherical partical density, kg/m^3

2. Problem Formulation

Consider a rigid body, single non-spherical particle is falling in an infinite extent of incompressible Radiated Newtonian fluid of density ρ and viscosity μ, u represents the velocity of the non-spherical particle at any instant time t, g is the acceleration due to gravity and H is a Radiation parameter [2, 3, 7]. Thus, the equation of the particle motion is given by

$$m\frac{du}{dt}=mg\left(1-\frac{\rho}{\rho_s}\right)-\frac{1}{8}\pi D^2\rho C_D u^2-\frac{1}{12}\pi D^3\rho\frac{du}{dt}+H \quad (2)$$

Where C_D the drag coefficient, equivalent volume diameter D, mass m and particle density ρ_s . In right hand side of the eq.(2), the 1st term represent the buoyancy effect, the 2nd term corresponds to drag resistance, 3rd term is associated with the added mass effect which is due to acc. of fluid around the particle and 4th term is Radiation Parameter.

The non-linear terms due to non-linearity nature of the drag coefficient C_D is the main difficulty in solving eq. (2) could be re-written as follows:

$$\left(m+\frac{1}{12}\pi D^3\rho\right)\frac{du}{dt}=mg\left(1-\frac{\rho}{\rho_s}\right)-\frac{1}{8}\pi D^2\rho C_D u^2+H$$

$$a\frac{du}{dt}+bu+cu^2-d-H=0 \quad (3)$$

Where $a=\left(m+\frac{1}{12}\pi D^3\rho\right)$, $b=3.75\pi D\mu$, $c=\frac{67.289e^{(-5.03\varphi)}}{8}\pi D^2\rho$, $d=mg\left(1-\frac{\rho}{\rho_s}\right)$,

H=Radiation parameter

3. Solution of Problem

3. (A). Diagonal Pade' [3/3] Approximants

A Pade' approximant is the ratio of two polynomials constructed from the coefficients of the Taylor series expansion of a function u (t). The technique was developed around 1890 by Henri Pade'. The Padé approximant often gives better closed form approximation of the function and it may still work where the Taylor series does not converge. For these reasons Padé approximants are used extensively in computer calculation. The [L/M] Pade' approximants to a function u (t) are given by [7, 12, 16]

$$\left[\frac{L}{M}\right]=\frac{P_L(t)}{q_M(t)}, \text{ where L=M (for diagonal pade' approximants)} \quad (4)$$

Where $P_L(t)$ and $q_M(t)$ are the polynomials of the degree of at most L and M respectively. The formal power series is give

$$u(t)=\sum_{i=1}^{\infty} a_i t^i \quad (5)$$

$$\text{From eq.(4), } u(t)-\frac{P_L(t)}{q_M(t)}=O(t^{L+M+1}) \quad (6)$$

$$\text{From eq.(6), } u(t)-\frac{P_L(t)}{Q_M(t)}=0 \quad (7)$$

$$p_L(t)=p_0+p_1t^1+p_2t^2+p_3t^3+\ldots+p_Lt^L \quad (7a)$$

$$q_M(t)=q_0+q_1t^1+q_2t^2+q_3t^3+\ldots+q_Mt^M \quad (7b)$$

Determine the coefficient of $p_L(t)$ and $q_M(t)$ by the help of eq.(6a) and take normalization condition $q_M(0)=1$

3. (A).1. For Diagonal pade' [3/3]

$$u(t)-\frac{P_3(t)}{Q_3(t)}=0 \text{ where } u(t)=a_0+a_1t^1+a_2t^2+a_3t^3+\ldots \quad (8)$$

$$a_0+a_1t^1+a_2t^2+a_3t^3+\ldots=\frac{p_0+p_1t^1+p_2t^2+p_3t^3}{q_0+q_1t^1+q_2t^2+q_3t^3} \quad (8a)$$

$$\text{Generally, } p_L=a_L+\sum_{i=1}^{min.(L,M)} q_i a_{L-i} \quad (8b)$$

$$a_0q_0=p_0$$
$$a_1q_0+a_0q_1=p_1$$
$$a_2q_0+a_1q_1+a_0q_2=p_2$$
$$a_3q_0+a_2q_1+a_1q_2+a_0q_3=p_3$$
$$a_4q_0+a_3q_1+a_2q_2+a_1q_3=0$$
$$a_5q_0+a_4q_1+a_3q_2+a_2q_3=0$$
$$a_6q_0+a_5q_1+a_4q_2+a_3q_3=0 \quad (8c)$$

