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On duplication of near Skolem difference mean graph P_n

S Shenbaga Devi and A Nagarajan

Abstract
Let G be a (p,q) graph and $f: V(G) \rightarrow \{1,2,\dots,p+q-1,p+q+2\}$ be an injection. For each edge $e = uv$, the induced edge labeling f^* is defined as follows:

$$f^*(\mathrm{e}) = \begin{cases} \dfrac{|f(u)-f(v)|}{2} & if\ |f(u)-f(v)|\ is\ even \\ \dfrac{|f(u)-f(v)|+1}{2} & if\ |f(u)-f(v)|\ is\ odd \end{cases}$$

Then f is called Near Skolem difference mean labeling if $f^*(e)$ are all distinct and from $\{1,2,3,\dots.q\}$. A graph that admits a Near Skolem difference mean labeling is called a Near Skolem difference mean graph. In this paper, we show that the graph obtained by duplicating a vertex or an edge in a Near Skolem Difference Mean graph P_n is also a Near skolem difference mean graph.

Keywords: Path, Duplication, Near Skolem Difference Mean labeling, Near Skolem difference mean graphs.

1. Introduction

All graphs considered in this paper are finite, undirected and simple. The vertex set and the edge set of a graph G are denoted by $V(G)$ and $E(G)$ respectively. Terms and notations not defined here are used in the sense of Harary [1].

A graph labeling is an assignment of integers to the vertices or edges or both vertices and edges subject to certain conditions. A detailed survey of various types of labeling is found in [2]. The notion of skolem difference mean labeling was due to Murugan and Subramanian [3]. It motivates us to define near skolem difference mean labeling.

In this paper, we show that the graph obtained by duplicating a vertex or an edge or a vertex by an edge or an edge by a vertex in a Near Skolem Difference mean graph P_n is again a Near Skolem Difference Mean graph. We use the following definitions in the present study.

Definition 1.1: Let v be a vertex of a graph G. Then the duplication of v is a graph $G(v)$ obtained from G by adding a new vertex v' with $N(v') = N(v)$.

Definition 1.2: Let $e = uv$ be an edge of G. Then duplication of an edge $e = uv$ is a graph $G(uv)$ obtained from G by adding a new edge $u'v'$ such that $N(u') = N(u) \cup \{v'\} - \{v\}$ and $N(v') = N(v) \cup \{u'\} - \{u\}$.

Definition 1.3: Duplication of a vertex v_k by a new edge $e' = u'v'$ in a graph G produces a new graph $G'\ such\ that\ N(v') = \{v_k, u'\}\ and\ N(u') = \{v_k, v'\}$.

Definition 1.4: Duplication of an edge $e = uv$ by a new vertex v' in a graph G produces a new graph G' such that $N(v') = \{u, v\}$.

Result 1.5: The path P_n is Near Skolem Difference Mean graph for every $n \geq 2$. [8]

2. Main Result

Definition 2.1: A graph $G = (V, E)$ with p vertices and q edges is said to have Near skolem difference mean labeling if it is possible to label the vertices $x \in V$ with distinct elements $f(x)$ from $\{1,2, \ldots\ldots, p+q-1, p+q+2\}$ in such a way that each edge e = uv , is labeled as $f^*(e) = \frac{|f(u)-f(v)|}{2}$ if $|f(u) - f(v)|$ is even and $f^*(e) = \frac{|f(u)-f(v)|+1}{2}$ if $|f(u) - f(v)|$ is odd. The resulting labels of the edges are distinct and from $\{1, 2, \ldots\ldots, q\}$. A graph that admits a Near skolem difference mean labeling is called a Near Skolem Difference Mean Graph.

Theorem 2.2: The graph obtained by duplicating an arbitrary vertex of P_n is Near Skolem Difference Mean.

