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Profit analysis of two unit poly wrapping system with preventive maintenance and Degradation

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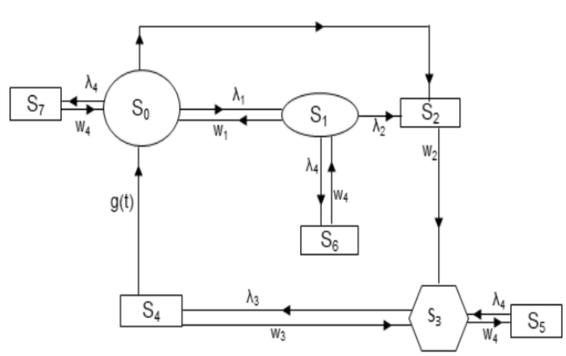
Abstract
Today most of the industries and plants are multi units systems i.e. not single unit containing. Various industries and processing systems are assembly of a number of units. In this paper we study a Polythene packing industry having a system comprising of two units namely Wrapper (A) and Bundler (B) having Preventive Maintenance in unit (A) only before partial failure and which undergoes Degradation on Complete Failure is analyzed. The failure of system collectively depends upon failure of each individual unit. Two units system is widely used in a number of process industries and many more others in which one unit is more important than second one and requires a more attention, so we apply preventive maintenance. If that single unit fails then the whole system fails. Considering the importance of such units in system Gupta, V. K. has discussed behavior with perfect and imperfect switch-over of systems using RPGT. So keeping in view the importance of every single unit in whole of the system, in this paper we have analyzed Two Unit System under Preventive Maintenance in main unit before complete failure and Degradation after Complete Failure. The unit may further deteriorate each time, so ultimately there will be stage when the unit will be beyond repair or it may not be advisable to repair it further due to higher cost maintenance or loss of production. Taking into account all the possibilities and path probabilities a transition diagram of system is traced out. As most of the units follow exponential failure rate, hence failure rate of individual is taken exponential. If the server reports that unit is not repairable then it is replaced by a new one. Fuzzy logic can be applied to determine the unit's working in full capacity or reduced capacity.

Keywords: Wrapper, Bundler, RPGT, Mean time to system failure, steady state availability, Profit optimization, Fuzzy Logic, Preventive maintenance

Introduction

Assumptions: - Here we have discussed the system for long run means for time taken is infinite. Preventive Maintenance is available for main unit Wrapper (A) only not to other unit. And switching of units is perfect. After complete failure replacement facility is immediate. There is single server facility. Repair of unit A is imperfect and repaired unit is not good as new one on complete failure i.e. system goes in degraded state after first complete failure.

Considering the various possibilities and assumptions the transition diagram of the system is drawn as under



Description of Model

Initially the two units A and B are working in full capacity initially in state S_0. The system can be in any of the following states with respect to the above symbols. States S_0, S_1, S_3 and S_4 are regenerative states.

S_0=AB, $S_1=\bar{A}$B, S_2=aB, $S_3=\bar{A}_1$B, $S_4=a_1$B, $S_5=\bar{A}_1$b, $S_6=\bar{A}$b, S_7=Ab

Determination of base-state

In the transition diagram there are four, two, two, four, three, one, one and one primary circuits at the vertices 0, 1, 2, 3, 4, 5, 6 & 7 respectively. As there are four primary circuits associated each of the vertices 0&3. But secondary circuited is minimum in state 0 so state 0 is base state.