Find the coefficients a_0, a_1, a_2, a_3, ...with help of Taylor's expansion for a=b=c=d=1

Equ.(3) becomes, $u'(t)=H+1-u^2-u$,$u(0)=0$ (8d)

3 (A).2 Diagonal pade' [3/3] for H=0

The given equation (8d) becomes

$u'(t)=1-u^2-u$, with initial condition $u(0)=0$ (9)

Taylor's series about zero of equ. (9) is given by

$$u(t)=u_0+tu'_0+\frac{t^2}{2!}u''_0+\frac{t^3}{3!}u'''_0+\frac{t^4}{4!}u^{iv}_0+\frac{t^5}{5!}u^{v}_0+\frac{t^6}{6!}u^{vi}_0+\frac{t^7}{7!}u^{vii}_0+\frac{t^8}{8!}u^{viii}_0+\ldots \quad (9a)$$

$$\text{Solution is } u(t)=t-\frac{t^2}{2!}-\frac{t^3}{3!}+7\frac{t^4}{4!}-5\frac{t^5}{5!}-85\frac{t^6}{6!}+335\frac{t^7}{7!}+1135\frac{t^8}{8!}+\ldots \quad (9b)$$

Now compare equ. (9b) with equ.(8), So $a_0=0$, $a_1=1$, $a_2=-\frac{1}{2!}$, $a_3=-\frac{1}{3!}$, $a_4=\frac{7}{4!}$,

$a_5=-\frac{5}{5!}$, $a_6=-\frac{85}{6!}$, $a_7=\frac{335}{7!}$, $a_8=\frac{1135}{8!}$ and so on (9c)

Solving the system of linear equations (8c) by Gauss elimination method and using eq. (9c)

$p_0=0$, $p_1=1$, $p_2=0$, $p_3=0.083$ and $q_0=1$, $q_1=0.5$, $q_2=0.5$, $q_3=0.0417$

$$\text{So Diagonal Pade' [3/3]}=\frac{t+0.083t^3}{1+0.5t+0.5t^2+0.0417t^3} \quad (9d)$$

For H=2, The given equation is $u'(t)=3-u^2-u$, with initial condition $u(0)=0$ (10)

$$\text{Diagonal Pade' [3/3]}=\frac{3t^1+0.65t^3}{1+(0.5)t^1+1.3t^2+(0.1083)t^3} \quad (10a)$$

For H=4, The given equation is $u'(t)=5-u^2-u$, with initial condition $u(0)=0$ ` (11)

$$\text{Diagonal Pade' [3/3]}=\frac{5t^1-0.7060t^2+2.1070t^3}{1+0.3588t^1+2.1008t^2-0.0365t^3} \quad (11d)$$

Table 1: The Velocity results of particle by diagonal Pade' [3/3] for without Radiation effect and with Radiation effect

t (sec)	Velocity(u)[m/s],H=0	Velocity(u)[m/s],H=2	Velocity(u)[m/s],H=4
0.0	0.000	0.0000	0.0000
0.1	0.0949	0.2828	0.4684
0.2	0.1791	0.5250	0.8556
0.3	0.2527	0.7225	1.1525
0.4	0.3160	0.8775	1.3686
0.5	0.3698	0.9954	1.5217
0.6	0.4150	1.0832	1.6301
0.7	0.4527	1.1476	1.7093
0.8	0.4838	1.1947	1.7708
0.9	0.5094	1.2292	1.8226
1.0	0.5304	1.2550	1.8699

