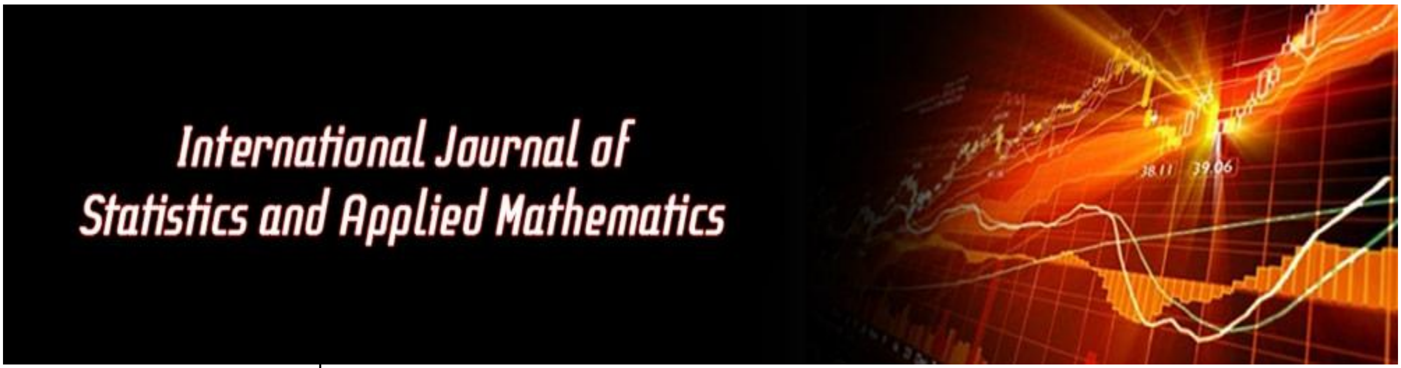

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Determination of variable sampling plans based on weighted expansion factor for a non-normal lomax process

D Ramya and S Devaarul

Abstract
This article deals with designing procedure of Lomax variable sampling plans by utilizing weighted factor for a heavy tailed non-normal family. Variable sampling plans are preferred because of its efficiency in sample size and also used when the nature of the quality characteristic is measurable. If the underlying distribution is non-normal then the standard normal procedures could not be used. In many industrial situations, the quality characteristics does not follow a normal or Gaussian pattern. Hence to develop new sampling plans, an attempt has been made by assuming a heavy tailed Lomax distribution. The selection of variable sampling plans based on AQL, LQL and minimum tangent angle technique using Lomax distribution has been designed using weighted factor method. Tables are constructed and illustration is given for easy selection of sampling plans.

Keywords: Heavy tailed, minimum tangent angle, Lomax distribution, variable sampling plans, weighted expansion factor, non normal process

1. Introduction
Even though variable sampling plans have wide range of advantages in sampling plan literature, the main aspect to be considered while employing variable sampling plans is that the distribution of the quality characteristic must be known. In many quality contro sections mostly the underlying distribution of the quality characteristic is assumed to be normal. But in industries there may be a chance for the distribution to be non-normal. It is always not possible to determine the parameters when the process is non-normal. However the procedure for non normal processes has been suggested by authors like, Zimmer and Burr [1], Owen [2] and Takagi [3]. In this paper a heavy tailed distribution namely Lomax distribution is used to determine the parameters of the sampling plans. It is a special case of generalised Pareto distribution. A Lomax distribution is also called as pareto distribution of second kind is a heavy tailed probability distribution used in life testing and reliability estimation. A heavy tailed distribution generally has a positive and huge kurtosis. Taking advantage of skewness and kurtosis, in this article sampling plans are determined.
In this article another approach to determine the sampling plans is by using minimum tangent angle. The smaller the value of the tan θ, closer is the angle θ approaches zero, and the chord length AB in Figure.1 approaches to AC and the ideal condition is reached. It can be understood that whenever the angle is minimised, then the abstract OC curve tends to ideal OC curve. Hence by using this advantage Lomax VSP are developed using minimum angle technique. Devaarul and Jemmy Joyce [4] have developed reliability sampling plans based on minimum angle technique. Soundararajan & Christina [5] have developed single sampling variables plans based on the minimum angle. Bush et.al [6] have considered two points on the Operating Characteristic curve as (AQL, 1-α) and (IQL, 0.5) for minimizing consumer's risk. Bander Al-Zahrani and Mashail Al-Harbi [7] considered the estimation problem of the probability S = P(Y < X) for Lomax distribution based on general progressive censored data. Artur J. Lemonte and Gauss M. Cordeiro [8] proposed a new five-parameter continuous distribution, called McDonald Lomax distribution that extends the Lomax distribution and some other distributions.

