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The theory of inference for statement calculus in reference of composition mappings and relation matrix

Jayesh Tiwari and Rajendra Tiwari

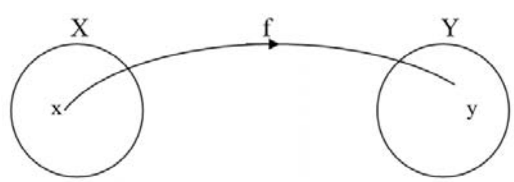
Abstract
In this article we prove that set of Complex numbers C= a+ib ; where a,b ϵ R and i= imaginary number also know as by iota (i= √-1), is the set of largest number, while set of Natural numbers N={1,2,3….n……} is the set of least numbers by compositions of mappings or product of mappings. We verify the inclusion relation N ⊂ W ⊂ I ⊂ Q ⊂ R ⊂ C or C⊃R ⊃ Q ⊃ I ⊃ W ⊃ N by relation matrix and graph of relation.

Keywords: Mappings, Composition of mappings Relation, Transitive relation, Relation matrix, Graph of a relation, Complex numbers, and Natural numbers.

Introduction

We know function is a particular class of relations we are primarily concern with discrete functions which transform a finite set into another finite set. There is several such transformations involved in the computer implementation of any program. Computer output can be considered as a function of the input.

Let X and Y be any two sets. A relation f from X to Y is called a function if for every x ϵ X there is a unique y ϵ Y such that (x ,y) ϵ f. Note that the definition of function requires that a relation must satisfy two additional condition in order to qualify as a function. The first condition is that every x ϵ X must be related to some to some y ϵ Y, that is domain of f must be X and not merely a subset of X. The second requirement of uniqueness can be expressed as (x, y) ϵ f ∧ (x, z) ϵ f ⇒ y= z. Terms such as "transformation" "map" or ("mappings") "correspondence" and "operation" are used as synonyms for "function". The notations f : X→ Y or $X \to^{f} Y$ are used to express f as a function from X to Y. Pictorially a function is generally shown as

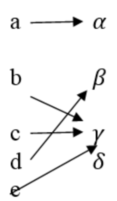


For any function f: X→ Y, if (x, y), then x is called an argument and corresponding y is called the image of x under f. Instead of writing (x, y) ϵ f, it is customary to write y= f(x) and call y the value of the function f at x. Other ways of expressing y=f(x) are f: X→ Y or $X \to^{f} Y$ and of curse (x, y) ϵ f. We sometimes denote the range of f, viz. R_f by f(x). The range of f is defined as

{y: ∃ x ϵ X ∧ y = f(x)}

It was mentioned that the domain of f is X, that is $D_f = X$. The range of f is denoted by R_f and $R_f \subseteq Y$. The set Y is called the co domain off Since a function is a relation, we can use a relation matrix or graph to represent it in some cases. Note that from the definition of a function it follows that every row of its relation matrix must have only one entry which is 1

while all other entries in the row are 0's. Therefore one can replace the relation matrix by a single column i.e. a vector consisting of entries which are images of the arguments. Thus the column consists of entries which show a correspondence between the argument and the function under the argument.
A function can often be represented in graphical form. Figure (a) shows a function R from A= {a, b, c, d, e} to B $=\{\alpha, \beta, \gamma, \delta\}$ following the conventional of representing A binary relation in tabular form in table (b) & (c) where left column contains all the elements in the domain and the right column contains their corresponding images.



(a)

	α	β	γ	δ
a	√			
b			√	
c			√	
d		√		
e			√	

(b)

	R
a	α
b	γ
c	γ
d	β
e	γ

(c)

Materials and Methods

Definition 1.1: One-One Mapping: A mapping f of X into Y is said to be one-one if distinct elements of X have distinct images in Y. Thus f: X→ Y is one-one mapping if whenever $x_1 \neq x_2$ then $f(x_1) \neq f(x_2)$, where x_1, x_2 ϵ X. Otherwise if $f(x_1)=f(x_2) \rightarrow x_1=x_2$. One –one mapping also know as by injective mapping.