In fig.1, u (vertically) denotes the velocity of particle w.r.t. time t (horizontally).Solution for the velocity of a single non-spherical particle during the acceleration motion is obtained by diagonal pade' approximants of order [3/3]. Fig.1.shows the Radiation effect on velocity of a single particle with Radiation parameter H=2, 4. Due to Radiation effect, the velocity of a particle is increasing with increment in the Radiation parameter

3. (B). Varinational iteration method (VIM) J. He, was introduced Varinational iteration Method (VIM) to solve the several nonlinear ordinary and partial differential equations in 1997 [6]. He's Varinational iteration method (VIM) has been extensively applied as a power tool for solving various kinds of problems [8, 9, 10]. Besides the He' VIM, Saeed. U. used Modified Varinational Iteration Method to solve Sine-Gordon Equation and comparison with numerical results. They conclude that VIM provides stable and accurate results. VIM also gives the useful result for solving WBK equation. Using VIM, Liu and Gurram have solved the problems of free vibration involving an Euler-Bernoulli beam and obtained accurate results and compare the result with ADM [20]. Slota use the VIM to obtained the results for the Heat equation which were same as the exact solution [4, 21]. He's VIM also used for solving a semi-linear inverse parabolic equation. A new application of Varinational iteration method to solve chaotic Rossler system [19]. From review of literature, it is confirmed that He's VIM is most accurate and useful method for solving the nonlinear problems.

$$Lu(t)+Nu(t)=g(t) \qquad (12)$$

Where L is a linear operator, N is a nonlinear operator and g(t) is a non-homogeneous term. By using the Varinational iteration method, a correction functional can be constructed as

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{Lu_n(\zeta)+N\tilde{u}(\zeta)-g(\zeta)\}d\zeta \qquad (12a)$$

Where λ is a general Lagrange multiplier, the subscript n means the nth approximation; u_n is restricted variation and $\delta\tilde{u}_n=0$. [11, 15]

According to VIM, firstly we will find Lagrange multiplier and then trial function u_0 to get the successive iterations u_{n+1}, n≥0

Which converge to the exact solution. The solution is $u=\lim_{n\to\infty} u_n$

To solve eq. (3) using VIM [22], the correction functional can be constructed as follows:

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{a\frac{du_n(s)}{ds}+b\,u_n(s)+cu^2{}_n(s)-d-H\}ds \qquad (13)$$

For a=b=c=d=1,

Eq. (13) becomes $u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{\frac{du_n(s)}{ds}+u_n(s)+u^2{}_n(s)-1-H\}ds$ (14)

When H=0, $u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{\frac{du_n(s)}{ds}+u_n(s)+u^2{}_n(s)-1\}ds$ (14a)

When H=2, $u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{\frac{du_n(s)}{ds}+u_n(s)+u^2{}_n(s)-3\}ds$ (14b)

When H=4, $u_{n+1}(t)=u_n(t)+\int_0^t \lambda\{\frac{du_n(s)}{ds}+u_n(s)+u^2{}_n(s)-5\}ds$ (14c)

The stationary conditions can be obtained as follows:

$\lambda_{s=t}-\lambda'_{s=t}=0$,$1+\lambda(t)_{s=t}=0$ (14d)

Subsequently, the Lagrangian multiplier is obtained as:

$\lambda=-e^{s-t}$ [9] (14e)

Substituting eq. (14e) in eq. (14) and assuming $u_0(t)=0$, solution will be gained for velocity variation w.r.t. time without Radiation effect (H=0) and with Radiation effect (H=2,4) $u_{n+1}(t)=u_n(t)-\int_0^t e^{s-t}\{\frac{du_n(s)}{ds}+u_n(s)+u^2{}_n(s)-1-H\}ds$, with condition $u_0(t)=0$ (15)

Solve equ.(15) by VIM 4th iteration for without Radiation effect and with Radiation effect .