Geetha and Vijayaraghavan [9] developed a procedure for determining the parameters of a certain plan to protect both consumer and producer. Wolfgang [10] discussed variable sampling plans based on generalised Pareto distribution. Montgomery [11] observed the effect of normality in variable sampling plans and the methods to detect normality. Jemmy Joyce, Devaarul and Rebecca Edna [12] proposed mixed sampling plans based on the tangent angle, AQL and LQL. Lomax [13] has introduced this distribution in the analysis of business failure data.
For any quality characteristic there exists an upper specification limit or lower specification limit or both for determination of parameters. Hence variable sampling plans based on upper and lower specification limits have been developed. Also the parameters for known standard deviation and unknown standard deviation are developed. Variable sampling plans for non normal process had been developed based on the procedure given by Takagi [3]. For any non normal process the skewness and kurtosis plays a vital role. Hence these plans were developed by using the expansion factor which is a function of skewness and kurtosis of the underlying distribution.

2. Formulation of the sampling plans

A random variable, 'X' is said to follow the Lomax distribution, if the probability density function (pdf) of t is as follows:

$$f(x,\alpha,\lambda) = \frac{\alpha}{\lambda}\left(1+\frac{x}{\lambda}\right)^{-(\alpha+1)}, x > 0, (\alpha,\lambda) > 0 \quad (1)$$

The cumulative distribution function (cdf) of Lomax distribution is given by,

$$F(x,\alpha,\lambda) = 1-\left(1+\frac{x}{\lambda}\right)^{-\alpha}, x > 0, (\alpha,\lambda) > 0 \quad (2)$$

where λ is the scale parameter and α is the shape parameter.

2.1 Determination of Parameters

The statistics $Y = \overline{X} \pm kS$ are asymptotically normally distributed with the means

$$\mu_Y = \mu \pm k\sigma \quad (3)$$

And the variances

$$\sigma_Y^2 = \frac{\sigma^2}{n}\left[1 + \left(k^2/4\right)\left(\beta_2 - 1\right) \pm k\beta_1\right] \quad (4)$$

Where β_1 and β_2 represent the skewness and kurtosis of the underlying distribution.
The underlying distribution is a Lomax distribution where λ is the scale parameter and α is the shape parameter.

2.1.1 Mean of the Lomax distribution

$$\mu_L = \frac{\lambda}{\alpha - 1} \; for\ \alpha > 1, \quad (5)$$

2.1.2 Variance of the Lomax distribution

$$\sigma_L^2 = \frac{\lambda^2\alpha}{(\alpha-1)^2(\alpha-2)} \; for\ \alpha > 2, \quad (6)$$

2.1.3 Skewness of the Lomax distribution

$$\beta_1 = \frac{2(1+\alpha)}{\alpha - 3}\sqrt{\frac{\alpha-2}{\alpha}} \; for\ \alpha > 3 \quad (7)$$

2.1.4 Kurtosis of the Lomax distribution

$$\beta_2 = \frac{3\left(\alpha^3 + 9\alpha^2 - 24\alpha - 4\right)}{\alpha(\alpha-3)(\alpha-4)} \; for\ \alpha > 4 \quad (8)$$

3. Variable sampling plans using Lomax distribution indexed through AQL and LQL

In this section variable sampling plans indexed through AQL and LQL are determined.
Step 1: Select a random sample $x_1, x_2, \ldots x_n$.

Step 2: From the sample identify the underlying distribution.

Step 3: Determine the skewness and kurtosis of the distribution.

Step 4: Find the expansion factor e_U which is a function of β_1 and β_2.

