International Journal of Statistics and Applied Mathematics 2018; 3(2): 107-110

**ISSN:** 2456-1452
Maths 2018; 3(2): 107-110

www.mathsjournal.com
Received: 17-01-2018
Accepted: 18-02-2018

**Dr. Prabhat Yadav**
Department of Mathematics, North Eastern Regional Institute of Science and Technology, Arunachal Pradesh, India

**Correspondence**
**Dr. Prabhat Yadav**
Department of Mathematics, North Eastern Regional Institute of Science and Technology, Arunachal Pradesh, India

# Extension of asymptotic approximations of continuous wavelet transform

**Dr. Prabhat Yadav**

**Abstract**
In this paper we just extend the previous result of asymptotic expansion of wavelet transform by using Ashish Pathak, Prabhat Yadav and M.M. Dixit technique. Here we find $(W_{\emptyset}^{-} f)\,(t,s)$ separately by recall earlier result and accumulate $(W_{\emptyset}^{+}f)$ (t, s) and$(W_{\emptyset}^{-} f)(t,s)$, in order to find asymptotic approximation of continuous wavelet transform $(W_{\emptyset}\, f)\,(t,s)$ for large value of dilation parameter.

**Keywords:** Asymptotic expansion, wavelet transform, Mellin convolution, integral transform, dilation

## Introduction

The concept of wavelet transform and asymptotic expansion can be seen as the synthesis of various ideas originating from different disciplines such as mathematics and other fields like physics and engineering etc.

As so many mathematicians have investigated in the field of an asymptotic expansion and wavelet transform like [1, 5, 6, 9-19] etc. Recently by using Mellin transform technique of Lopez and Pagola and an asymptotic expansion of the Fourier transform of the function and the wavelet, Pathak, Yadav, Dixit [2, 3, 4] obtained the asymptotic expansion of the continuous wavelet transform for large and small values of dilation and translation parameter and Jose L. Lopez [5] derived asymptotic expansion of Mellin convolutions by means of analytic continuation instead of distribution technique.

The continuous wavelet transform of a function $f \in L^2(R)$ with respect to the wavelet $\emptyset \in L^2(R)$ is defined by [1].

$$(W_{\emptyset}\, f)\,(t,s) = \frac{1}{\sqrt{s}} \int_{-\infty}^{\infty} f(u)\ \overline{\emptyset(\tfrac{u-t}{s})}\, du,\ s > 0, t \in R, \tag{1}$$

Provided the integral exists. The asymptotic expansion for Mellin convolution

$$I(\upsilon) = \int_0^{\infty} f(u) g(\upsilon u) du\ ,\ \text{as}\ \upsilon \to 0^+, \tag{2}$$

Was proposed by Pathak, Yadav and Dixit [1] under dyadic conditions on $f$ and $g$. As asymptotic behaviour of $g(u)$ and $f(u)$ at $u \to 0^+$ and $u \to +\infty$ has satisfied [(3), (4), (5), (6) and (7) of [1].

Let us remind earlier results (8), (9), (10), of Theorem 1 [1]. The asymptotic expansion of (2) at the origin is given below by the following three cases of Theorem 1[1] as:

**Case I:** For any $n = 1,2,3, \ldots$ and $m = n + \lfloor p + q \rfloor$ with $+q\ \notin Z$, we have

$$\int_0^{\infty} f(u)\, g(\upsilon u) du = \sum_{i=0}^{n-1} t_i\, M[\, g(u); 1 - i - q]\, \upsilon^{i+q-1} + \sum_{i=0}^{m-1} s_i\, M[\, f(u); 1 + i - p]\, \upsilon^{i-p} + O\,(\,\upsilon^{n+q-1}). \tag{3}$$

**Case II:** For any $n = 1,2,3, \ldots$ and $m = n + p + q - 1$ with $p + q\ \in N$, we have

$$\int_0^{\infty} f(u)\, g(\upsilon u) du = \sum_{i=0}^{p+q-2} s_i\, M[\, f(u)\,; 1 + i - p]\upsilon^{i-p}$$