Table 2: The Velocity results of particle by VIM 4th iteration for without Radiation effect and with Radiation effect.

t	Velocity(u)[m/s], H=0	Velocity(u)[m/s], H=2	Velocity(u)[m/s], H=4
0.0	0.0000	0.0000	0.000
0.1	0.0949	0.2828	0.4684
0.2	0.1791	0.5250	0.8554
0.3	0.2527	0.7225	1.1512
0.4	0.3160	0.8773	1.3629
0.5	0.3698	0.9943	1.5026
0.6	0.4150	1.0793	1.5786
0.7	0.4526	1.1368	1.5903
0.8	0.4837	1.1698	1.5289
0.9	0.5091	1.1790	1.3813
1.0	0.5296	1.1637	1.1371

In Table 2, Solution for velocity of a vertically falling single non-spherical particle during the acceleration motion with Radiation effect and without Radiation effect is obtain by Varinational Iteration Method (VIM) of 4th iteration. IV. Runge-kutta 4th order method (Numerical Method), It is clear that the current problem is initial value problem (IVP) of 1st order. So far a solution, we can apply numerical methods like trapezoidal method, Euler's method (1st order R-K method), and R-K 4th order method. Trapezoidal method is generally used for typical problems. The R-K 4th order method is the modification in Euler's method by adding midpoint in the step which increase the accuracy. Thus R-K 4th order method is a suitable numerical technique in present problem [14]

Equ. (3) ,u' (t)=H+1-u^2 –u , is a 1st order differential equation with initial condition u(0)=0 (16)

For H=0, f (t,u)= 1-u^2 –u ,u(0)=0 (16a)

For H=2, f (t,u)= 3-u^2 –u ,u(0)=0 (16b)

For H=4, f (t,u)= 5-u^2 –u ,u(0)=0 (16c)

Table 3: The Velocity results of particle by R-K 4th order for without Radiation effect and with Radiation effect

t (sec)	Velocity(u)[m/s],H=0	Velocity(u)[m/s], H=2	Velocity(u)[m/s], H=4
0	0	0	0
0.1	0.0949	0.2828	0.4684
0.2	0.1791	0.5249	0.8554
0.3	0.2527	0.7225	1.1513
0.4	0.3160	0.8774	1.3646
0.5	0.3698	0.9952	1.5119
0.6	0.4150	1.0826	1.6105
0.7	0.4527	1.1464	1.6753
0.8	0.4838	1.1922	1.7172
0.9	0.5094	1.2250	1.7442
1.0	0.5303	1.2482	1.7614

In fig.3, velocity variation of a vertically falling single non-spherical particle in an infinite incompressible Radiated Newtonian fluid is investigated by numerical method. With increase in Radiation parameter, the velocity is also increasing.

Table 4: Acceleration motion of a single non- spherical particle at different time t by R-K 4th order method for without Radiation effect (H=0) and with Radiation effect (H=2, 4)

Time (sec)	Acc.[m/s^2], H=0	Acc.[m/s^2], H=0	Acc.[m/s^2], H=4
0.2	0.8955	2.6237	4.2729
0.4	0.6844	1.7610	2.5403
0.6	0.4951	1.0253	1.2283
0.8	0.3440	0.5485	0.5364
1.0	0.2325	0.2803	0.2240
1.2	0.1541	0.1399	0.0918
1.4	0.1009	0.0690	0.0373
1.6	0.0655	0.0338	0.0151
1.8	0.0425	0.0165	0.0060
2.0	0.0275	0.0079	0.0025
2.2	0.0175	0.0040	0.0001
2.4	0.0110	0.0020	0.0000
2.6	0.0070	0.0000	0.0000
2.8	0.0050	0.0000	0.0000
3.0	0.0025	0.0000	0.0000

In fig.4. the Acceleration variation of a single particle without Radiation effect and with Radiation effect as a Radiation parameter H=2 &H=4 is investigated by numerical (R-K 4th order) methods. It shows that the particle with Radiation effect H=4 reaches early at steeling velocity as compare to others like H=2,H=0. So it is clear that acceleration motion of a particle with high Radiation effect becomes zero (not accelerated) in short time interval as compared to and low Radiation effect and without Radiation effect

