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Jayasree G
Department of Statistics,
University College of Science,
Osmania University, Hyderabad,
Telangana, India

Bhatra Charyulu N.Ch.
Department of Statistics,
University College of Science,
Osmania University, Hyderabad,
Telangana, India

Correspondence
Jayasree G
Department of Statistics,
University College of Science,
Osmania University, Hyderabad,
Telangana, India

Properties of new derived power series distribution

Jayasree G, Bhatra Charyulu N.Ch.

Abstract
Jayasree and Bhatra charyulu (2017) [2] provided a new derived power series using the power series distributions of Kulasekera and Tonkyn (1992) [5] and Geometric distribution. In this paper, the new derived power series distribution moment estimators and maximum likelihood estimators are obtained and also shown that it belongs to the two-parameter exponential family of distributions.

Keywords: Derived Power Series Distribution (DPSD), JS-NB Distribution, Maximum likelihood estimator

1. Introduction

Suppose that $\underline{a} = (a_0, a_1, a_2, \ldots)$ is a sequence of nonnegative real numbers. The partial sum of order $n \in N$ is $g_n(\theta) = \Sigma\, a_k \theta^k$, k=0, 1 … n for all $\theta \in R$. The power series is then defined by $g(\theta) = \lim_{n\to\infty} g_n(\theta)$ for $\theta \in R$ for which the limit exists, and is denoted $g_n(\theta) = \Sigma a_k\theta^k$, k = 0,1 … $\forall\ \theta \in R$. A random variable n with values in N has the power series distribution associated with the function g (or equivalently with the sequence a), with parameter $\theta \in [\,0, r\,)$, if N has discrete probability density function $f_\theta(n) = a_n\theta^n / g(\theta)$, $n \in N$.

Let $P_1(s)$ and $P_2(s)$; $|\, s \,| \le 1$ are the probability generating functions of two power series distributions. Jayasree and Swamy (2006) [2] defined a family of new power series distributions with the convolution of $P_1(s)$ and $[P_2(s)]^{-1}$ called Derived Power Series Distributions (DPSD). Consider a DPSD for which the probability mass function is given by

$$P[X=x] = [\{a_0 g(\theta_2)\}/\ \{b_0 f(\theta_1)\}]d_x \ ;\ x = 1,2, \ldots \ldots \qquad (1.1)$$

where $d_x = a'_x - \sum_{j=1}^{x} b'_j\, d_x$; x =1, 2, … with $a'_x = [a_x(\theta_1)^x] / a_0$ and $b'_j = [b_j\,(\theta_2)^j]/b_0$; $\theta_1, \theta_2 > 0$

The mean and variance of derived power series distributions are $\mu = \mu_1 - \mu_2$ and $\sigma^2 = \sigma_1^2 - \sigma_2^2$ where μ_1, σ_1^2 are the mean and variance of $P_1(s)$ and μ_2, σ_2^2 are the mean and variance of $P_2(s)$ power series distributions. A power series distribution derived by Kulasekera and Tonkyn (1992) [5] is presented in definition 1.1.

Definition 1.1: A random variable X is said be Kulasekera and Tonkyn probability distribution if it satisfies the probability law $P\,[X = x] = x^\alpha q^x / (\Sigma\, x^\alpha q^x)$; $x = 1, 2, \ldots$; $\infty < \alpha < \infty$; $0 < q < 1$; p+q =1. For fixed value of α, the distribution belongs to the family of power series distribution. The mean and variance of the distribution are $(1+q)\,p^{-1}$ and $2qp^{-2}$.