Proof: Let P_n be the path.
Let v be the new vertex which is adjacent to both v_{i-1} and v_{i+1}, thus forming a new graph G.
The graph G is obtained by duplicating an arbitrary vertex $v_i, 1 \le i \le n$ of the given path.
Let $V(G) = \{v, v_i / 1 \le i \le n\}$ and

$$E(G) = \begin{cases} vv_{i+1}, v_i v_{i+1} / 1 \le i \le n-1 \; if \; v_i \; is \; a \; pendant \; vertex \\ vv_{i-1}, vv_{i+1}, v_i v_{i+1} / 1 \le i \le n-1 \; if \; v_i \; is \; not \; a \; pendant \; vertex \end{cases}$$

Then $|V(G)| = n+1$ and $|E(G)| = \begin{cases} n, & if \quad v_i \; is \; pendant \; vertex \\ n+1, & if \quad v_i \; is \; not \; pendant \; vertex \end{cases}$

Case (i): When v_i is a pendant vertex

Define $f: V(G) \to \{1,2, \ldots, 2n, 2n+3\}$ as follows:

$$f(v_{2i+1}) = 1 + 2i, \quad \begin{matrix} 0 \le i \le \dfrac{n-1}{2}, when \; n \; is \; odd. \\ 0 \le i \le \dfrac{n-2}{2}, when \; n \; is \; even \end{matrix}$$

$$f(v_{2i}) = 2n - 2i + 2, \quad \begin{matrix} 1 \le i \le \dfrac{n-1}{2}, when \; n \; is \; odd. \\ 1 \le i \le \dfrac{n}{2}, when \; n \; is \; even \end{matrix}$$

$$f(v) = \begin{cases} 2n-1 \; if \; v \; is \; the \; duplicating \; vertex \; of \; v_1 \\ n-2 \; if \; v \; is \; the \; duplicatng \; vertex \; of \; v_n \end{cases}$$

Let f^* be the induced edge labeling of f. Then,
$f^*(v_i v_{i+1}) = n + 1 - i, 1 \le i \le n-1$
$f^*(vv_2) = f^*(vv_{n-1}) = 1.$
The induced edge labels are distinct and are $\{1,2, \ldots, n\}$.

Case (ii): When v_i is not a pendant vertex

Let $f: V(G) \to \{1,2, \ldots, 2n+1, 2n+4\}$ be defined as follows:

$$f(v_{2i+1}) = 2 + 2i, \quad \begin{matrix} 0 \le i \le \frac{n-1}{2} \; if \; n \; is \; odd \\ 0 \le i \le \frac{n-2}{2} \; if \; n \; is \; even \end{matrix}.$$

$$f(v_2) = 2n + 4$$

$$f(v_{2i}) = (2n+1) - 2i + 4, \quad \begin{matrix} 2 \le i \le \frac{n-1}{2}, n \; if \; is \; odd \\ 2 \le i \le \frac{n}{2}, n \; if \; is \; even \end{matrix}.$$

$f(v) = \max\{f(v_{i-1}), f(v_{i+1})\} + 1, i \; is \; even.$
$f(v) = \min\{f(v_{i-1}), f(v_{i+1})\} - 1, i \; is \; odd.$
Let f^* be the induced edge labeling of f. Then,
$f^*(v_i v_{i+1}) = (n+2) - i, 1 \le i \le n-1$
$f^*(vv_{i-1}) = 2$
$f^*(vv_{i+1}) = 1$
In both cases the induced edge labels are distinct and are $\{1,2, \ldots n+1\}$.

Example 2.3: The Near skolem difference mean labeling obtained by duplicating the pendant vertex of P_9 *and* P_{10} are shown in fig 1 and fig 2 respectively.

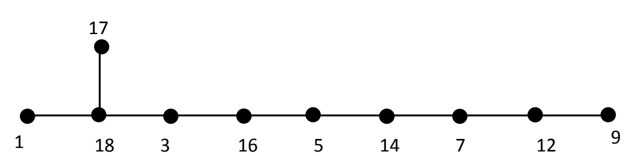

Fig 1

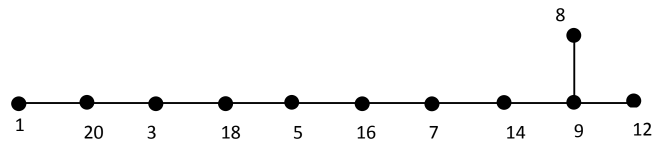

Fig 2

Example 2.4: The Near skolem difference mean labeling obtained by duplicating a non-pendant even vertex and odd vertex of P_{11} *and* P_{12} are shown in fig 3 and fig 4 respectively.