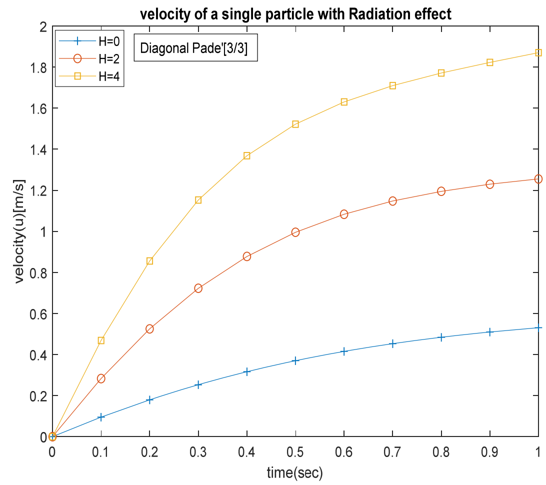

Fig 1: Velocity of particle by Diagonal Pade' [3/3]

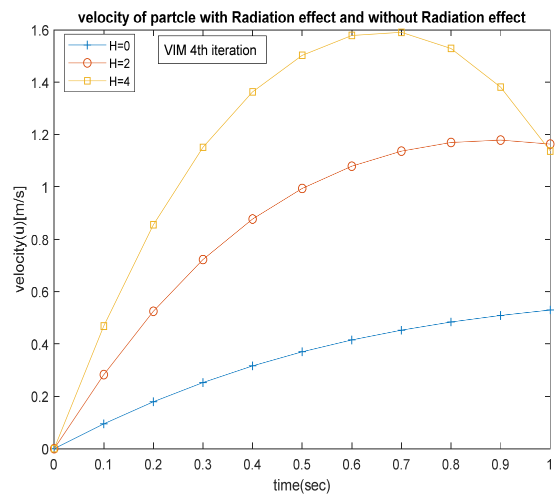

Fig 2: Velocity of particle by VIM 4th iteration

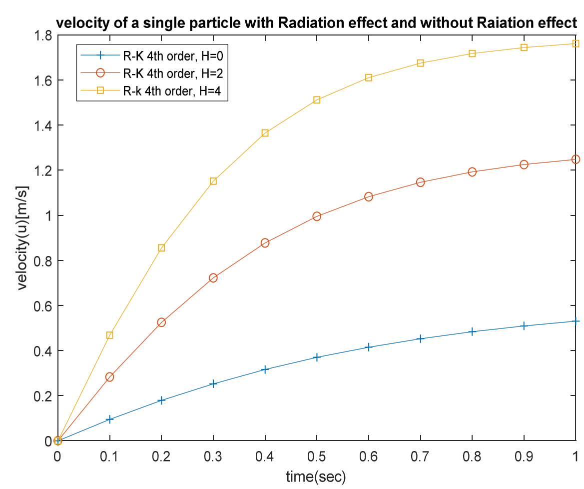

Fig 3: Velocity of particle by R-K 4th order method

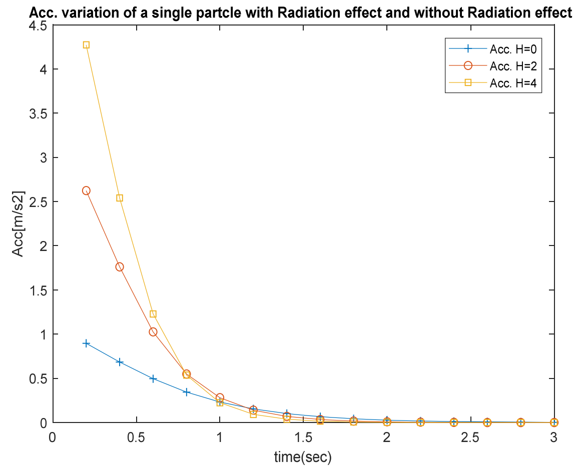

Fig 4: Acceleration variation particle without Radiation effect and with Radiation effect