Transition Probabilities

$q_{i,j}^{(t)}$	$P_{ij} = q^*_{i,j}{}^{(t)}$
$q_{0,1}= \lambda_1 e^{-(\lambda_1+\lambda+\lambda_4)t}$ $q_{0,2}= \lambda e^{-(\lambda_1+\lambda+\lambda_4)t}$ $q_{0,7}= \lambda_4 e^{-(\lambda_1+\lambda+\lambda_4)t}$	$p_{0,1}= \lambda_1/(\lambda+\lambda_1+\lambda_4)$ $p_{0,2}= \lambda/(\lambda+\lambda_1+\lambda_4)$ $p_{0,7}= \lambda/(\lambda+\lambda_1+\lambda_4)$
$q_{1,0}= w_1 e^{-(w_1+\lambda_2+\lambda_4)t}$ $q_{1,2}= \lambda_2 e^{-(w_1+\lambda_2+\lambda_4)t}$ $q_{1,6}= \lambda_4 e^{-(w_1+\lambda_2+\lambda_4)t}$	$p_{1,0}= w_1/(w_1+\lambda_2+\lambda_4)$ $p_{1,2}= \lambda_2/(w_1+\lambda_2+\lambda_4)$ $p_{1,6}= \lambda_4/(w_1+\lambda_2+\lambda_4)$
$q_{2,3}= w_2 e^{-w_2 t}$	$p_{2,3}= w_2/w_2 = 1$
$q_{3,4}= \lambda_3 e^{-(\lambda_3+\lambda_4)t}$ $q_{3,5}= \lambda_4 e^{-(\lambda_3+\lambda_4)t}$	$p_{3,4}= \lambda_3/(\lambda_3+\lambda_4)$ $p_{3,5}= \lambda_4/(\lambda_3+\lambda_4)$
$q_{4,0}= g(t) e^{-w_3 t}$ $q_{4,3}= w_3 e^{-w_3 t}\overline{g(t)}$	$p_{4,0}= g^*w_3$ $p_{4,3}= 1-g^*w_3$
$q_{5,3}= w_4 e^{-w_4 t}$	$p_{5,3}= w_4/w_4 = 1$
$q_{6,1}= w_4 e^{-w_4 t}$	$p_{6,1}= w_4/w_4 = 1$
$q_{7,0}= w_4 e^{-w_4 t}$	$p_{7,0}= w_4/w_4 = 1$

Mean Sojourn Times

$R_i(t)$	$\mu_i=R_i^*(0)$
$R_0(t)= e^{-(\lambda_1+\lambda+\lambda_4)t}$	$\mu_0 = 1/(\lambda+\lambda_1+\lambda_4)$
$R_1(t)= e^{-(w_1+\lambda_2+\lambda_4)t}$	$\mu_1 = 1/(w_1+\lambda_2+\lambda_4)$
$R_2(t)= e^{-w_2 t}$	$\mu_2 = 1/w_2$
$R_3(t)= e^{-(\lambda_3+\lambda_4)t}$	$\mu_3 = 1/(\lambda_3+\lambda_4)$
$R_4(t)= e^{-w_3 t}\overline{g(t)}$	$\mu_4 = [1-g^*[(w_3)]/w_3$
$R_5(t)= e^{-w_4 t}$	$\mu_5 = 1/w_4$
$R_6(t) = e^{-w_4 t}$	$\mu_6 = 1/w_4$
$R_7(t)= e^{-w_4 t}$	$\mu_7 = 1/w_4$