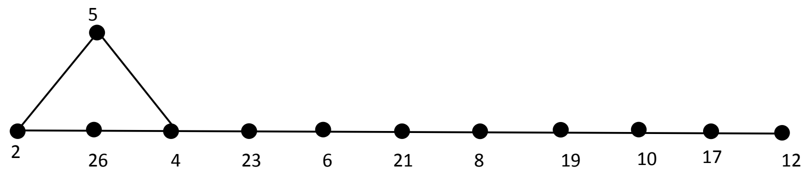

Fig 3

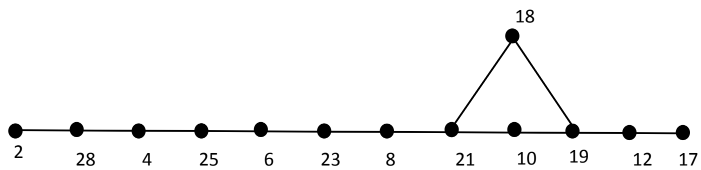

Fig 4

Theorem 2.5: The graph obtained by duplicating an arbitrary edge of P_n is Near skolem difference mean.
Proof: Let P_n be the path.
Let $e = uv$ be the duplicated edge of $v_k v_{k+1}$.
Let G be the resulting graph.
Let $V(G) = \{u, v, v_i / 1 \le i \le n\}$ and
$E(G) = \{uv, v_i v_{i+1} / 1 \le i \le n-1\} \cup \{vv_3$ *or* vv_{n-2} when uv is the duplication of $v_1 v_2$ or $v_{n-1} v_n$ respectively$\}$.
$= \{uv, uv_{k-1}, vv_{k+2}, v_i v_{i+1} / 1 \le i \le n-1\}$ when uv is the duplication of an edge $v_k v_{k+1}$which is not a pendant edge$\}$.

Case (i): When uv is the duplication of a pendant edge $\mathbf{v_1 v_2}$ or $\mathbf{v_{n-1} v_n}$
Then $|V(G)| = n + 2$ and $|E(G)| = n + 1$.
Define $f: V(G) \to \{1, 2, \dots, 2n+2, 2n+5\}$ as follows:

$$f(v_{2i+1}) = 3 + 2i, \begin{matrix} 0 \le i \le \frac{n-1}{2}, n\ odd \\ 0 \le i \le \frac{n-2}{2}, n\ even \end{matrix}.$$

$$f(v_2) = 2n + 5$$

$$f(v_{2i}) = (2n+6) - 2i, \begin{matrix} 2 \le i \le \frac{n-1}{2}, n\ odd \\ 2 \le i \le \frac{n}{2}, n\ even \end{matrix}.$$

$$f(u) = \begin{cases} 1, when\ v_1 v_2 \text{ is the pendant edge} \\ n-4, \text{when } v_{n-1} v_n \text{ is the pendant edge} \end{cases}$$

$$f(v) = \begin{cases} 2, when\ v_1 v_2 \text{ is the pendant edge} \\ n-3, \text{when } v_{n-1} v_n \text{ is the pendant edge} \end{cases}$$

Let f^* be the induced edge labeling of f. Then,
$f^*(v_i v_{i+1}) = (n+2) - i, 1 \le i \le n-1$
$f^*(uv) = 1$
$f^*(vv_3) = 2$, *when* $v_1 v_2$ is the pendant edge
$f^*(vv_{n-2}) = 2$, when $v_{n-1} v_n$ is the pendant edge
The edge labels are all distinct and are $\{1, 2, \dots, n+1\}$.