Definition 1.2: If the mappings f : X→ Y is such that two different elements of X have the same f- image in Y i.e. $f(x_1)=f(x_2)$ even if $x_1 \neq x_2$ then the mapping is called many one mapping.

Definition 1.3: If the mapping f: X→ Y is such that there is at least one element of Y which is not the f-image of any element of X, then we say f is a mapping of X into Y. In such a mapping f- image of X is a proper subset of Y i.e. f(x) ⊂ Y; x as into is called many one into mapping.
ϵ X.

Definition 1.4: If the mapping f: X→ Y is such that every element of Y is the f-image of at least one element of X, then the mapping is called an onto mapping or surjective mapping. Since f-image of X is equal Y i.e. f(X) =Y. Thus in case of an onto mapping, the range of f is equal to the co-domain.

Definition 1.5: Any mapping which is an one-one as well into is called one-one into mapping.

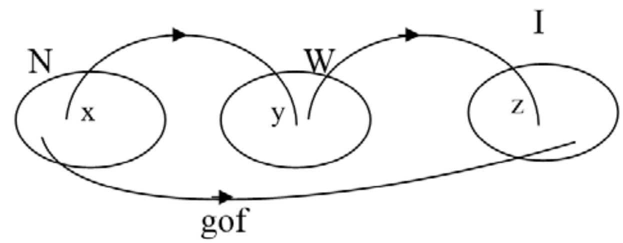
Definition 1.6: A mapping which is one-one as well as onto is called one–one onto mapping it is also called bijective mapping.

Definition 1.7: A mapping which is many one as well as into is called many one into mapping.

Definition 1.8: A mapping which is many one as well as onto mapping is called many-one onto mapping.

Definition 1.9: If X be a set and f : *X*→ *X* is defined by f(x)=x, that is f assign to each element to each element in X the element itself, then f is called the identity mapping on X. It is denoted by I_A. Thus I_A: X→X is given by $I_A(x)$= x ∀ x ϵ X.

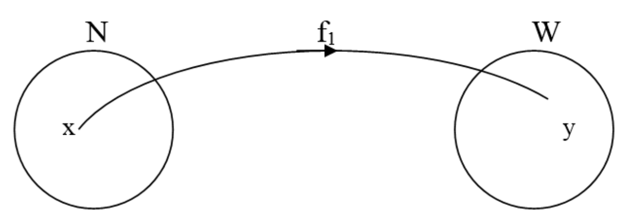
Definition 2.0: Let X, Y, Z be three non-empty sets. Consider tow mappings f: *X*→ Y and g: *Y*→ Z, and consider two mappings f : *X*→ Y defined by y=f(x) where x ϵ X, y ϵ Y and g : *Y*→ Z defined by z=g(y) where y ϵ Y and z ϵ Z. The composition of mappings f and g (in this order) is the composite mapping denoted by gof and is a function f : *X*→ Z denoted by gof : *X*→ Z where (gof)(x)= g{f(x)}, ∀ x ϵ X.



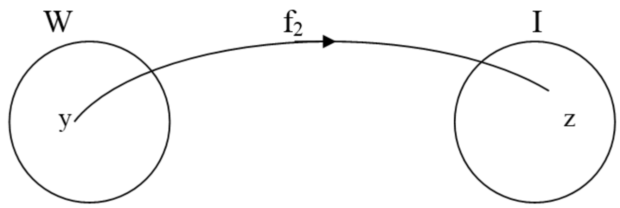
Results and Discussions

(i) Here we shall prove that set of natural no's N = {1,2,3,4…………+ ∞} is the least set of numberss while complex number C= a+ib where a,b ϵ R and i^2=-1 i.e. i= √-1 is called largest set of numbers, by composition of mappings or product of mappings.