The path probabilities for the steady state are given w.r.t. Base State '0' are

$V_{0,0} = \{(0,1,0)/[1-(1,6,1)\}+\{(0,7,0)/1\}+\{(0,1,2,3,4,0)/[1-(1,6,1)1-(3,4,3)1-(3,5,3)\}]$
$+\{(0,2,3,4,0)/[1-(3,5,3)[1-(3,4,3)\}]$
$= (p_{0,1}p_{1,0})/(1-p_{1,6}p_{6,1})+p_{0,7}p_{7,0}+(p_{0,1}p_{1,2}p_{2,3}p_{3,4}p_{4,0})/(1-p_{1,6}p_{6,1})(1-p_{3,4}\ p_{4,3})$
$(1-p_{3,5}p_{5,3})+ (p_{0,2}p_{2,3}p_{3,4}p_{4,0})/[(1-p_{3,4}\ p_{4,3})(1-p_{3,5}p_{5,3})] =1$
$V_{0,1} = \{p_{0,1}/1-(1,6,1)\} = \{\lambda/(\lambda_1+\lambda+\lambda_4)\}/1-\lambda_4(\lambda_2+w_1+\lambda_4) = \{\lambda_1(\lambda_2+w_1+\lambda_4)/(\lambda_2+w_1)$
$(\lambda_1+\lambda+\lambda_4)\}$
$V_{0,2} = \{(0,1,2)/1-(1,6,1)\}+(0,2) = \{p_{0,1}p_{1,2}/1-p_{1,6}p_{6,1}\}+ p_{0,2}$
$= [\{\lambda_1/(\lambda_1+\lambda+\lambda_4)\lambda_2/(\lambda_2+w_1+\lambda_4)\}/1-\{\lambda_4(w_1+\lambda_2+\lambda_4)\}]+\{\lambda/(\lambda_1+\lambda+\lambda_4)\}$
$= \{\lambda_1\lambda_2+\lambda(\lambda_2+w_1)\}/(\lambda_1+\lambda+\lambda_4)(\lambda_2+w_1)\}$
$V_{0,3} = \{(0,1,2,3)/1-(1,6,1)1-(3,5,3)1-(3,4,3)\}+\{(0,2,3)/1-(3,5,3)1-(3,4,3)\}$
$= \{p_{0,1}p_{1,2}\ p_{2,3}/(1-p_{1,6}p_{6,1})(1-p_{3,5}p_{5,3})(1-p_{3,4}\ p_{4,3})\}+\{p_{0,2}\ p_{2,3}/(1-p_{3,5}p_{5,3})(1-p_{3,4}\ p_{4,3})\}$
$= \{(\lambda_3+\lambda_4)^2(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/\lambda_3\lambda_4(\lambda_1+\lambda+\lambda_4)(\lambda_2+w_1)\}$
$V_{0,4} = \{(0,1,2,3,4)/[1-(1,6,1)1-(3,5,3)1-(3,4,3)\}+\{(0,2,3,4)]/[1-(3,5,3)1-(3,4,3)\}$
$= \{p_{0,1}p_{1,2}\ p_{2,3}p_{3,4}/(1-p_{1,6}p_{6,1})(1-p_{3,5}p_{5,3})(1-p_{3,4}\ p_{4,3})\}+\{p_{0,2}\ p_{2,3}p_{3,4}/(1-p_{3,5}p_{5,3})$
$(1-p_{3,4}\ p_{4,3})\}$
$= \{(\lambda_3+\lambda_4)/\lambda_4(\lambda_1+\lambda+\lambda_4)\}\{(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)/(w_1+\lambda_2)\}$
$V_{0,5} = \{(0,1,2,3,5)/1-(1,6,1)1-(3,5,3)1-(3,4,3)\}+\{(0,2,3,5)/1-(3,5,3)1-(3,4,3)\}$
$= \{p_{0,1}p_{1,2}\ p_{2,3}p_{3,5}/(1-p_{1,6}p_{6,1})(1-p_{3,5}p_{5,3})(1-p_{3,4}\ p_{4,3})\}+\{p_{0,2}\ p_{2,3}p_{3,5}/(1-p_{3,5}p_{5,3})$
$(1-p_{3,4}\ p_{4,3})\}$
$= \{(\lambda_3+\lambda_4)/\lambda_3(\lambda_1+\lambda+\lambda_4)\}\{(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)/(w_1+\lambda_2)\}$
$V_{0,6} = \{(0,1,6)/1-(1,6,1)\} = p_{0,1}p_{1,6}/(-\ p_{1,6}\ p_{6,1}) = [\{\lambda_1/(\lambda_1+\lambda+\lambda_4)\}\{\lambda_4/(w_1+\lambda_2+\lambda_4)\}]/\ [1-\{\lambda_4/(w_1+\lambda_2+\lambda_4)\}]$

Evaluation of parameters of the system

The reliability measures of the system (under steady state conditions) are evaluated by determining a 'base-state'. The parameter MTSF is determined w. r. t. the initial state '0' and the other parameters are obtained by using base-state.

(i). Mean Time to System Failure (MTSF) (T_0)

The regenerative un-failed states to which the system can transit (initial state '0'), before entering any failed state are: 'i'

MTSF (T_0) = =, 1, 2, 3, 4, 5, 6, 7 taking 'ξ' = '0'.