Case (ii): When uv is the duplication of some non-pendant edge $\mathbf{v_k v_{k+1}}$
Then $|V(G)| = n + 2$ and $|E(G)| = n + 2$
Let $f: V(G) \to \{1, 2, \dots, 2n+3, 3n+6\}$ be defined as follows:

Subcase (i): When k is an even number

$f(v_1) = 2n + 6$

$$f(v_{2i+1}) = (2n+5) - 2i, \quad \begin{matrix} 1 \le i \le \frac{n-1}{2}, n\ odd \\ 1 \le i \le \frac{n-2}{2}, n\ even \end{matrix}.$$

$$f(v_{2i}) = \begin{cases} 2i, & 1 \le i \le \frac{k}{2} \\ 2i+2, & \begin{matrix} \frac{k}{2} \le i \le \frac{n-1}{2}, & n\ odd. \\ \frac{k}{2} \le i \le \frac{n}{2}, n\ even \end{matrix} \end{cases}$$

$f(u) = k$
$f(v) = k + 1$

Subcase (ii): When k is an odd number

$$f(v_{2i+1}) = \begin{cases} 2i+2, & 0 \le i \le \frac{k-1}{2} \\ 2i+4, & \begin{matrix} \frac{k-1}{2} \le i \le \frac{n-1}{2}, n\ odd. \\ \frac{k-1}{2} \le i \le \frac{n}{2}, n\ even \end{matrix} \end{cases}$$

$f(v_2) = 2n + 6$

$$f(v_{2i}) = (2n+7) - 2i, \quad \begin{matrix} 2 \le i \le \frac{n-1}{2}, n\ odd \\ 2 \le i \le \frac{n}{2}, n\ even \end{matrix}.$$

$f(u) = k + 1$
$f(v) = k + 2$
Let f^* be the induced edge labeling of f.

$$f^*(v_i v_{i+1}) = \begin{cases} (n+3) - i, & 1 \le i \le k-2 \\ (n+2) - i, & k-1 \le i \le n-1 \end{cases}.$$

$f^*(uv_{k-1}) = n - k + 4$

$f^*(vv_{k+2}) = 2$

$f^*(uv) = 1$
The edge labels are all distinct and are $\{1, 2, \ldots, n+2\}$.

Example 2.6: The Near skolem difference mean labeling obtained by duplicating the pendant edge v_1v_2 and $v_{n-1}v_n$ of P_{10} *and* P_{11} are shown in fig 5 and fig 6 respectively.

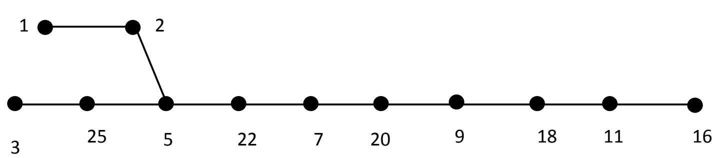

Fig 5

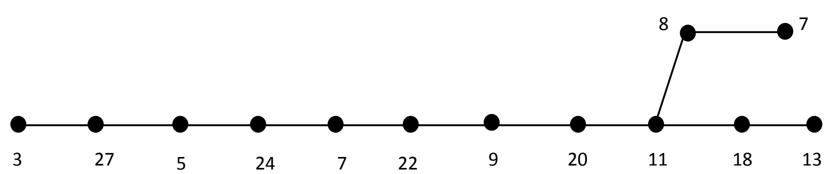

Fig 6

Example 2.7: The Near skolem difference mean labeling obtained by duplicating a non-pendant edge $v_k v_{k+1}$, (where k is an even number) of P_{11} is shown in fig 7.

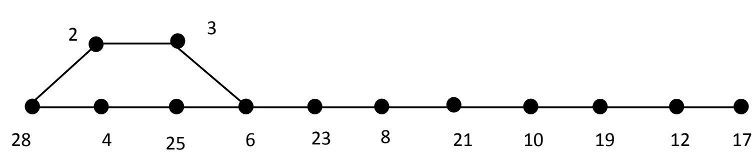

Fig 7

Example 2.8: The Near skolem difference mean labeling obtained by duplicating a non-pendant edge $v_k v_{k+1}$ (where k is an odd number) of P_{11} is shown in fig 8.