4. Results and discussion

Table 5: Compare velocity results of vertically falling single non-spherical particle by Pade' [3/3], VIM (3rd iteration) with Runge-kutta 4th order method without Radiation effect

t	$u(t)_{pade'[3/3]}$	$u(t)_{VIM}$	$u(t)_{R-K}$	$Error_{(pade'[3/3]}\%$	$Error_{(VIM)}\%$
0.0	0	0.0000	0	0.0000	0.0000
0.1	0.0949	0.0949	0.0949	0.0000	0.0000
0.2	0.1791	0.1791	0.1791	0.0000	0.0000
0.3	0.2527	0.2527	0.2527	0.0000	0.0000
0.4	0.3160	0.3160	0.3160	0.0000	0.0000
0.5	0.3698	0.3698	0.3698	0.0000	0.0000
0.6	0.4150	0.4150	0.4150	0.0000	0.0000
0.7	0.4527	0.4526	0.4527	0.0000	0.0220
0.8	0.4838	0.4837	0.4838	0.0000	0.0206
0.9	0.5094	0.5091	0.5094	0.0000	0.0589
1.0	0.5304	0.5296	0.5303	0.0189	0.1320

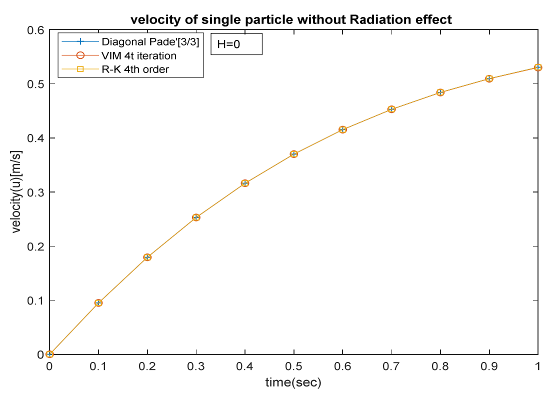

Fig 5: Velocity variation of a particle without Radiation effect

In table 5, the velocity results of vertically falling single non-spherical particle in Newtonian fluid was investigated by different methods at different time without any Radiation effect. The results of Pade'[3/3] and VIM(4th iteration) are compared with Numerical Method (R-k 4th order method). In short time both methods gives accurate results but Diagonal Pade'[3/3] is more accurate at any time as compared to VIM 4th iteration i.e the absolute error % of Pade' [3/3] is less as compared to VIM (4th iteration).

Table 6: Compare velocity results of vertically falling single non-spherical particle by Pade' [3/3], VIM (3rd iteration) with Runge-kutta 4th order method with Radiation effect, H=2

t	$u(t)_{pade'[3/3]}$	$u(t)_{VIM}$	$u(t)_{R-K}$	$Error_{(pade'[3/3]}\%$	$Error_{(VIM)}\%$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.2828	0.2828	0.2828	0.0000	0.0000
0.2	0.5250	0.5250	0.5249	0.0190	0.0190
0.3	0.7225	0.7225	0.7225	0.0000	0.0000
0.4	0.8775	0.8773	0.8774	0.0114	0.0114
0.5	0.9954	0.9943	0.9952	0.0201	0.0904
0.6	1.0832	1.0793	1.0826	0.0554	0.3048
0.7	1.1476	1.1368	1.1464	0.1047	0.8374
0.8	1.1947	1.1698	1.1922	0.2097	1.8789
0.9	1.2292	1.1790	1.2250	0.3429	3.7551
1.0	1.2550	1.1637	1.2482	0.5448	6.7697