$T_0 = [(0,0)\mu_0+(0,1)\mu_1]/[1-(0,1,0)$

$= (p_{0,0}\mu_0+p_{0,1}\mu_1)/(1-p_{0,1}p_{1,0})$

$= (\lambda_2+\lambda_4+w_1+\lambda_1)/[(\lambda_1+\lambda+\lambda_4)(w_1+\lambda_2+\lambda_4)-\lambda_1 w_1]$

(ii). Availability of the System

The regenerative states at which the system is available are 'j' = 0, 1, 3 and the regenerative states are 'i' = 0 to 4 taking 'ξ' = '0' the total fraction of time for which the system is available is given by

$A_0 = \left[\sum_j V_{\xi,j}, f_j, \mu_j\right] \div \left[\sum_i V_{\xi,i}, f_j, \mu_i^1\right]$

$=(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,3}\mu_3)(V_{0,0}\mu_0^1+V_{0,1}\mu_1^1+V_{0,2}\mu_2^1+V_{0,3}\mu_3^1+V_{0,4}\mu_4^1+V_{0,5}\mu_5^1+V_{0,6}\mu_6^1$
$+V_{0,7}\mu_7^1)$

$= [\{(\lambda_2+w_1)/1\}+\lambda_1+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)/\lambda_3\lambda_4\}]/[\{\lambda_2+w_1+\lambda_1(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/w_1\}$
$+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/\lambda_3\lambda_4\}+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)(1-g^*(w_3))/\lambda_4 w_3\}+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)/\lambda_3 w_4\}+\{\lambda_1\lambda_4/w_4\}+\{\lambda_4(\lambda_2+w_1)/$
$w4\}]$

(iii). Proportional Busy Period of the Server

$B_0 = \left[\sum_j V_{\xi,j}, n_j\right] \div \left[\sum_i V_{\xi,i}, \mu_i^1\right]$

$=(V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,4}\mu_4+V_{0,5}\mu_5+V_{0,6}\mu_6+V_{0,7}\mu_7)/(V_{0,0}\mu_0+V_{0,1}\mu_1+V_{0,2}\mu_2+V_{0,3}\mu_3$
$+V_{0,4}\mu_4+V_{0,5}\mu_5+V_{0,6}\mu_6+V_{0,7}\mu_7)$

$= [(\lambda_1/1)+(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/w_2\}+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)(1-g^*w_3)/\lambda_4 w_3\}+\{(\lambda_3+\lambda_4)$
$(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/\lambda_3\lambda_4\}+\{\lambda_1\lambda_4/w_4\}+\{\lambda_1(\lambda_2+w_1)/w_4\}]/[\{(\lambda_2+w_1)+\lambda_1(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)$
$/w_2\}+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/\lambda_4 w_3\}+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)(1-g^*w_3)/\lambda_3 w_4\}$
$+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/\lambda_4 w_3\}+\{\lambda_1\lambda_4/w_4\}\{\lambda_4(\lambda_2+w_1)/w_4\}]$

Profit function of the system

Profit analysis of the system can be done by using the profit function

P_0 = Mean Revenue Earning Rate *Availability of the system - mean server cost that server is busy *total busy period - mean cost per visit which the server charges *number of visits server called in a unit time.

$P_0 = C_1A_0 - C_2B_0 - C_3V_0$

$=C_1[\{(\lambda_2+w_1)/1\}+\lambda_1+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)/\lambda_3\lambda_4\}]/[\{\lambda_2+w_1+\lambda_1(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/w_1\}+$
$\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/\lambda_3\lambda_4\}+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)(1-g^*(w_3))/\lambda_4 w_3\}+\{(\lambda_3+\lambda_4)$
$(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)/\lambda_3 w_4\}+\{\lambda_1\lambda_4/w_4\}+\{\lambda_4(\lambda_2+w_1)/ w4\}]$

$= [\{(\lambda_2+w_1)/1\}+\lambda_1+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)/\lambda_3\lambda_4\}]/[\{\lambda_2+w_1+\lambda_1(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/w_1\}$
$+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda\lambda_2+\lambda w_1)/\lambda_3\lambda_4\}+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)(1-g^*(w_3))/\lambda_4 w_3\}+\{(\lambda_3+\lambda_4)(\lambda_1\lambda_2+\lambda w_1+\lambda\lambda_2)/\lambda_3 w_4\}+\{\lambda_1\lambda_4/w_4\}+\{\lambda_4(\lambda_2+w_1)/$
$w4\}]$

Particular Cases

Now we discuss special cases when $\lambda_i = \lambda$ for i = 1, 2, 4 and $w_i = w$, i = 1, 2, 4 and repair rate after degradation is given by recurrence relation.

