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Extreme value control charts based on Rayleigh distribution

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Abstract
Variable control charts are based on subgroup statistics and variation in the values of the subgroups. In this paper extreme order statistic of the subgroup are considered to develop the control limits to decide upon the in control status of the process. Here, the quality characteristic is assumed to follow Rayleigh (Weibull with shape parameter 2) and gamma with shape parameter 2 distributions are considered. Relevant comparisons are presented with examples.

Keywords: Extreme value control chart, extreme order statistic, variable control charts.

Introduction
In order to monitor the variability in the quality control data by developing graphical procedures for subgroup statistics such as mean, range, standard deviation etc., the variable control charts are popularly used. Depending on the subgroup size and the sampling distribution of the subgroup statistics separate control chart constants and hence control limits would be constructed in practice. However some research works appeared in literature that deals with control charts for non-normally distributed process variates (Edgemen (1989), Kantam and SriRam (2001) [2] developed control charts for process variates which are non-normally distributed and the references therein). Control charts for individual observations in the case of normal process variates are developed and the principle was made use of to propose a procedure for comparison of multiple means known as Analysis of Means (OTT (1967). On similar lines the well known gamma and exponential distributions are assumed as models of process variate and the corresponding ANOM procedure along with control charts for individual observations are developed by SriRam (2004) [5]. Motivated from these aspects we extend the same principle to two probability models of Rayleigh distribution (Weibull with shape parameter 2) and gamma with shape parameter 2 developed alternative pairs of control limits for variable control charts and compare their appropriateness with that of existing pairs of limits in literature. The percentiles of extreme order statistics in samples from the models and their use in developing limits for variable control charts are presented in the section 2. In section 3, we compare our control limits with those existing in literature with examples.

Extreme Order Statistics Based Control Limits
If X is a process variate having a Rayleigh distribution (Weibull distribution with shape parameter 2) has the probability density function and distribution function given as

$$f(x) = \frac{x}{\sigma^2}\, e^{-x^2/2\sigma^2};\ x \geq 0 \quad (2.1)$$

$$F(x) = 1 - e^{-x^2/2\sigma^2};\ for\ x \in [0, \infty) \quad (2.2)$$

In the standard form

f (z) = z. $e^{-\frac{z^2}{2}}$; z≥0 (2.3)

F(z) = 1- $e^{-\frac{z^2}{2}}$ for z€[0, ∞) (2.4)

Similarly in case of gamma distribution (2) the probability density function and cumulative distribution function

f (x) = $\frac{x}{\sigma^2} e^{-\frac{x}{\sigma}}$ (2.5)

F(x) = 1- $e^{-\frac{x}{\sigma}}$ (1+ $\frac{x}{\sigma}$) (2.6)

In the standard form

f (z) = z. e^{-z} ; z≥0 (2.7)

F(z) = 1- e^{-z} (1+z) for z€[0, ∞) (2.8)

Here we made an attempt to get the pair of control limits using extreme order statistics in samples from Rayleigh distribution and gamma distribution with shape parameter 2. The percentiles of extreme order statistics for standard Rayleigh distribution can be obtained from the following steps. The cumulative distribution function of smallest order statistic

$X_{(1)}$ is given by

$F_1(x) = 1-(1-F(x))^n$ (2.9)

The cumulative distribution function of the largest order statistic $X_{(n)}$ given by

$F_n(x) = (F(x))^n$ (2.10)

In case of Rayleigh distribution CDF of the smallest order statistics $X_{(1)}$ and largest order statistics $X_{(n)}$ are given by

$F_1(x) = P(X_{(i)}<L) = 0.00135$ (2.11)

$F_1(x) = P(X_{(i)}<U) = 0.99865$ (2.12)

With as the conjecture, we made an attempt to get the pair of control limits using extreme order statistics in samples from Rayleigh and gamma distribution.

