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Bayesian analysis of truncated skew Laplace distribution

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Abstract
This paper describes the Bayesian inference and prediction of the Truncated Skew Laplace Distribution. The aim of this paper is to obtain the Bayesian inference of the unknown parameters under different loss functions. The Bayes estimates can be obtained and it has been used to compute the Bayes estimates and also to construct symmetric loss function. We consider the posterior predictive density of the future observations and also asymmetric loss function

Keywords: Lifetime distributions, reliability, failure rate, order statistics, exponential distribution, truncated logarithmic distribution

1. Introduction

The quality of the procedures used in a statistical analysis depends heavily on the assumed probability model or distributions. Because of this, considerable effort over the years has been expended in the development of large classes of standard distributions along with relevant statistical methodologies, designed to serve as models for a wide range of real world phenomena. However, there still remain many important problems where the real data does not follow any of the classical or standard models. Very few real world phenomenon's that we need to statistically study are symmetrical. Thus the popular normal model would not be a useful model for studying every phenomenon. The normal model at a time is a poor description of observed phenomena. Skewed models, which exhibit varying degrees of asymmetry, are a necessary component of the modeler's tool kit. Genton, M. [8] mentions that actually an introduction of non-normal distributions can be traced back to the nineteenth century. Edgeworth examined the problem of fitting asymmetrical distributions to asymmetrical frequency data. The aim of the present study is to investigate a probability distribution that can be derived from the Laplace probability distribution and can be used to model various real world problems. In fact, we will develop two probability models namely the skew Laplace probability distribution and the truncated skew Laplace probability distribution and show that these models are better than the existing models some of the real world problems. The Laplace distribution is also known as the law of the difference between two exponential random variables consequently, it is also known as double exponential distribution, as well as the two tailed distribution. It is also known as the bilateral exponential law. In life testing experiments, for example, it is sometimes impossible or inconvenient to measure the life length of a device on a continuous scale; in many practical situations, in fact, reliability data are measured in terms of the number of runs, cycles, or shocks the device sustains before failing.

In survival analysis, one may record the number of days of survival for patients since therapy, or the time from remission to relapse is also usually recorded in the number of days. In all cases, a discrete random variable is the most appropriate model to fit the data. Various families of skew Laplace distributions appeared in the literature in recent years, see, e.g., McGill (1962) [2], Holla and Bhattacharya (1968) [1], Kozubowski and Podg Borski (2001) [3], Among these, the asymmetric Laplace (AL) distributions given by, density over a flexible and natural extension of the traditional symmetric Laplace distribution. The AL laws proved valuable for stochastic modeling in a variety of fields including finance, economics, and sciences Additionally, they arise naturally as limits of random sums with geometrically distributed number of summands.

Thus, if the variable or process of interest can be viewed as a sum of a geometric number of independent increments with finite variance (which is the case in numerous applications, see, its distribution can be approximated by an AL law. Furthermore, AL laws are infinitely divisible, have maximum entropy property under specified mean and first absolute moment conditions, extend naturally to the multivariate setting, and are relatively simple to apply in practice, since their moments are finite and computationally reasonable estimators for all parameters are available. Our test of symmetry can be extended to accommodate other classes of skew distributions defined by the same procedure as the skew Laplace model (1). One such family consists of two-piece normal, or skew normal distributions introduced by Fechner (1897) [7] and rediscovered by Gibbons and Mylroie (1973) [9] in connection with fitting impurity profiles in ion-implantation research (for further applications see, e.g., Here, different scale factors are used on each side of the mode of a normal distribution. Statistical issues and properties of this family were Jrst considered by who obtained maximum likelihood and moment estimators of parameters, and proposed a test of symmetry. also considered the skew normal model and obtained asymptotic distributions of the estimators, developed a likelihood ratio test of symmetry, and compared it with that proposed by Only the asymptotic distributions of the test statistics were considered in these works. The skew normal laws also appeared recently in the context of Bayesian analysis. In fields such as petroleum engineering, civil engineering, geography and geology, spatial prediction is an important problem. Usually, data collected from these disciplines are thought as samples from realizations of random fields. The prevalent technique for statistical spatial prediction is Cressie, 1993. Most of the theories related to spatial prediction assume that the data are generated from a Gaussian random field. However, non-Gaussian characteristics, such as nonnegative continuous variables with skewed distribution, often with a heavy right or left tail, appear in many data sets.