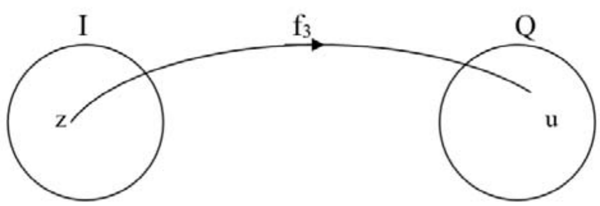
We shall adopt the following notations and derive following identity.
N = The set of Natural numbers ={ 1,2,3……..n….}.
W= The set of Whole numbers ={0,1,2,3…..n…….}
I = The set of Integers = {……-3,-2,-1,0,1,2,3……..}.
Q= The set of Rational numbers.
I_r = The set of Irrational numbers.
R= The set of Real numbers.
C = The set of Complex numbers.
Clearly we observed the following identity relation.
N ⊂ W ⊂ I ⊂ Q ⊂ R ⊂ C or C⊃R ⊃ Q ⊃ I ⊃ W ⊃N.
Here we take two sets N(set of natural no's) and W (set of whole no's) and we defined a mapping f_1: N→W by a rule $f_1(x)=x$. We observed N and W are two non-empty sets and a mapping f from N into W by a rule $f_1(x)=x$ each element x of N exactly one element y of W, means for every value of N there exist unique image in whole number i.e. f_1: N→ W, ∀ x ϵ N $f_1(x)=x$



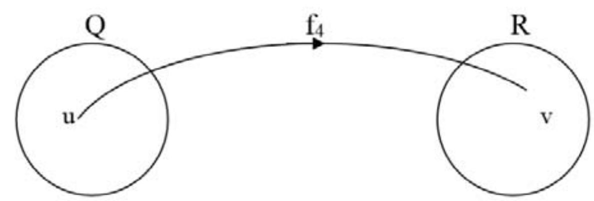
Now we take two sets W(set of whole no's) and I(set of integers) and we defined a mapping f_2: W→ I are two non-empty sets and a mapping f_2 from W into I by a rule f(x)=x each element y of W exactly one element z of I. Means for every value of W there exist unique image in integer I. Hence f_2 is mapping from W into I; ∀ y ϵ W, $f_2(y)=y$



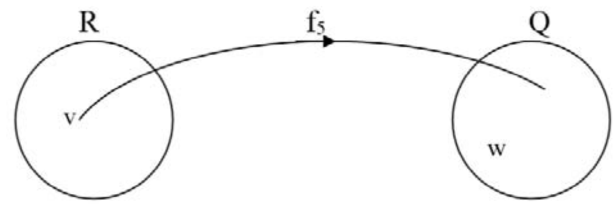
Now we take two sets I(set of Integer no's) and Q(set of rational no's) and we defined a mapping f_3: I→ Q are two non-empty sets and a mapping f_3 from I into Q by a rule $f_3(z)=z$ each element z of W exactly one element u of I. Means for every value of I there exist unique image in set of rational no's Q. Hence f_3 is mapping from I into Q; ∀ z ϵ I, $f_3(z)=z$



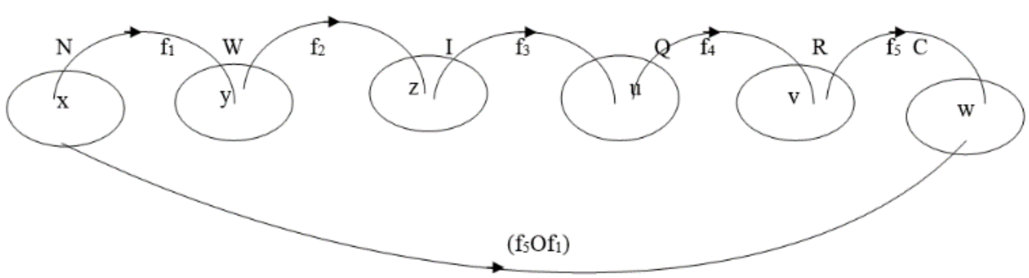
Now we take two sets Q(set of rational no's) and R(set of real no's) and we defined a mapping f_4: Q→ R are two non-empty sets and a mapping f_4 from Q into R by a rule f(u)=u each element u of W exactly one element v of R. Means for every value of Q there exist unique image in set of Real no's R. Hence f4 is mapping from Q into R; ∀ u ϵ Q, $f_4(u)=u$