$\lambda n = (1+\alpha)\lambda_{n-1}$

MTSF (T_0) = $(3\lambda+w)/[\{3\lambda/(2\lambda+w)-\lambda w\} = (3\lambda+w)/(6\lambda^2+2\lambda w) = 1/2\lambda$

$dT_0/d\lambda = -1/2\lambda^2 < 0$

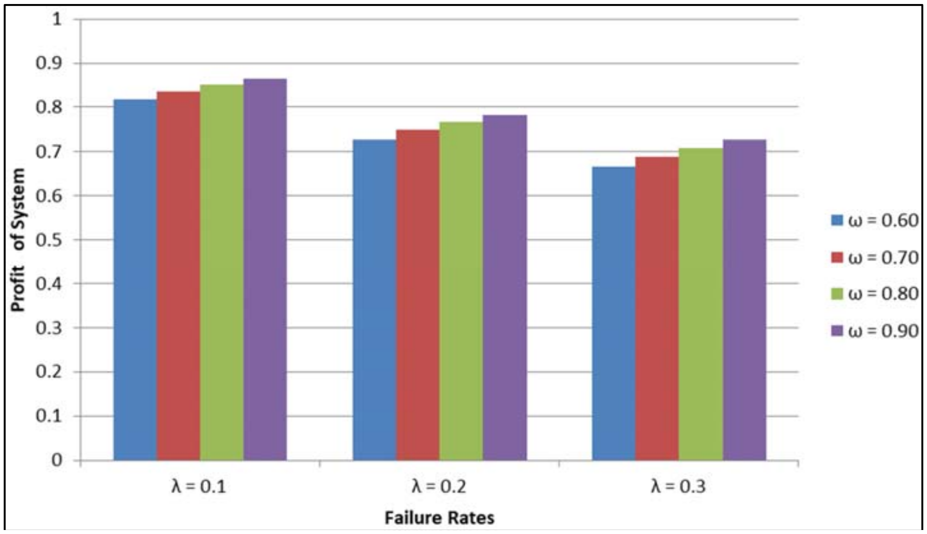
i.e. MTSF decreases with increases in failure rate λ.

Profit Function = P_0

$[(\lambda+w)+\lambda+2(2\lambda+w)]/[(2\lambda+w)+\{\lambda(2\lambda+w)/w\}+(4\lambda+w)+(6\lambda+w)$
$+\{2(2\lambda^2+\lambda w)/w\}\{w/(r+w)\}+\{(4\lambda^2+2\lambda w)/w\}+\{\lambda^2/w\}\{\lambda(\lambda+w)/w\}]$
$= [12\lambda w+6w^2]/[22\lambda w+6w^2+4\lambda^2]$

Table 4.7: Table for Profit function of the system

A	ω = 0.60	ω = 0.70	ω = 0.80	ω = 0.90
λ = 0.1	0.8181	0.8362	0.8510	0.8634
λ = 0.2	0.7258	0.7476	0.7660	0.7817
λ = 0.3	0.6667	0.6894	0.7089	0.7258
λ = 0.4	0.6000	0.6262	0.6486	0.6681



Profit function Graph for Different Repair Rates

Result Discussion and Conclusions:

In present time, competitions between different industries have increased demands on production systems. Customer satisfaction or reliability of a brand depends upon the production system's capability to provide goods and services on time and meeting the required quality specification. To fulfill the customer's requirement, the system must be reachable. Here we have studied the system for maximization of net profit and obtained that it is independent of base state and mean time to system failure. We have seen that profit function is directly influenced by repair rate and highly sensitive to rate of repair. As repair rate is increased then profit function have gradual change for small values of failure rates. Also we see that if study state availability increases and replacement factor increases then also profit function increases. Hence, we conclude that for optimization of profit replacement of faulty unit is more beneficial then successive repairs of degraded unit.

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