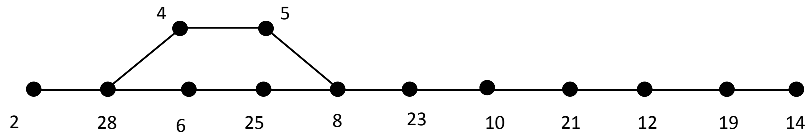

Fig 8

Theorem 2.9: The graph obtained by duplicating a vertex by an edge in P_n is Near skolem difference mean.
Proof: Let P_n be the path.
Let $e = uv$ be the duplicated edge of the vertex v_k.
Let G be the resulting graph.
Let $V(G) = \{u, v, v_i / 1 \le i \le n\}$ and
$E(G) = \{uv, uv_k, vv_k, v_i v_{i+1} / 1 \le i \le n-1\}$.
Then $|V(G)| = n + 2$ and $|E(G)| = n + 2$
Define $f: V(G) \to \{1,2, \dots, 2n+3, 2n+6\}$ as follows:

Case (i): Let k be an odd number

$f(v_1) = 2n + 6$

$$f(v_{2i+1}) = (2n+5) - 2i, \quad \begin{matrix} 1 \le i \le \frac{n-1}{2} \ if\ n\ is\ odd \\ 1 \le i \le \frac{n-2}{2} \ if\ n\ is\ even \end{matrix}$$

$$f(v_{2i}) = 2i, \quad \begin{matrix} 1 \le i \le \frac{n-1}{2} \ if\ n\ is\ odd \\ 1 \le i \le \frac{n}{2} \ if\ n\ is\ even \end{matrix}$$

$$f(u) = \begin{cases} f(v_k) - 6 \ if & f(v_k) = 2n+6 \\ f(v_k) - 5 \ if & f(v_k) \ne 2n+6 \end{cases}.$$

$$f(v) = \begin{cases} f(v_k) - 4 \ if & f(v_k) = 2n+6 \\ f(v_k) - 3 \ if & f(v_k) \ne 2n+6 \end{cases}.$$

Case (ii): Let k an even number

$$f(v_{2i+1}) = 2 + 2i, \quad \begin{matrix} 0 \le i \le \frac{n-1}{2} \ if\ n\ is\ odd \\ 0 \le i \le \frac{n-2}{2} \ if\ n\ is\ even \end{matrix}$$

$f(v_2) = 2n + 6$

$$f(v_{2i}) = (2n+3) - 2i + 4, \quad \begin{matrix} 2 \le i \le \frac{n-1}{2} \ if\ n\ is\ odd \\ 2 \le i \le \frac{n}{2} \ if\ n\ is\ even \end{matrix}$$

$$f(u) = \begin{cases} f(v_k) - 6 \ if & f(v_k) = 2n+6 \\ f(v_k) - 5 \ if & f(v_k) \ne 2n+6 \end{cases}.$$

$$f(v) = \begin{cases} f(v_k) - 4 \ if & f(v_k) = 2n+6 \\ f(v_k) - 3 \ if & f(v_k) \ne 2n+6 \end{cases}.$$

Let f^* be the induced edge labeling of f. Then,
$f^*(uv) = 1$
$f^*(uv_k) = 3$
$f^*(vv_k) = 2$
$f^*(v_i v_{i+1}) = (n+3) - i, 1 \le i \le n-1$
Thus, the induced edge labels are all distinct and are $\{1,2, \dots, n+2\}$.

Example 2.10: The Near skolem difference mean labeling obtained by duplicating a vertex v_k, (where k is an odd number) by an edge in P_9 is shown in fig 9 and fig 10 respectively.

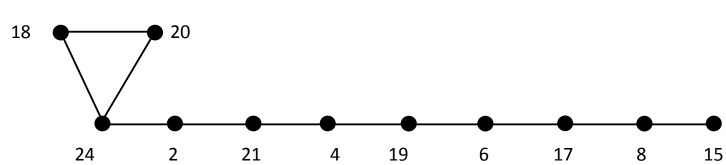

Fig 9

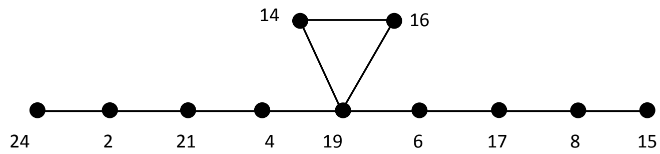

Fig 10

Example 2.11: The Near skolem difference mean labeling obtained by duplicating a vertex v_k (where k is an even number) by an edge in P_{10} is shown in fig 11 and fig 12 respectively.