Step 5: If U is specified determine the corresponding expansion factor e_U using the formula:

$$e_U = 1 + \left(\frac{k_U^2}{4}\right)\left[\frac{3(\alpha^3 + 9\alpha^2 - 24\alpha - 4)}{\alpha(\alpha-3)(\alpha-4)} - 1\right] + \left[k_U\left(\frac{2(1+\alpha)}{\alpha-3}\sqrt{\frac{\alpha-2}{\alpha}}\right)\right] \quad (9)$$

Step 6: If L is specified determine the corresponding expansion factor e_L using the formula:

$$e_L = 1 + \frac{k_L^2}{4}(\beta_2 - 1) - k_L\beta_1 \quad (10)$$

3.1. Algorithm for selection of variable sampling plans when U is specified

Step 1: The scale parameter λ and shape parameter α of Lomax distribution are determined using past experience in the production process.
Step 2: Draw a random sample of n items say $x_1, x_2, x_3 \ldots x_n$ from a lot of size N.
Step 3: Determine the Skewness (β_1) and Kurtosis (β_2) from the observed data.
Step 4: Now obtain the corresponding mean $\bar{x}$ and standard deviation σ.
Step 5: If $\bar{x} + k\sigma \le U$, accept the lot or otherwise reject it. (when σ is known).
Step 6: In case of unknown σ, if $\bar{x} + kS \le U$, accept the lot or otherwise reject it.

3.2 Designing procedure of the variable sampling plans using the expansion factor (U specified)

The OC function for unknown-sigma plan for a situation when U is specified is

$$P_a(p) = \Pr\left[\bar{x} + k_U S \le U \mid p\right] \quad (11)$$

$$K_{L_U(p)} = \frac{\sqrt{n_U}\left(k_U - K_p^*\right)}{\sqrt{e_U}} \quad (12)$$

where K_p^* is defined as

$$K_p^* = \frac{U - \mu}{\sigma} \quad (13)$$

The n_U and k_U values for unkown sigma plans are

$$n_U = e_U\left[\frac{K_\alpha + K_\beta}{K_{p_1}^* - K_{p_2}^*}\right]^2 \quad (14)$$

$$k_U = \frac{K_\alpha K_{p_2}^* + K_\beta K_{p_1}^*}{K_\alpha + K_\beta} \quad (15)$$

Where K_α and K_β are the standardised deviates exceeded with the probabilities α and β respectively.
The n' and k' values for known sigma plans are

$$n'_U = \frac{n_U}{e_U} \text{ and } k'_U = k_U \quad (16)$$

3.3 Algorithm for selection of variable sampling plans when L is specified

Step 1: The scale parameter λ and shape parameter α of Lomax distribution are determined using past experience in the production process.
Step 2: Draw a random sample of n items say $x_1, x_2, x_3 \ldots x_n$ from a lot of size N.
Step 3: Determine the Skewness (β_1) and Kurtosis (β_2) from the observed data.
Step 4: Now obtain the corresponding mean $\bar{x}$ and standard deviation σ.
Step 5: If $\bar{x} - k\sigma \ge L$, accept the lot or otherwise reject it. (When σ is known).
Step 6: In case of unknown σ, if $\bar{x} - kS \ge L$, accept the lot or otherwise reject it.

3.4 Designing procedure of the variable sampling plans using the expansion factor (L specified)

In a similar way when L is specified the OC function for unknown-sigma plan is

$$P_a(p) = \Pr\left[\overline{X} - k_L S \geq L \mid p\right] \tag{17}$$

Also

$$K_{L_L(p)} = \frac{\sqrt{n_L}\left(k_L + K^*_{1-p}\right)}{\sqrt{e_L}} \tag{18}$$

where K^*_{1-p} is defined as

$$K^*_{1-p} = \frac{L-\mu}{\sigma}, 1-p = \Pr\left(X > L\right) = \Pr\left(Z^* > K^*_{1-p}\right) \tag{19}$$

The n_L and k_L values for unkown sigma plans are

$$n_L = e_L\left[\frac{K_\alpha + K_\beta}{K^*_{1-p_1} - K^*_{1-p_2}}\right]^2 \tag{20}$$

$$-k_L = \frac{K_\alpha K^*_{1-p_2} + K_\beta K^*_{1-p_1}}{K_\alpha + K_\beta} \tag{21}$$

Where K_α and K_β are the standardised deviates exceeded with the probabilities α and β respectively.

The n' and k' values for known sigma plans are

$$n'_L = \frac{n_L}{e_L} \text{ and } k'_L = k_L \tag{22}$$

4. Construction of tables

Table 1

(i) The values of p_1 and p_2 are chosen at fixed intervals.

(ii) When U is specified the corresponding values of $K^*_{p_1}$ and $K^*_{p_2}$ are obtained from equation 13 (for $p=p_1$ and $p=p_2$).