The extreme value control charts based on Rayleigh distribution

$$L= \sqrt{\left(\frac{-2}{n}\log(0.99865)\right)} \quad (2.13)$$

$$U = \sqrt{\left(-2\log(1-(0.99865)^{1/n})\right)} \quad (2.14)$$

The extreme value control charts based on gamma (2) distribution

$1-e^{-L}(1+L) = 1-(0.00135)^{1/n}$ (2.15)

$1-e^{-U}(1+U) = (0.99865)^{1/n}$ (2.16)

By using equations (2.13), (2.14) we can calculate percentiles of Rayleigh distribution and for percentiles of gamma (2) distribution we use cumulative distribution function tables for gamma (2) distribution from SriRam (2004) [5] and solve the equations (2.15), (2.16) and respective percentiles are tabulated in Tables (2.1) and (2.2).

In this case gamma distribution (2), if σ is not known, it is estimated by $\frac{\bar{R}}{d_2}$ where d_2 =E[Z_n-Z_1] taken from Gupta (1960).

In case of Rayleigh Distribution σ is estimated by MLE i.e $\sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$ say s under repeated sub-grouping $\hat{\sigma} = \bar{s}$

For Rayleigh distribution the control limits are $\left[D_2^*\bar{R}, D_3^*\bar{R}\right]$

Where $D_2^* = \frac{L}{d_2}$ and $D_3^* = \frac{U}{d_2}$, D_2^* and D_3^* are given in the following table

For Rayleigh distribution the control limits are $\left[\sqrt{L^*}\bar{s}, \sqrt{U^*}\bar{s}\right]$, $\sqrt{L^*}$ and $\sqrt{U^*}$ are given in the following table

Control chart constants for R-Chart

Rayleigh distribution Rayleigh distribution

Table 2.1

n	D_2^*	D_3^*
2	0.825	1.128
3	0.748	1.693
4	0.709	2.059
5	0.670	2.326
6	0.648	2.534
7	0.627	2.704
8	0.614	2.847
9	0.600	2.970
10	0.588	3.076

Table 2.2

n	$\sqrt{L^*}$	$\sqrt{U^*}$
2	0.03675	3.82112
3	0.03001	3.92576
4	0.02599	3.99836
5	0.02325	4.05378
6	0.02122	4.09850
7	0.01965	4.13594
8	0.01838	4.16809
9	0.01733	4.19625
10	0.01644	4.22129

Constants of R-chart gamma (2) distribution gamma (2) distribution

Table 2.3

n	M^*_1	M^*_2
2	1.27875	2.41750
3	0.62727	1.68667
4	0.38135	1.30250
5	0.26034	1.06800
6	0.19109	0.90417
7	0.14739	0.7900
8	0.11786	0.69875
9	0.09685	0.62889
10	0.08132	0.56950

Example 1

Consider the following data

Sample No	Sample Observations	Total	Sample Range	Sample S.D (s)
1	42 65 75 78 87	347	45	71.09
2	42 45 68 72 90	317	48	65.87
3	19 24 80 81 81	285	62	63.97
4	36 54 69 77 84	320	48	66.27
5	42 51 57 59 78	287	36	58.62
6	51 74 75 78 132	410	81	86.27
7	60 60 72 95 138	425	78	89.95
8	18 20 27 42 60	167	42	39.92
9	15 30 39 62 84	230	69	52.05
10	69 109 113 118 153	562	84	115.54
11	64 90 93 109 112	468	48	95.15
12	61 78 94 109 136	478	75	99.01
	Total		716	903.71

For Sub-group size 5 the control chart constants from the above table are

The control limits for Rayleigh distribution $\left[D_2^*\overline{R}, D_3^*\overline{R}\right]$,

D_2^* = 0.670, D_3^* = 2.326

$\left[D_2^*\overline{R}, D_3^*\overline{R}\right]$ = [35.2889, 122.5104]

The Control limits for Rayleigh distribution are $\left[\sqrt{L^*}\,\overline{s}, \sqrt{U^*}\,\overline{s}\right]$

s = $\sqrt{\dfrac{\sum_{i=1}^{n} x_i^2}{n}}$, $\overline{s}$ = $\dfrac{\sum s}{k}$, k is the Sample number

$\sqrt{L^*}$ = 0.02325, $\sqrt{U^*}$ = 4.05378

From above Rayleigh distribution distribution table at n=5,

$\overline{s}$ = 75.31

$\left[\sqrt{L^*}\,\overline{s}, \sqrt{U^*}\,\overline{s}\right]$ = [1.2379, 215.8725]

From the above considered example all the sample points are within the control limits.