$+ \sum_{i=0}^{n-1} \upsilon^{i-q-1} \{-t_i\, s_{i+p+q-1} \log \upsilon\}$
$+ \lim_{z\to 0}[t_i\, M[g(u)\,; z+1-i-q]$
$+ s_{i+p+q-1}\, M[\, f(u); z+i+q]\,]$ + O $(\upsilon^{m-p} \log \upsilon)$. (4)

**Case III:** For any $m = 1,2,3, \ldots$ and $n = m+1-p-q$ with $1-p-q \in N$, we have
$\int_0^\infty f(u)\, g(\upsilon u) du = \sum_{i=0}^{-p-q} t_i\; M[\, g(u)\,;\; 1-i-q]\upsilon^{i+q-1}$
$+ \sum_{i=0}^{m-1} \upsilon^{i-p}\{-\, s_i\, t_{i+1-p-q} \log \upsilon$
$+ \lim_{z\to 0}$ $[\, t_{i+1-p-q}\, M[\, g(u)\,; z+p-i]$
$+ s_i\; M[\, f(u)\,; z+i+1-p]\,]\,\}$
+ O $(\upsilon^{m-p} \log \upsilon)$. (5)

By using aforesaid techniques (3), (4) and (5), we derive asymptotic approximation of continuous wavelet transform $(W_\emptyset f)\,(t,s)$ for large value of $s$, by recall Pathak, Yadav and Dixit technique.

Asymptotic Approximation of Continuous Wavelet Transform for Large Value of $s$.
In this section, we obtain asymptotic approximation of the continuous wavelet transform (1), when $s \to +\infty$. Now, let us rewrite (1) in the form [1]:

$(W_\emptyset f)\,(t,s) = \vartheta^{\frac{1}{2}} \int_{-\infty}^{\infty} f(u+t)\, \overline{\emptyset(\vartheta u)}\, du$
$= \vartheta^{\frac{1}{2}}\, \{ \int_0^\infty f(u+t)\, \overline{\emptyset(\vartheta u)}\, du + \int_0^\infty f(-u+t)\, \overline{\emptyset(-\vartheta u)}\, du\}$ (6)
$= (W_\emptyset{}^{+} f)(t, s) + (W_\emptyset{}^{-} f)(t, s)$, (7)

where, $\vartheta = \frac{1}{s}$ and $t$ is assumed to be a fixed real number. Setting $f(u+t) = \chi(u)$ and assume that $\chi(u)$ and $\overline{\emptyset(u)}$ are locally integrable on $(0,\infty)$. Further assume that asymptotic approximation of the form [1]:

$\overline{\emptyset(u)} = \sum_{i=0}^{n-1} s_i\, u^{i-p} + \overline{\emptyset_n\,(u)}$, as $u \to 0^+$, (8)

$\chi(u) = \sum_{i=0}^{n-1} t_i\, u^{-i-q} + \overline{\chi_n\,(u)}$, as $u \to +\infty$. (9)

Also assume that
$\overline{\emptyset(u)}$ = O $(u^{-\tau})$, as $u \to +\infty$, $\tau \in R$, (10)

and $\chi(u)$ = O $(u^{-\rho})$, as $u \to 0^+$, $\rho \in R$. (11)

With parameters $p, q, \tau$ and $\rho$ satisfying the following condition [1]:
$p+\tau < 1 < q+\rho,\ \tau < q$ and $< \rho$. (12)

Then by using aforesaid techniques (3), (4) and (5), we obtain asymptotic approximation of $(W_\emptyset f)\,(t,s)$ for large value of dilation parameter when $s \to +\infty$ as [1]:
Case I: When $n = 1,2,3, \ldots$ and $m = n + \lfloor p+q \rfloor$ with $+q \notin Z$, we have
$(W_\emptyset{}^{+} f)(t, s) = \sum_{i=0}^{n-1} t_i\, M\big[\, \overline{\emptyset(u)}\,; 1-i-q\big] s^{-i-q+\frac{1}{2}}$
$+ \sum_{i=0}^{n-1} s_i\, M[\, \chi(u); 1+i-p]\, s^{-i+p-1/2}$
+ O $(s^{-n-q+1/2})$. (13)