(iii) The values of n_U and k_U for unknown sigma plans are obtained using equations 14 & 15.

(iv) Similarly the n'_u and k'_u values of known sigma plans are obtained using equation 16.

Table 2

(i) The values of p_1 and p_2 are chosen.

(ii) When L is specified the corresponding values of $K^*_{1-p_1}$ and $K^*_{1-p_2}$ are obtained from equation 19 (for $p=p_1$ and $p=p_2$).

(iii) The values of n_L and k_L for unknown sigma plans are obtained using equations 20 & 21. Similarly the n'_L and k'_L values of known sigma plans are obtained using equation 22.

Table 3

(i) When U is specified the values of n_U, k_U, n'_u and k'_u are obtained using the equations 14, 15 & 16 for known values of p_1 and p_2.

(ii) The probability of acceptance are obtained using equation 11 for the values of p_1 and p_2.

(iii) The corresponding mininimum tangent angle is calculated using equation 23.

Table 4

(i) When L is specified the values of n_L, k_L, n'_L and k'_L are obtained using the equations 20, 21 & 22 for known values of p_1 and p_2.

(ii) The probability of acceptance are obtained using equation 17 for the values of p_1 and p_2.

(iii) The corresponding mininimum tangent angle is calculated using equation 23.

Table 1: Values of n_U, k_U, n'_U and k'_U for the upper specification limit U

p_1	p_2	$K^*_{p_1}$	$K^*_{p_2}$	β_1	β_2	e_U	n_U	k_U	n'_U	k'_U
0.005	0.01	4.3205	3.6171	2.0304	6.2458	29.1747	505	3.9251	17	3.9251
0.006	0.02	4.1449	2.9181	2.0459	6.3733	24.1072	137	3.4553	6	3.4553
0.007	0.03	3.9956	2.5076	2.0615	6.5039	21.2460	82	3.1592	4	3.1592
0.008	0.04	3.8655	2.2152	2.0774	6.6377	19.2689	61	2.9379	3	2.9379
0.009	0.05	3.7502	1.9876	2.0935	6.7749	17.7707	49	2.7595	3	2.7595
0.01	0.02	3.6579	2.9291	2.1431	7.2081	24.3372	392	3.2483	16	3.2483
0.01	0.03	3.6635	2.5081	2.1601	7.3601	21.9557	141	3.0141	6	3.0141
0.01	0.04	3.6692	2.2097	2.1773	7.5162	20.4241	82	2.8489	4	2.8489
0.01	0.05	3.6748	1.9783	2.1948	7.6764	19.3325	58	2.7212	3	2.7212
0.01	0.06	3.6804	1.7892	2.2125	7.8410	18.5074	44	2.6174	2	2.6174
0.01	0.07	3.6860	1.6292	2.2304	8.0101	17.8598	36	2.5299	2	2.5299
0.01	0.08	3.6915	1.4906	2.2486	8.1839	17.3386	31	2.4544	2	2.4544
0.02	0.03	2.9402	2.5071	2.2671	8.3626	20.4997	936	2.6967	46	2.6967
0.02	0.04	2.9416	2.2028	2.2859	8.5464	18.8156	295	2.5263	16	2.5263
0.02	0.05	2.9429	1.9677	2.3049	8.7354	17.6102	159	2.3948	9	2.3948
0.02	0.06	2.9442	1.7762	2.3242	8.9299	16.6924	105	2.2877	6	2.2877
0.02	0.07	2.9455	1.6146	2.3438	9.1301	15.9648	77	2.1974	5	2.1974
0.02	0.08	2.9467	1.4749	2.3637	9.3363	15.3716	61	2.1194	4	2.1194
0.02	0.09	2.9478	1.3520	2.3839	9.5488	14.8778	50	2.0508	3	2.0508
0.02	0.1	2.9489	1.2422	2.4044	9.7676	14.4607	43	1.9896	3	1.9896
0.03	0.04	2.5033	2.1929	2.4253	9.9933	18.8422	1675	2.3289	89	2.3289
0.03	0.05	2.5027	1.9537	2.4465	10.2260	17.4717	497	2.1941	28	2.1941
0.03	0.06	2.5019	1.7595	2.4680	10.4660	16.4292	255	2.0847	16	2.0847
0.03	0.07	2.5012	1.5962	2.4899	10.7138	15.6023	163	1.9925	10	1.9925
0.03	0.08	2.5003	1.4554	2.5121	10.9696	14.9267	117	1.9130	8	1.9130
0.03	0.09	2.4995	1.3317	2.5347	11.2338	14.3626	90	1.8431	6	1.8431
0.03	0.1	2.4986	1.2215	2.5577	11.5068	13.8839	73	1.7807	5	1.7807
0.03	0.11	2.4976	1.1222	2.5811	11.7891	13.4723	61	1.7245	5	1.7245
0.03	0.12	2.4966	1.0319	2.6048	12.0810	13.1150	52	1.6733	4	1.6733
0.03	0.13	2.4955	0.9491	2.6290	12.3831	12.8024	46	1.6263	4	1.6263
0.03	0.14	2.4944	0.8728	2.6536	12.6958	12.5272	41	1.5829	3	1.5829
0.04	0.05	2.1732	1.9304	2.6787	13.0197	18.9213	2750	2.0367	145	2.0367
0.04	0.06	2.1711	1.7328	2.7042	13.3554	17.6477	787	1.9247	45	1.9247
0.04	0.07	2.1690	1.5672	2.7302	13.7035	16.6426	394	1.8307	24	1.8307
0.04	0.08	2.1668	1.4250	2.7566	14.0645	15.8247	246	1.7499	16	1.7499
0.05	0.09	1.9200	1.3005	2.7836	14.4393	13.6756	305	1.5718	22	1.5718