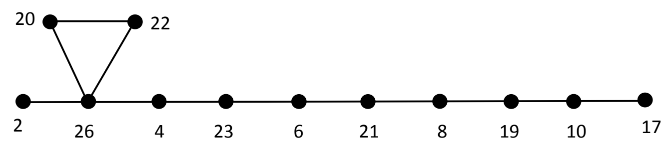

Fig 11

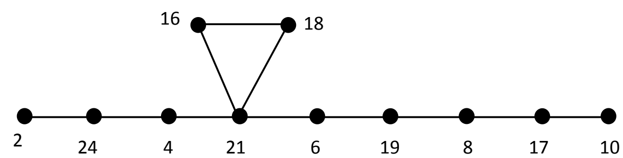

Fig 12

Theorem 2.12: The graph obtained by duplicating an edge by a vertex in P_n is Near skolem difference mean.
Proof: Let P_n be the path.
Let v be the duplicated vertex of the edge $v_k v_{k+1}$.
Let G be the resulting graph.
Let $V(G) = \{v, v_i / 1 \le i \le n\}$ and
$E(G) = \{vv_k, vv_{k+1}, v_i v_{i+1} / 1 \le i \le n-1\}$
Then $|V(G)| = n+1$ and $|E(G)| = n+1$
Define $f: V(G) \rightarrow \{1, 2, \ldots, 2n+1, 2n+4\}$ as follows:

Case (i): Let k be an odd number
Subcase (i):n is odd
$f(v_1) = 2n+4$
$f(v_{2i+1}) = (2n+3) - 2i, 1 \le i \le \frac{n-1}{2}$.
$$f(v_{2i}) = \begin{cases} 2i, & 1 \le i < \frac{k+1}{2} \\ 2i+2, & \frac{k+1}{2} \le i \le \frac{n-1}{2} \end{cases}.$$
$f(v) = k+1$

Subcase (ii):n is even
$f(v_1) = 2n+4$
$f(v_{2i+1}) = (2n+3) - 2i, 1 \le i \le \frac{n-2}{2}$.
$$f(v_{2i}) = \begin{cases} 2i, & 1 \le i < \frac{k+1}{2} \\ 2i+2, & \frac{k+1}{2} \le i \le \frac{n}{2} \end{cases}.$$
$f(v) = k+1$

Case (ii): Let k be an even number
Subcase (i):n is odd
$$f(v_{2i+1}) = \begin{cases} 2i+2, & 0 \le i < \frac{k}{2} \\ 2i+4, & \frac{k}{2} \le i \le \frac{n-1}{2} \end{cases}.$$
$f(v_2) = 2n+4$
$f(v_{2i}) = (2n+5) - 2i, 2 \le i \le \frac{n-1}{2}$.
$f(v) = k+2$

Subcase (ii):n is even

$$f(v_{2i+1}) = \begin{cases} 2i+2, & 0 \le i < \frac{k}{2} \\ 2i+4, & \frac{k}{2} \le i \le \frac{n-2}{2} \end{cases}.$$

$f(v_2) = 2n+4$

$f(v_{2i}) = (2n+5) - 2i, 2 \le i \le \frac{n}{2}.$

$f(v) = k+2$

Let f^* be the induced edge labeling of f in both the subcases. Then,

$f^*(vv_k) = n-k+2$

$f^*(vv_{k+1}) = 1$

$$f^*(v_iv_{i+1}) = \begin{cases} (n+2)-i, & 1 \le i \le k-1 \\ (n+1)-i, & k \le i \le n-1 \end{cases}.$$

Thus, the edge labels are all distinct and are $\{1,2,..,n+1\}$.

Example 2.13: The Near skolem difference mean labeling obtained by duplicating an edge by a vertex v_k, (where k is an odd number) of P_{11} is shown in fig 13 and fig 14 respectively.