Similarly on setting $\chi(u) = \chi(-u)$ and $\overline{\emptyset(u)} = \overline{\emptyset(-u)}$ to obtain $(W_\emptyset{}^{-} f)(t, s)$, we have
$(W_\emptyset{}^{-} f)(t, s) = \sum_{i=0}^{n-1} (-1)^{-i-q}\, t_i\, M\big[\overline{\emptyset(-u)}\,; 1-i-q\big]\, s^{-i-q+\frac{1}{2}}$
$+ \sum_{i=0}^{n-1} (-1)^{-i-p}\, s_i\, M[\, \chi(-u); 1+i-p]\, s^{-i+p-1/2}$
+ O $(s^{-n-q+1/2})$. (14)

From (13) and (14) in (7), we get the required asymptotic approximation of wavelet transform $(W_\emptyset f)\,(t,s)$, when $s \to +\infty$ as:
$(W_\emptyset f)\,(t,s) = \sum_{i=0}^{n-1} t_i\, \big[M\big[\, \overline{\emptyset(u)}\,; 1-i-q\big] + (-1)^{-i-q}\, M\big[\, \overline{\emptyset(-u)}\,; 1-i-q\big]\,\big]\, s^{-i-q+\frac{1}{2}}$
$+ \sum_{i=0}^{n-1} s_i\, [M[\, \chi(u); 1+i-p] + (-1)^{-i-p}\, M[\, \chi(-u); 1+i-p]\,] \times s^{-i+p-1/2}$+O$(s^{-n-q+1/2})$. (15)

Case II: When $n = 1,2,3, \ldots$ and $m = n+p+q-1$ with $p+q \in N$, we have
$(W_\emptyset{}^{+} f)\,(t,s) = \sum_{i=0}^{p+q-2} s_i\, M[\, \chi(u)\,; 1+i-p]\, s^{-i-p+1/2}$
$+ \sum_{i=0}^{n-1} s^{-i-q+1/2}\, \{\, -t_i\, s_{i+p+q-1} \log(1/s)$
$+ \lim_{z\to 0}$ $[t_i\, M\big[\overline{\emptyset(u)}\,; z+1-i-q\big] + s_{i+p+q-1}\, M[\, \chi(u); z+i+q\,]\,]\,\}$
+ O $(s^{-m+p-1/2} \log(1\backslash s))$. (16)

Similarly on setting $\chi(u) = \chi(-u)$ and $\overline{\emptyset(u)} = \overline{\emptyset(-u)}$ in order to obtain $(W_\emptyset{}^{-} f)$(t, s), we have

$(W_\emptyset{}^{-} f)\ (t,s) = \sum_{i=0}^{p+q-2} (-1)^{i-p}\, s_i\, M[\,\chi(-u)\,;1+i-p]\, s^{-i+p-1/2}$
$+ \sum_{i=0}^{n-1} s^{-i-q+1/2} \{ -(-1)^{-i-q+i+p+q-1-p}\, t_i\, s_{i+p+q-1} \log(1/s)$
$+ \lim_{z\to 0} [\,(-1)^{-i-q}\, t_i\, M\big[\overline{\emptyset(-u)}\,;z+1-i-q\big] + (-1)^{i+p+q-1-p}$
$\times\, s_{i+p+q-1}\, M[\,\chi(-u);z+i+q\,]\,]\,\}$
$+ O\,(s^{-m+p-1/2}\ \log(1\backslash s)).$ (17)

From (16) and (17) in (7), we get the required asymptotic approximation of wavelet transform $(W_\emptyset\, f)\ (t,s)$, when $s \to +\infty$ as:

$(W_\emptyset\, f)\ (t,s) = \sum_{i=0}^{p+q-2} s_i\ [\ M[\,\chi(u)\,;1+i-p] + (-1)^{i-p}\ M[\,\chi(-u)\,;1+i-p]\,] \times s^{-i+p-1/2} + \sum_{i=0}^{n-1} s^{-i-q+1/2}\ \{\lim_{z\to 0} [t_i$
$M\big[\,\overline{\emptyset(u)}\,;z+1-i-q\big] + (-1)^{-i-q}\, M\big[\,\overline{\emptyset(-u)}\,;z+1-i-q\big] + s_{i+p+q-1}\,[\,M[\,\chi(u);z+i+q\,]$
$+ (-1)^{i+q-1}\, M[\,\chi(-u);z+i+q\,]\,]\,]\,\}$
$+ O\,(s^{-m+p-1/2}\ \log(1\backslash s)).$ (18)

Case III: When $m = 1,2,3,\ldots$ and $n = m+1-p-q$ with $1-p-q\ \in N$, we have

$(W_\emptyset{}^{+} f)\ (t,s) = \sum_{i=0}^{-p-q} t_i\ M\big[\,\overline{\emptyset(u)}\,;\ 1-i-q\big]\, s^{-i-q+1/2} + \sum_{i=0}^{m-1} s^{-i+p-1/2}$
$\{-\, s_i\ t_{i+1-p-q}\ \log(1\backslash s) + \lim_{z\to 0} [\,t_{i+1-p-q}\, M\big[\,\overline{\emptyset(u)}\,;z+p-i\big]$
$+ s_i\ M[\,\chi(u)\,;z+1+i-p]]\,\}$
$+ O\,(s^{-m+p-1/2}\ \log(1\backslash s)).$ (19)

Similarly on setting $\chi(u) = \chi(-u)$ and $\overline{\emptyset(u)} = \overline{\emptyset(-u)}$ in order to obtain $(W_\emptyset{}^{-} f)$(t, s), we have

$(W_\emptyset{}^{-} f)\ (t,s) = \sum_{i=0}^{-p-q} (-1)^{-i-q}\, t_i\ M\big[\,\overline{\emptyset(-u)}\,;\ 1-i-q\big] s^{-i-q+1/2} + \sum_{i=0}^{m-1} s^{-i+p-1/2}$
$\{-\,(-1)^{i-p-i-1+p+q-q}\ s_i\ b_{i+1-p-q}\ \log(1\backslash s)$
$+ \lim_{z\to 0} [\,(-1)^{-i-1+p+q-q}\ t_{i+1-p-q}\, M\big[\,\overline{\emptyset(-u)}\,;z+p-i\big]$
$+ (-1)^{i-p}\, s_i\ M[\,\chi(-u)\,;z+1+i-p]]\,\}$
$+ O\,(s^{-m+p-1/2}\ \log(1\backslash s)).$ (20)

From (19) and (20) in (7), we get the required asymptotic approximation of wavelet transform $(W_\emptyset\, f)\ (t,s)$, when $s \to +\infty$ as:

$(W_\emptyset\, f)\ (t,s) = \sum_{i=0}^{-p-q} t_i\,[\,M\big[\,\overline{\emptyset(u)}\,;\ 1-i-q\big] + (-1)^{-i-q}\ M\big[\,\overline{\emptyset(-u)}\,;\ 1-i-q\big]\,] \times s^{-i-q+1/2} + \sum_{i=0}^{m-1} s^{-i+p-1/2}\ \{\ \lim_{z\to 0}$
$[\,t_{i+1-p-q}\,[M\big[\,\overline{\emptyset(u)}\,;z+p-i\big]$
$+ [\,[\,(-1)^{-i-1+p}\, M\big[\,\overline{\emptyset(-u)}\,;z+p-i\big]\,] + s_i\,[\,M[\,\chi(u)\,;z+1+i-p]$
$+ (-1)^{i-p}\, M[\,\chi(-u)\,;z+1+i-p]]]\,\}$
$+ O\,(s^{-m+p-1/2}\ \log(1\backslash s)).$ (21)