Table 2: Values of n_L, k_L, n'_L and k'_L for the Lower Specification limit L

p_1	p_2	$K^*_{1-p_1}$	$K^*_{1-p_2}$	β_1	β_2	e_L	n_L	k_L	n'_L	k'_L
0.005	0.01	-4.3205	-3.6171	2.0304	6.2458	13.2357	229	3.9251	17	3.9251
0.006	0.02	-4.1449	-2.9181	2.0459	6.3733	9.9690	57	3.4553	6	3.4553
0.007	0.03	-3.9956	-2.5076	2.0615	6.5039	8.2202	32	3.1592	4	3.1592
0.008	0.04	-3.8655	-2.2152	2.0774	6.6377	7.0621	22	2.9379	3	2.9379
0.009	0.05	-3.7502	-1.9876	2.0935	6.7749	6.2166	17	2.7595	3	2.7595
0.01	0.06	-3.6464	-1.8010	2.1098	6.9157	5.5629	14	2.6091	3	2.6091
0.01	0.03	-3.6635	-2.5081	2.1601	7.3601	8.9343	57	3.0141	6	3.0141
0.01	0.04	-3.6692	-2.2097	2.1773	7.5162	8.0184	32	2.8489	4	2.8489
0.01	0.05	-3.6748	-1.9783	2.1948	7.6764	7.3875	22	2.7212	3	2.7212
0.01	0.06	-3.6804	-1.7892	2.2125	7.8410	6.9256	17	2.6174	2	2.6174
0.01	0.07	-3.6860	-1.6292	2.2304	8.0101	6.5742	13	2.5299	2	2.5299
0.01	0.08	-3.6915	-1.4906	2.2486	8.1839	6.3003	11	2.4544	2	2.4544
0.02	0.03	-2.9402	-2.5071	2.2671	8.3626	8.2721	378	2.6967	46	2.6967
0.02	0.04	-2.9416	-2.2028	2.2859	8.5464	7.2659	114	2.5263	16	2.5263
0.02	0.05	-2.9429	-1.9677	2.3049	8.7354	6.5708	59	2.3948	9	2.3948
0.02	0.06	-2.9442	-1.7762	2.3242	8.9299	6.0583	38	2.2877	6	2.2877
0.02	0.07	-2.9455	-1.6146	2.3438	9.1301	5.6641	27	2.1974	5	2.1974
0.02	0.08	-2.9467	-1.4749	2.3637	9.3363	5.3521	21	2.1194	4	2.1194
0.02	0.09	-2.9478	-1.3520	2.3839	9.5488	5.0998	17	2.0508	3	2.0508
0.02	0.1	-2.9489	-1.2422	2.4044	9.7676	4.8929	14	1.9896	3	1.9896
0.03	0.04	-2.5033	-2.1929	2.4253	9.9933	7.5459	671	2.3289	89	2.3289
0.03	0.05	-2.5027	-1.9537	2.4465	10.2260	6.7360	191	2.1941	28	2.1941
0.03	0.06	-2.5019	-1.7595	2.4680	10.4660	6.1394	95	2.0847	16	2.0847
0.03	0.07	-2.5012	-1.5962	2.4899	10.7138	5.6801	59	1.9925	10	1.9925