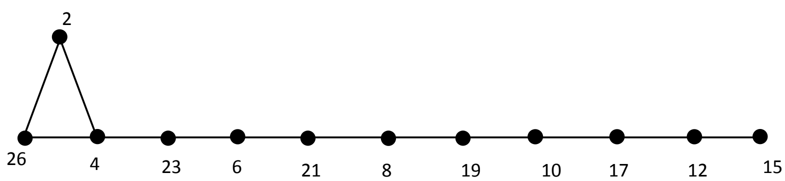

Fig 13

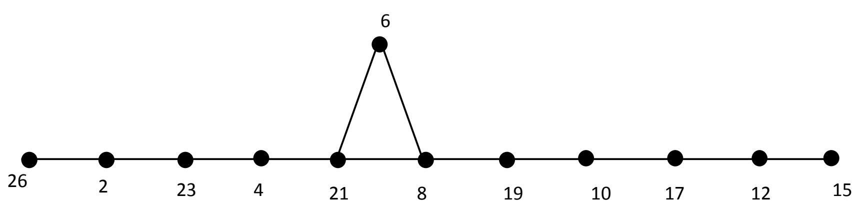

Fig 14

Example 2.14: The Near skolem difference mean labeling obtained by duplicating an edge by a vertex v_k, (where k is an even number) of P_{11} is shown in fig 15 and fig 16 respectively.

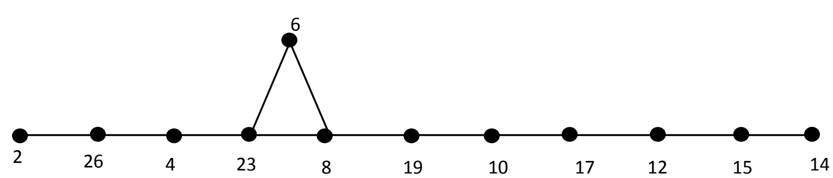

Fig 15

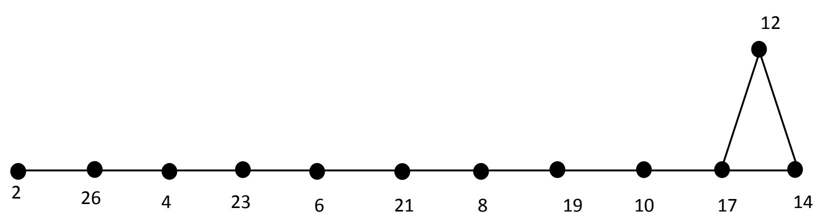

Fig 16

3. Conclusion

We have investigated some results on Near Skolem Difference mean labeling for the graphs resulting from the duplication of the path P_n. We are working to extend the study to other families of graphs in our forth coming papers.

4. References

1. Harary F. Graph Theory, Narosa Publishing House, New Delhi, 2001.
2. Gallian JA. A Dynamic Survey of Graph Labeling, The Electronic Journal of Combinatorics, 2008, 15. #DS6.

3. Murugan K, Subramanian A. Skolem difference mean labelling of H-graphs, International Journal of Mathematics and Soft Computing, 2011; 1(1):115-129.
4. Shenbaga Devi S, Nagarajan A. Near Skolem Difference Mean Labeling of cycle related Graphs, International Journal for Science and Advance Research in Technology, 2017; 3(12):1037-1042
5. Shenbaga Devi. S, Nagarajan. A, Near Skolem Difference Mean labeling of some special types of trees, International Journal of Mathematics Trends and Technology, 2017; 52(7):474-478.
6. Shenbaga Devi S, Nagarajan A. On Changing behavior of vertices of some graphs, International Journal of Scientific Research in Science, Engineering and Technology, 2018; 4(1):400-405.
7. Shenbaga Devi. S, Nagarajan. A, On Near Skolem Difference Mean Graphs (Communicated).
8. Shenbaga Devi S, Nagarajan A. Some Results on Duplication of Near Skolem Difference Mean graph C_n. (Communicated)
9. Shenbaga Devi S, Nagarajan A. Near Skolem Difference Mean Labeling of some Subdivided graphs. (Communicated)
10. Shenbaga Devi S, Nagarajan A. On Changing behavior of edges of some special classes of graphs I (Communicated)