Finally we take two sets R(set of real no's) and C(set of complex no's) and we defined a mapping f_5: R→ C are two non-empty sets and a mapping f_5 from R into C by a rule $f_5(v)=v$ each element v of R exactly one element w of C. Means for every value of R there exist unique image in set of complex no's C. Hence f_5 is mapping from R into C; ∀ vϵ R, $f_5(v)=v$



Clearly we observed that N,W,I,Q,R,C all are six non-empty set and f_1 is a mapping N into W, f_2 is a mapping from W into I, f_3 is mapping from I into Q,F_4 is mapping from Q into R and f_5 is a mapping from R into C. Now we shall obtained composition of all these mappings f_1, f_2, f_3, f_4, f_5.



∀ x ϵ N $y = f_1(x)$

⇒ $z = f_2\{f_1(x)\}$
⇒ $u = f_3\{f_2\{f_1(x)\}\}$
⇒ $v = f_4\{f_3\{f_2\{f_1(x)\}\}\}$
⇒ $w = f_5\{f_4\{f_3\{f_2\{f_1(x)\}\}\}\}$
⇒ $w = f_5\{f_4\{f_3\{f_2\ (y)\}\}\}$
⇒ $w = f_5\{f_4\{f_3\{z\}\}\}$
⇒ $w = f_5\{f_4\{f_3(z)\}\}$
⇒ $w = f_5\{f_4\{u\}\}$
⇒ $w = f_5\{f_4\{u\}\}$
⇒ $w = f_5\{f_4(u)\}$
⇒ $w = f_5\{v\}$
⇒ $w = f_5(v)$

Now we shall find out compositions of all these mappings f_1 ,f_2, f_3 ,f_4 f_5.

⇒ $[f_5of_4of_3of_2of_1](x)$ where x ϵ N

⇒ $\{f_5of_4of_3of_2\}f_1(x)$ {Definition of composition of mappings}

⇒ $\{f_5of_4of_3of_2\}y$ where y ϵ W

⇒ $\{f_5of_4of_3\}$f2(y) {Definition of composition of mappings}

⇒ $\{f_5of_4of_3\}z$ where z ϵ I

⇒ $\{f_5of_4\}$f3(z) {Definition of composition of mappings}

⇒ $\{f_5of_4\}(u)$ where u ϵ Q

⇒ $\{f_5\ \}$f4(u) {Definition of composition of mappings}

⇒ $\{\ f_5\ \}v$ where v ϵ R

⇒ $f_5(v)$ {Definition of composition of mappings}

⇒ w where w ϵ C

We have shown that every natural numbers is mapped on set of complex numbers according to our rule defined by f(x)=x. Hence for every natural number N there exist unique image in the set of complex numbers. Therefore set of natural numbers is the least set of numbers while set of complex numbers is the largest set of numbers.

Now we shall prove with the help of relation matrix and graph of a relation the set of natural no's is the least set of no's while set of complex no's is the largest set of no's. We know that a relation matrix R from a finite set X into finite set Y can also represented by a matrix called the relation matrix of R.

Let $X = \{x_1, x_2, x_3 \ldots\ldots x_m\}$ and $Y = \{y_1, y_2. y_3 \ldots\ldots. y_n\}$ and R be a relation from X into Y. The relation matrix of R can be obtained by first constructing a table whose columns are preceded by column consisting of successive elements X and whose rows are headed by a rows consisting of successive elements of Y. If $x_i R y_j$, then we enter a 1 in the i^{th} row and j^{th} column. If $x_k R x_l$, then we enter a zero in the k^{th} row and l^{th} column. If we assume that the elements of X and Y appear in a certain order, then the relation R can be represented by a matrix whose elements are 1's and 0's. This matrix can be written down from the table constructed or can be defined in the following manner.

$$r_{ij} = \begin{cases} 1 & \text{if } x_i R y_j \\ 0 & \text{if } x_i R y_j \end{cases}$$

Where r_{ij} is the elements in the i^{th} row and j^{th} column. The matrix obtained in this way is called the relation matrix. If X has m elements and Y has n elements then the relation matrix is an m×n matrix. Here we shall show that $N \subset W \subset I \subset Q \subset R \subset C$. If R is the relation of proper inclusion set of N, W, I, Q, R, C then the relation matrix can be obtained as.