0.03	0.08	-2.5003	-1.4554	2.5121	10.9696	5.3154	42	1.9130	8	1.9130
0.03	0.09	-2.4995	-1.3317	2.5347	11.2338	5.0193	32	1.8431	6	1.8431
0.03	0.1	-2.4986	-1.2215	2.5577	11.5068	4.7748	25	1.7807	5	1.7807
0.03	0.11	-2.4976	-1.1222	2.5811	11.7891	4.5703	21	1.7245	5	1.7245
0.03	0.12	-2.4966	-1.0319	2.6048	12.0810	4.3978	18	1.6733	4	1.6733
0.03	0.13	-2.4955	-0.9491	2.6290	12.3831	4.2512	15	1.6263	4	1.6263
0.03	0.14	-2.4944	-0.8728	2.6536	12.6958	4.1260	13	1.5829	3	1.5829
0.04	0.05	-2.1732	-1.9304	2.6787	13.0197	8.0096	1164	2.0367	145	2.0367
0.04	0.06	-2.1711	-1.7328	2.7042	13.3554	7.2380	323	1.9247	45	1.9247
0.04	0.07	-2.1690	-1.5672	2.7302	13.7035	6.6461	157	1.8307	24	1.8307
0.04	0.08	-2.1668	-1.4250	2.7566	14.0645	6.1772	96	1.7499	16	1.7499
0.05	0.09	-1.9200	-1.3005	2.7836	14.4393	4.9252	110	1.5718	22	1.5718

5. Designing Procedure by Minimum Angle Method

Whenever the angle is minimized, then the abstract OC curve tends to ideal OC curve which leads to best sampling plans. Generally a portion of the abstract OC curve is compared with the ideal OC curve in minimum angle technique. The approach of minimum angle method by considering the tangent of the angle between the lines joining the points (AQL, 1-α) & (LQL, β) is shown in Figure 1.

Here $$\tan\theta = \frac{p_2 - p_1}{P_a(p_1) - P_a(p_2)} \qquad (23)$$

Thus when the two points on the OC curve such as AQL and LQL are known, the minimum values of the angle tanθ could be calculated. This minimum angle provides a better sampling plan with good discriminating power. It minimizes the angle between the abstract and ideal OC cuves. For a minimum tanθ, the angle θ approaches to zero as the chord AB approaches to AC, hence the ideal condition is reached. This approach minimizes both producers and consumers risk simultaneously. Thus both are benefitted by choosing these plans. The minimum angle method of Lomax variable sampling plans are presented in Table 3 and Table 4. The parameters of Lomax variable sampling plans are chosen from Table 3 and Table 4 corresponding to the minimum angle.

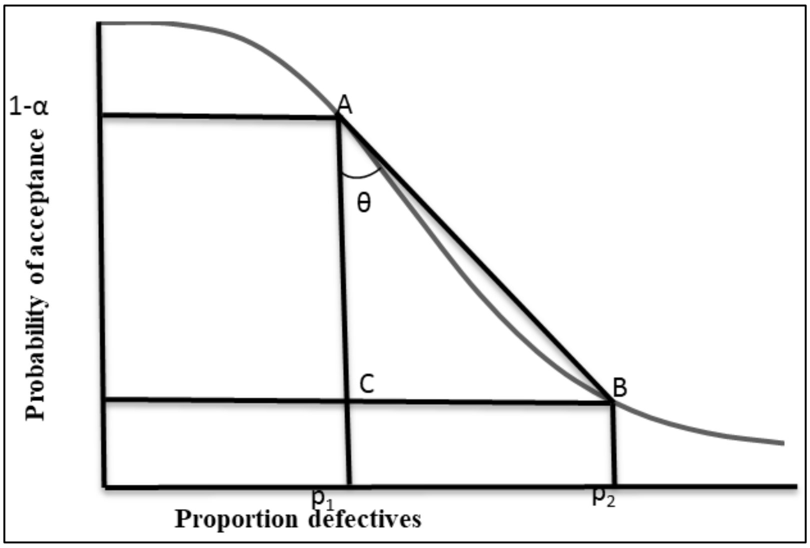

Fig 1: Minimum Tangent Angle

5.1 Illustration

For a production process, when U is specified we are provided with (0.02,0.95), (0.06,0.1). Also the process was observed to follow a heavy tailed Lomax distribution with the corresponding β_1=2.3242 and β_2=8.9299. Determine the corresponding sampling plan and the minimum tangent angle.