A power series distribution derived by Jayasree and Bhatra Charyulu (2017) [2] were presented is like as follows. Let X_1 be the random variable follows a power series probability distributions of Kulasekera and Tonkyn (1992) [5], and X_2 be the random variable follows Geometric distribution. Let $P_1(s)$ and $P_2(s)$ are the probability generating functions of the two power series distributions in s and convergent for $|\, s \,| \le 1$,

$$P_1(s) = b^{-1} \sum_{x=1}^{\infty} x q_1^x\, s^x \text{ and } P_2\,(s) = p_2 \sum_{y=1}^{\infty} q_2^{y-1} s^y \ ;\ |\, s \,| \le 1 \qquad (1.2)$$

Where $P[X_1 = x_1] = x^{\alpha}q_1^{x} / (\sum_{x=1}^{\infty} x^{\alpha} q_1^{x})$; and $P[X_2= x_2] = p_2 q_2^{x2-1}$; $x_2 = 1, 2, \ldots$ The convolution of $P_1(s)$ and $[P_2(s)]^{-1}$ provides a power series distribution with random variable X satisfies the probability law

$P[X=x] = q_1(bp_2)^{-1} [(x+1)(q_1)^{x} - x(q_1)^{x-1}q_2]$; (1.3)

$0 < p_1 < p_2 < 1$, such that $p_1 + q_1 = 1$ and $p_2 + q_2 = 1$.

Under appropriate conditions for the coefficients of the power series P(s) to be non-negative, one can consider this as the probability generating function of a random variable, where from, the p.m.f. can be obtained by identifying the coefficients in P(s). The P(s) is a proper Power series probability distribution for specified vector of probabilities with The mean $(1+q_1) p_1^{-1} - p_2^{-1}$ and variance $\sigma^2 = 2q_1p_1^{-2} - q_2p_2^{-2}$.

2. Properties of the Power Series Distribution

Some properties of new derived power series distribution of Jayasree and Bhatra Charyulu (2017) [2] are studied and presented below.

Theorem 2.1: If $u_1(u_1-1)^{-1} > c^2$ then the moment estimators of p_1 and p_2 are given by

$\hat{p}_1 = 2 [u_1 + u_2\{-3 u_1 + u_3\}^{-1}]$ (2.1)

and $\hat{p}_2 = u_2^{-1}\{-3u_1+u_3\}$ (2.2)

where $u_1= \mu+1$, $u_2 = 2(u_1^2 - 2u_1 - 2\sigma^2)$, $u_3= (13u_1^2 - 8u_1 - 8\sigma^2)^{1/2}$, and $c = \sigma\mu^{-1}$ which is the coefficient of variation of the distribution .

Proof: The moment equations for the estimators are given by

$\mu = 2p_1^{-1}-1- p_2^{-1}$ (2.3)

$\sigma^2 = p_2^{-1}(1- p_2^{-1}) -2 p_1^{-1}(1- p_1^{-1})$ (2.4)

The solution of these equations yields $\hat{p}_2 = u_2^{-1}\{-3u_1+u_3\}$. Substituting $\hat{p}_2$ in (2.3), one can obtain (2.4).

Further $\hat{p}_2$ is real if $13u_1^2 - 8u_1 - 8\sigma^2 > 0$. Equivalently, if $\mu + \mu^2 > \sigma^2$ .In terms of u_1 and c, the moment estimators are given by (2.3) and (2.4) if $u_1(u_1-1)^{-1} > c^2$.

Theorem 2.2: The derived power series distribution belongs to the two-parameter exponential family of distributions, if the k^{th} power of $\omega(x; q_1, q_2)$ is negligible for $k \geq 3$, where $\omega(x; q_1, q_2) = |x^{-1} - q_1^{-1} q_2|$.