Control limits for gamma distribution with shape parameter 2 is given by

$\left[M_1^*\overline{R}, M_2^*\overline{R}\right]$ for n=5, $\overline{R}$ = 52.67, M_1^* = 0.26034,

M_2^* = 1.06800

$\left[M_1^*\overline{R}, M_2^*\overline{R}\right]$ = [13.71211, 56.25156]

From above data six points are outside the limits and the respective control chart based on examples are given below.

Range control Chart based on Rayleigh distribution

Range control Chart based Range values Range control Chart based S.D

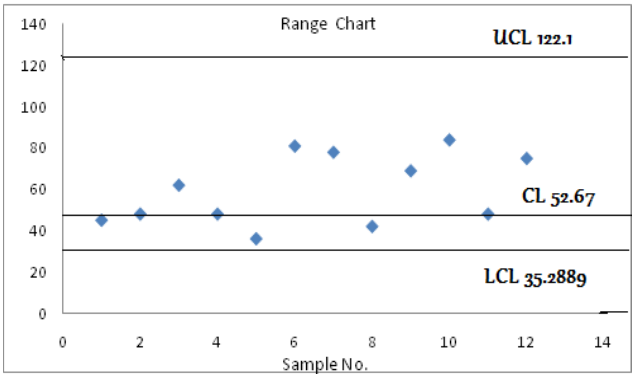

Fig 1.1

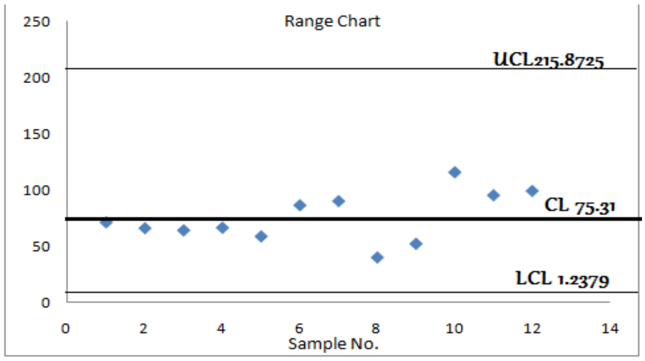

Fig 1.2

Range control Chart based on gamma (2) distribution

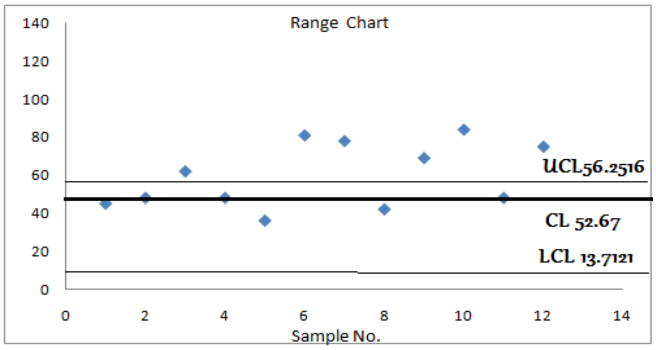

Fig 1.3

Example 2

Consider the following data

Sample No	Sample Observations	Total	Sample Range	Sample S.D (s)
1	14 8 12 12	46	6	11.70
2	11 10 13 8	42	5	10.65
3	11 12 16 13	52	5	13.13
4	15 12 14 11	52	4	13.09
5	10 10 8 8	36	2	9.05
	Total		22	57.62