Solution: Since the production process has β_1=2.3242 and β_2=8.9299 from table 1, it can observed that n_U=105 and k_U=2.2877. Hence the required plan is (105,2.2877) and from table 3 it has the minimum tangent angle θ=1.1040. The above plan is for known standard deviation and for unknown standard deviation, n'_U =6, k'_U =2.2877 and hence the required plan is (6, 2.2877). On the otherhand if L is specified then from table 2, n_L=38 and k_L=2.2877, the required plan is (38,2.2877) and from table 4 it has the minimum tangent angle θ =14.76. The above plan is for known standard deviation and for unknown standard deviation, n'_L =6, k'_L =2.2877 and hence the required plan is (6, 2.2877).

Table 3: The values of the sample size n, acceptance constant k, min tanθ for the known and unknown sigma when U is specified, given values of p_1 and p_2.

p_1	p_2	β_1	β_2	n_U	k_U	n'_U	k'_U	$P_a(p_1)$	$P_a(p_2)$	$\tan\theta$
0.01	0.02	2.1431	7.2081	392	3.2483	16	3.2483	0.9999	0.9983	6.3597
0.01	0.07	2.2304	8.0101	36	2.5299	2	2.5299	0.9999	0.9484	1.1646
0.01	0.08	2.2486	8.1839	31	2.4544	2	2.4544	0.9999	0.9320	1.0306
0.02	0.03	2.2671	8.3626	936	2.6967	46	2.6967	0.9984	0.9939	2.2490
0.02	0.04	2.2859	8.5464	295	2.5263	16	2.5263	0.9984	0.9862	1.6432
0.02	0.05	2.3049	8.7354	159	2.3948	9	2.3948	0.9984	0.9754	1.3086
0.02	0.06	2.3242	8.9299	105	2.2877	6	2.2877	0.9984	0.9621	1.1040
0.02	0.07	2.3438	9.1301	77	2.1974	5	2.1974	0.9984	0.9468	0.9693
0.02	0.08	2.3637	9.3363	61	2.1194	4	2.1194	0.9984	0.9299	0.8758
0.02	0.09	2.3839	9.5488	50	2.0508	3	2.0508	0.9984	0.9118	0.8084
0.02	0.1	2.4044	9.7676	43	1.9896	3	1.9896	0.9984	0.8929	0.7583
0.03	0.04	2.4253	9.9933	1675	2.3289	89	2.3289	0.9938	0.9858	1.2493
0.03	0.05	2.4465	10.2260	497	2.1941	28	2.1941	0.9938	0.9746	1.0415
0.03	0.06	2.4680	10.4660	255	2.0847	16	2.0847	0.9938	0.9608	0.9072
0.03	0.07	2.4899	10.7138	163	1.9925	10	1.9925	0.9938	0.9448	0.8158
0.03	0.08	2.5121	10.9696	117	1.9130	8	1.9130	0.9938	0.9272	0.7510
0.03	0.09	2.5347	11.2338	90	1.8431	6	1.8431	0.9938	0.9085	0.7037
0.03	0.1	2.5577	11.5068	73	1.7807	5	1.7807	0.9938	0.8890	0.6684
0.03	0.11	2.5811	11.7891	61	1.7245	5	1.7245	0.9937	0.8691	0.6418
0.03	0.12	2.6048	12.0810	52	1.6733	4	1.6733	0.9937	0.8489	0.6215
0.03	0.13	2.6290	12.3831	46	1.6263	4	1.6263	0.9937	0.8287	0.6061
0.03	0.14	2.6536	12.6958	41	1.5829	3	1.5829	0.9937	0.8086	0.5944
0.04	0.05	2.6787	13.0197	2750	2.0367	145	2.0367	0.9851	0.9732	0.8409
0.04	0.06	2.7042	13.3554	787	1.9247	45	1.9247	0.9850	0.9584	0.7517
0.04	0.07	2.7302	13.7035	394	1.8307	24	1.8307	0.9850	0.9415	0.6898
0.04	0.08	2.7566	14.0645	246	1.7499	16	1.7499	0.9849	0.9229	0.6456
0.05	0.09	2.7836	14.4393	305	1.5718	22	1.5718	0.9726	0.9033	0.5773