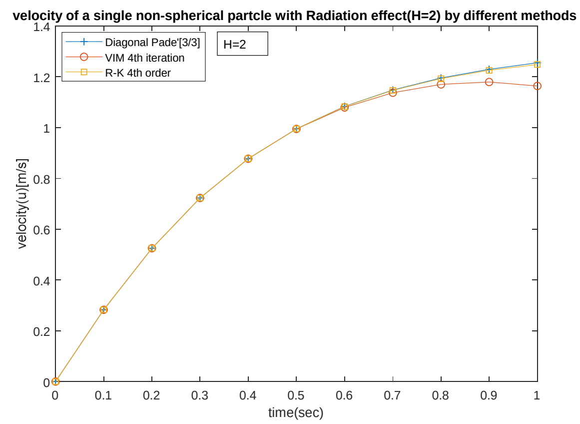

Fig 6: Velocity variation of a particle with Radiation effect, H=2

In table 6, the velocity results of vertically falling single non-spherical particle in Newtonian fluid was investigated by different methods at different time with Radiation effect as Radiation parameter H=2. When t∈[0,0.6] sec both methods are good but in long time Diagonal Pade'[3/3] gives the results with high accuracy as compared to VIM 4th iteration i.e the absolute error % of Pade' [3/3] is less as compared to VIM (4th iteration).

Table 7: Compare velocity results of vertically falling single non-spherical particle by Pade' [3/3], VIM (3rd iteration) with Runge-kutta 4th order method with Radiation effect, H=4

t	$u(t)_{pade'[3/3]}$	$u(t)_{VIM}$	$u(t)_{R-K}$	$Error_{(pade'[3/3]}\%$	$Error_{(VIM)}\%$
0	0.0000	0.0000	0.0000	0.0000	0.0000
0.1	0.4684	0.4684	0.4684	0.0000	0.0000
0.2	0.8556	0.8554	0.8554	0.0234	0.0000
0.3	1.1525	1.1512	1.1513	0.1042	0.0087
0.4	1.3686	1.3629	1.3646	0.2931	0.1246
0.5	1.5217	1.5026	1.5119	0.6482	0.6151
0.6	1.6301	1.5786	1.6105	1.2170	1.9808
0.7	1.7093	1.5903	1.6753	2.0295	5.0737
0.8	1.7708	1.5289	1.7172	3.1214	10.9655
0.9	1.8226	1.3813	1.7442	4.4949	20.8061
1.0	1.8699	1.1371	1.7614	6.1599	35.4434

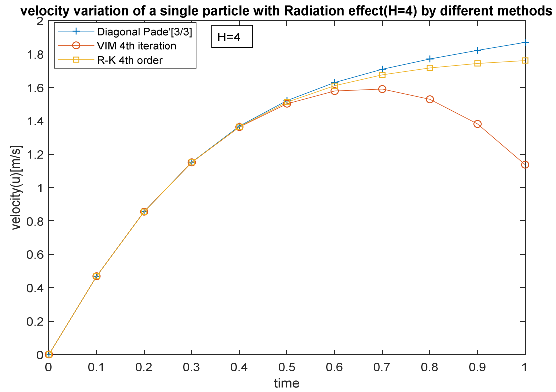

Fig 7: Velocity variation of a particle with Radiation effect, H=4

In table 7, the velocity results of vertically falling single non-spherical particle in Newtonian fluid was investigated by different methods at different time with Radiation parameter H=4. Upto t∈[0,0.4] sec both methods give the accurate results but Diagonal Pade'[3/3] gives results with high accuracy as compared to VIM 4th iteration at time above t∈[0,0.4] sec also.

5. Conclusion

The achievement of this work is to apply the current methods i.e. diagonal pade' and VIM in order to study the nonlinear differential equation of 1st order with initial condition that governed from the Radiation effect on accelerated motion of vertically falling single non-spherical particle in incompressible Radiated Newtonian fluid. The current methods are applied without using any linearization, discretization, restrictions or transformations. From above discussion, it is clear that due to increase in Radiation parameter velocity is also increasing and particle with high Radiation effect reaches early at terminal velocity. The diagonal pade' method [3/3] has a good agreement with numerical method and provides highly reliable results. In addition, this method does not require many iterations like VIM to reach accurate results. Both methods gives the accurate results in short time, but Diagonal pade' method is also suitable for long time. Also, the current method (diagonal pade') can be used to develop the valid solution of other nonlinear differential equation of order more than one.

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