⊂	N	W	I	Q	R	C
N	0	1	1	1	1	1
W	0	0	1	1	1	1
I	0	0	0	1	1	1
Q	0	0	0	0	1	1
R	0	0	0	0	0	1
C	0	0	0	0	0	0

From relation matrix we get the following information's

Since $a_{12} \in R$; $a_{23} \in R \Rightarrow a_{13} \in R$. Since $(a,b) \in R$; $(b,c) \in R \Rightarrow (a,c) \in R$.

Therefore relation is a transitive. Hence Natural no's included in Integers.

Again $a_{13} \in R$; $a_{34} \in R \Rightarrow a_{14} \in R$. Since $(a,b) \in R$; $(b,c) \in R \Rightarrow (a,c) \in R$.

Therefore relation is a transitive. Hence Natural no's included in rational no's.

Again $a_{14} \in R$; $a_{45} \in R \Rightarrow a_{15} \in R$. Since $(a,b) \in R$; $(b,c) \in R \Rightarrow (a,c) \in R$.

Therefore relation is a transitive. Hence Natural no's included in Real no's.

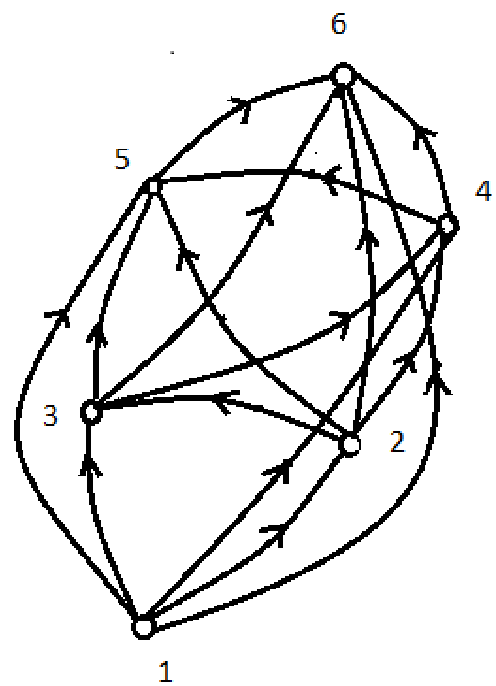
Again $a_{15} \in R$; $a_{56} \in R \Rightarrow a_{16} \in R$. Since $(a,b) \in R$; $(b,c) \in R \Rightarrow (a,c) \in R$.

Therefore relation is a transitive. Hence Natural no's included in Complex no's.

Here we shall see some more transitive relations.

$a_{23} \in R$; $a_{34} \in R \Rightarrow a_{24} \in R$
$a_{24} \in R$; $a_{45} \in R \Rightarrow a_{25} \in R$
$a_{25} \in R$; $a_{56} \in R \Rightarrow a_{26} \in R$
$a_{34} \in R$; $a_{45} \in R \Rightarrow a_{35} \in R$
$a_{35} \in R$; $a_{56} \in R \Rightarrow a_{36} \in R$
$a_{45} \in R$; $a_{56} \in R \Rightarrow a_{46} \in R$

If we combine all the results we obtained $N \subset W \subset I \subset Q \subset R \subset C$. Hence natural no's are the least set of the numbers while complex no's are the largest set of the numbers. Now we shall show transitive relations through graphs.



Figure

Conclusion

Hence from both the proof we can conclude that set of Complex number is the largest set of numbers and set of Natural numbers is the least set of the number.

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