Table 4: The values of the sample size n, acceptance constant k, min tanθ for the known and unknown sigma when L is specified, given values of p_1 and p_2.

p_1	p_2	β_1	β_2	n_L	k_L	n'_L	k'_L	$P_a(p_1)$	$P_a(p_2)$	$\tan\theta$
0.01	0.02	2.1601	7.3601	57	3.0141	16	3.0141	1.0000	0.9998	44.6171
0.01	0.07	2.2304	8.0101	13	2.5299	2	2.5299	1.0000	0.9957	14.0255
0.01	0.08	2.2486	8.1839	11	2.4544	2	2.4544	1.0000	0.9936	10.9807
0.02	0.03	2.2671	8.3626	378	2.6967	46	2.6967	1.0000	0.9998	53.8184
0.02	0.04	2.2859	8.5464	114	2.5263	16	2.5263	1.0000	0.9993	31.2491
0.02	0.05	2.3049	8.7354	59	2.3948	9	2.3948	1.0000	0.9985	20.5500
0.02	0.06	2.3242	8.9299	38	2.2877	6	2.2877	1.0000	0.9972	14.7600
0.02	0.07	2.3438	9.1301	27	2.1974	5	2.1974	1.0000	0.9955	11.2952
0.02	0.08	2.3637	9.3363	21	2.1194	4	2.1194	1.0000	0.9933	9.0586
0.02	0.09	2.3839	9.5488	17	2.0508	3	2.0508	1.0000	0.9907	7.5284
0.02	0.1	2.4044	9.7676	14	1.9896	3	1.9896	1.0000	0.9875	6.4329
0.03	0.04	2.4253	9.9933	671	2.3289	89	2.3289	0.9998	0.9993	21.0783
0.03	0.05	2.4465	10.2260	191	2.1941	28	2.1941	0.9998	0.9984	14.9314
0.03	0.06	2.4680	10.4660	95	2.0847	16	2.0847	0.9998	0.9971	11.2646
0.03	0.07	2.4899	10.7138	59	1.9925	10	1.9925	0.9998	0.9953	8.9259
0.03	0.08	2.5121	10.9696	42	1.9130	8	1.9130	0.9998	0.9930	7.3480
0.03	0.09	2.5347	11.2338	32	1.8431	6	1.8431	0.9998	0.9901	6.2332
0.03	0.1	2.5577	11.5068	25	1.7807	5	1.7807	0.9998	0.9868	5.4155
0.03	0.11	2.5811	11.7891	21	1.7245	5	1.7245	0.9998	0.9831	4.7968
0.03	0.12	2.6048	12.0810	18	1.6733	4	1.6733	0.9998	0.9789	4.3168
0.03	0.13	2.6290	12.3831	15	1.6263	4	1.6263	0.9998	0.9744	3.9363
0.03	0.14	2.6536	12.6958	13	1.5829	3	1.5829	0.9998	0.9695	3.6293
0.04	0.05	2.6787	13.0197	1164	2.0367	145	2.0367	0.9992	0.9983	10.6554
0.04	0.06	2.7042	13.3554	323	1.9247	45	1.9247	0.9992	0.9969	8.3999
0.04	0.07	2.7302	13.7035	157	1.8307	24	1.8307	0.9992	0.9949	6.8790
0.04	0.08	2.7566	14.0645	96	1.7499	16	1.7499	0.9992	0.9923	5.8108
0.05	0.09	2.7836	14.4393	110	1.5718	22	1.5718	0.9982	0.9893	4.4643

6. Conclusion

In this paper the parameters of a nonnormal heavy tailed Lomax distribution were obtained using the weighted expansion factor. The parameters for both known and unknown standard deviation were obtained. A procedure for minimizing the angle between the AQL and LQL values. This makes the consumer and producer to easily select the apt variable sampling plans. Lomax distribution was used as a heavy tailed failure distribution for minimizing the angle between the points. Similarly for any non normal distribution this method could be employed and hence the minimum tangent angle also could be obtained.

7. References

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