## Conclusion

In this paper we can easily calculate the approximation terms with their exact error terms. The result obtained in the previous paper Asymptotic Expansion of Wavelet Transform [1] is little different from the result obtaining in the present paper. In the Previous paper [1], we obtained asymptotic expansion of wavelet transform for $(W_\emptyset{}^{+} f)$(t, s) only, whereas in the present paper we obtaining asymptotic approximations of both $(W_\emptyset{}^{+} f)$(t, s) and $(W_\emptyset{}^{-} f)\ (t,s)$ in order to derive asymptotic approximation of the continuous wavelet transform $(W_\emptyset\, f)\ (t,s)$ (7) for large value of dilation parameter $s$. By using aforesaid results (3), (4) and (5) and remind Pathak, Yadav and Dixit technique, we get the following results (15), (18) and (21) respectively.

## References


1. Pathak A, Yadav P, Dixit MM. Asymptotic Expansion of Wavelet Transform, Advances in Pure Mathematics, Scientific Research Publishing, 2015; 5:21-26.
2. Pathak A, Yadav P, Dixit MM. An Asymptotic Expansion of Continuous Wavelet Transform for Large Values of Dilation Parameter, Boletim da Sociedade Paranaense de Matematica, 2018; 36(3):27-39, 2175-1188(online)/ 00378712 IN PRESS.
3. Pathak A, Yadav P, Dixit MM. On Convolution for General Novel Fractional Wavelet Transform. Journal of Advanced Research in Scientific Computing. 2015; 7(1):30-37, Online ISSN: 1943-2364.
4. Pathak A, Yadav P, Dixit MM. An Asymptotic Expansion of Continuous Wavelet Transform for Large Values of b; Investigation in Mathematical Sciences. 2016; 5(87-92):2250-1436.
5. Jose Lopez L. Asymptotic Expansions of Mellin Convolutions by Means of Analytic Continuation. Journal of Computational and Applied Mathematics. 2007; 200:628-636.
6. Jos Lopez L, Pedro P. Asymptotic Expansions of Mellin Convolution Integrals: An Oscillatory Case. Journal of Computational and Applied Mathematics. 2010; 233:1562-1569.
7. Debnath L. Wavelet Transforms and their Applications, Birkhauser, 2002.
8. Chui CK. An Introduction To Wavelets, Academic Press, New York, 1992.
9. Armstrong JA, Bleistein N. Asymptotic Expansions of Integral with Oscillatory Kernels and Logarithmic Singularities, SIAM Journal on Mathematical Analysis, 1980.

10. Erde'lyi A, Magnus W, Oberhettinger F, Tricomi FG. Tables of Integral Transforms, McGraw-Hill, New York, 1954, 1.
11. Pathak RS, Pathak A. Asymptotic Expansion of Wavelet Transform for Small Value a, The Wavelet Transform, World Scientific, 2009, 164-169.
12. Pathak RS, Pathak A. Asymptotic Expansion of Wavelet Transform for Small Value a, The Wavelet Transform, World Scientific, 2009, 164-169.
13. Pathak RS, Pathak A. Asymptotic Expansion of Wavelet Transform with Error Term, World Scientific, 2009, 154-164.
14. McClure JP, Wong R. Exact Remainders for Asymptotic Expansions of Stieltjes Transforms, J Inst. Math. Appl. 22,129-149.
15. Wong R. Asymptotic Approximations of Integrals, Academic Press, New York, 1996.
16. Wong R. Explicit Error Terms for Asymptotic Expansions of Mellin Convolutions. J Math., Anal., Appl. 19797; 2(2):740-756.
17. Wong R. Asymptotic Approximations of Integrals, Acad., Press, New York, 1989.
18. Wong JS, Wong R. On Asymptotic Solutions of the Renewal Equation, J., Math., anal., Appl. 1976; 53:243-250.
19. Yadav P. A Brief Description of Wavelet and Wavelet Transforms and their Applications. International Journal of Statistics and Applied Mathematics. 2018; 3(1):266-271.