For Sub-group size 4 the control chart constants from the above table are

From above Rayleigh distribution table at n=4, $\overline{R}$ = 4.4

$\left[D_2^*\overline{R}, D_3^*\overline{R}\right]$, D_2^* = 0.709, D_3^* = 2.059

$\left[D_2^*\overline{R}, D_3^*\overline{R}\right]$ = [3.1196, 9.0596]

The Control limits for Rayleigh distribution are $\left[\sqrt{L^*}\,\overline{s}, \sqrt{U^*}\,\overline{s}\right]$

$s = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n}}$, $\bar{s} = \frac{\sum s}{k}$, k is the Sample number

$\sqrt{L^*} = 0.02599$, $\sqrt{U^*} = 3.99836$

From above Rayleigh distribution table at n=4, $\bar{s} = 11.524$

$\left[\sqrt{L^*}\,\bar{s}, \sqrt{U^*}\,\bar{s}\right] = [0.29951, 46.0771]$

All the sample points within the control limits.

Control limits for gamma distribution with shape parameter 2 is given by

$\left[M_1^*\bar{R}, M_2^*\bar{R}\right]$ for n=4, $\bar{R} = 4.4$, $M_1^* = 0.38135$, $M_2^* = 1.3025$

$\left[M_1^*\bar{R}, M_2^*\bar{R}\right] = [1.67794, 5.734]$

From the above considered example one sample point is outside the control limits and the respective control charts based examples are given below.

Range control Chart based on Rayleigh distribution

Range control Chart based Range values Range control Chart based S.D

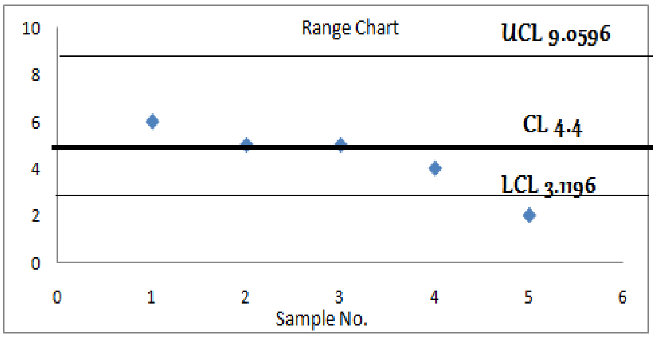

Fig 2.1

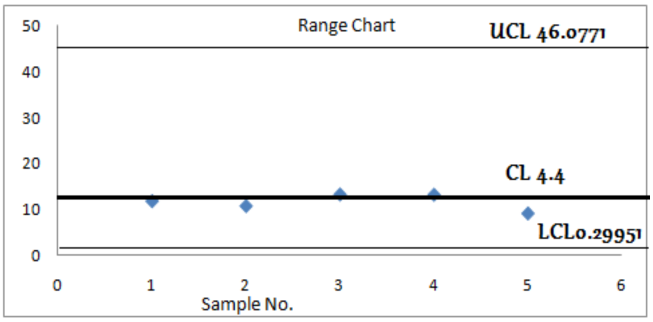

Fig 2.2

Range control Chart based on gamma (2) distribution

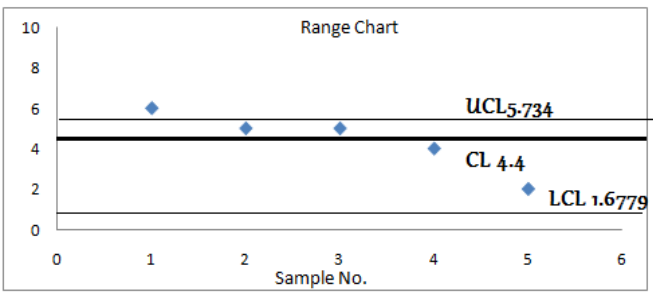

Fig 2.3

Conclusions

Since the whole computation is carried out with examples. The conclusions from the above calculations about preferability of one over the other cannot generalize. However, it gives a hint that our control chart limits are preferable.

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