Proof: The probability mass function of the new Derived Power series distribution specified in (1.1) can be expressed as

$P_x = \exp[x \log_e q_1 + \log_e x + \log_e \{(1-q_1)^2 (1-q_2)^{-1}\} + \log_e\{1+(x^{-1}-q_2q_1^{-1})\}]$, $x=1,2,\ldots$ (2.5)

When the k th power of $\omega(x; q_1, q_2) = |x^{-1} - q_1^{-1} q_2|$ is negligible for $k \geq 3$, considering $\log_e\{1+(x^{-1}-q_2q_1^{-1})\}$, one has

$P_x = \exp[x \log_e q_1 + \log_e x + q_2 (xq_1)^{-1} + x^{-1} - (2x^2)^{-1} + \lambda(q_1,q_2)]$ (2.6)

where $\lambda(q_1,q_2) = \log_e \{(1-q_1)^2 (1-q_2)^{-1}\} - q_2 q_1^{-1} - q_2^2(2q_1^2)^{-1}$

Let $\theta_1 = \log_e q_1$; $\theta_2 = q_2q_1^{-1}$, so that $q_1 = e^{\theta 1}$ and $q_2 = \theta_2 e^{\theta 1}$ then by the re-parameterization, one has,

$P_x = (1- \theta_2 e^{\theta 1})^{-1}x(1- e^{\theta 1})^2 \exp[\theta_1 x + \theta_2 x^{-1} + x^{-1} - (2x^2)^{-1} - \theta_2 - 2^{-1} \theta_2^2]$; $x=1,2,\ldots$ (2.7)

From (2.7), we can obtain maximum likelihood estimators for the parameters θ_1 and θ_2.

Theorem 2.3: If the $\max_{1\leq r\leq n} \omega^k(x_r;q_1,q_2)$ is negligible for $k \geq 3$, the maximum likelihood equation estimators of q_1 and q_2 are given by the solutions of the equations

$A=[e^{\theta 1} \{(1-\theta_2) + (1-\theta_2 e^{\theta 1})\}] [(1-e^{\theta 1}) (1-\theta_2 e^{\theta 1})]^{-1}$ and $H^{-1}= (1+ \theta_2) - \{e^{\theta 1}(1- \theta_2 e^{\theta 1})^{-1}\}$ (2.8)

Proof From (1.1) the likelihood based on the sample $\mathbf{x} = (x_1, x_2,\ldots,x_n)$ is given by

$$L(x; \theta_1, \theta_2) = \exp [\theta_1 \sum_{i=1}^{n} x_i + \theta_2 \sum_{i=1}^{n} x_i^{-1} + \sum_{i=1}^{n} H(x_i) + n \lambda(\theta_1, \theta_2)$$

where $H(x) = \log x + x^{-1} - (2x^2)^{-1}$ and $\lambda(\theta_1, \theta_2) = 2\log(1- e^{\Theta 1}) - \log(1- \theta_2 e^{\theta 1}) - \theta_2 - (\theta_2^2)/2$

Thus, the maximum likelihood equations are given by

$A = [e^{\theta 1} \{(1- \theta_2) + (1- \theta_2 e^{\theta 1})\}] [(1- e^{\theta 1}) (1- \theta_2 e^{\theta 1})]^{-1}$

and $H^{-1}= (1+ \theta_2) - \{e^{\theta 1} (1- \theta_2 e^{\theta 1})^{-1}\}$

where, A is the arithmetic mean and H is the Harmonic mean of the sample. Since the solution of these equations seem to be difficult, one can adopt an iterative procedure like the Newton-Raphson, to obtain the same.

Example 2.1: Consider distribution of spiders among patches of habitat or prey data presented in table 2.1. The fitting of derived distribution is presented below.

Table 2.1: Frequency Distribution

x	0	1	2	3	4	5
$f(x)$	85	53	30	16	8	8

When a distribution is fit, using the method of moments for estimating the parameters one can obtain the estimates of $\hat{p}_1 = 0.6277$, $\hat{p}_2 = 0.9792$, $\hat{q}_1 = 0.3723$ and $\hat{q}_2 = 0.0208$ and the expected frequencies are obtained as 0.1975, 0.4310, 0.125, 0, 0.1428 and 0.6666, so that the observed value of χ^2 is given by $\chi^2 = 1.5629$.
Note: It can be noted that the derived power series distribution of Swamy and Jayasree (2006) SJS-4 is also an equally good fit to the same data.

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