The present chapter considers the Bayesian analysis of skew symmetric distribution using progressive sampling specially related to the Laplace distribution. In this chapter we use some known skew distributions and investigate different characteristics of members of this class such as different characteristics of truncated skew Laplace distribution using modeling of progressive sampling. Using progressive sampling model find the Bayes posterior analysis of truncated skew Laplace distribution. Under the squared error loss function and LINEX loss function, the Bayes estimator of mean and survival function and the failure rate function (hazard function) have been derived.

2. Model and assumption

The probability density function (pdf) and cumulative distribution function of standard Laplce distribution are given by

$$g(x) = \frac{1}{2\phi}\exp\left(\frac{-|x|}{\phi}\right) \tag{2.1}$$

$$G(x) = \begin{cases} \frac{1}{2}\exp\left(\frac{x}{\phi}\right) \; if \; x \le 0 \\ 1 - \frac{1}{2}\exp\left(\frac{-x}{\phi}\right) \; if \; x \ge 0 \end{cases} \tag{2.2}$$

A random variable X follows the skew Laplace distribution if its pdf is given by

$$f(x) = 2g(x)G(\lambda x) \tag{2.3}$$

where $x \epsilon R \; and \; \lambda \epsilon R$

It follows from (2.2.2) and (2.2.3) that the pdf of X is

$$f(x) = \begin{cases} \frac{1}{2\phi}\text{ep}\left(\frac{-(1+|\lambda|)|x|}{\phi}\right) \; if \; \lambda x \le 0 \\ \frac{1}{\phi}\exp\left(\frac{-|x|}{\phi}\right)\left\{1 - \frac{1}{2}\exp\left(\frac{-\lambda x}{\phi}\right)\right\} \; if \; \lambda x > 0 \end{cases} \tag{2.4}$$

Suppose first n_1 observations $x_1, x_2 \cdots x_{n_1}$ are negative and next n_2 observations $x_{n_1+1}, x_{n_1+2}, \cdots x_{n_{1+n_2}}$ $(n_1 + n_2 = n)$ are positive. Further we assume that $\lambda > 0$. Then the likelihood function is given by

$$f(\underline{x}, \lambda, \phi) = \prod_{i=1}^{n_1}\left[\frac{1}{2\phi}\exp\left(\frac{-(1+\lambda)|x_i|}{\phi}\right)\right]\prod_{i=n_1+1}^{n_1+n_2}\left[\frac{1}{\phi}\exp\left(-\frac{x_i}{\phi}\right)\left\{1 - \frac{1}{2}\exp\left(-\frac{\lambda x_i}{\phi}\right)\right\}\right] \tag{2.5}$$

$$= \frac{1}{\emptyset^{n_2+n_1}}\exp\left(-\frac{\sum_{i=1}^{n}|x_i|}{\phi}\right)\exp\left(-\frac{\lambda\sum_{i=1}^{n}|x_i|}{\phi}\right)\prod_{i=n_1+1}^{n_1+n_2}\left\{1 - \frac{1}{2}\exp\left(-\frac{\lambda x_i}{\phi}\right)\right\} \tag{2.6}$$

We observe that

$$I = \prod_{i=n_1+1}^{n_1+n_2}\left\{1 - \frac{1}{2}\exp\left(-\frac{\lambda x_i}{\phi}\right)\right\}$$

$$= \sum_{r=0}^{n}(-\frac{1}{2})^r \sum_{\substack{i_1,\dots\dots i_r = n_1+1 \\ i_1 < i_2 < \cdots\dots < i_r}}^{n} \exp\left[-\frac{\lambda}{\phi}(x_{i_1} + \cdots \dots + x_{i_r})\right]$$

$$= \sum_{r=0}^{n}(-\frac{1}{2})^r \sum_{\substack{i_1,\dots\dots i_r = n_1+1 \\ i_1 < i_2 < \cdots\dots < i_r}}^{n} \exp\left(-\frac{\lambda}{\phi}(S_{i_1\dots\dots\dots i_r})\right)$$

where,

$S_{i_1\dots\dots\dots i_r} = x_{i_1} + x_{i_2} \cdots x_{i_r} \; for \; every \; r > 0, i_1, i_2, \cdots i_r$

and for $r = 0, S_0 = 0$ further we write

$$S = \sum_{i=1}^{n} |x_i|$$

Hence the likelihood function (2.2.6) can be written as

$$f(\underline{x}, \lambda, \phi) = \sum_{r=0}^{n} \sum_{\substack{i_1,\ldots\ldots,i_r=n_1+1 \\ i_1<i_2<\cdots\ldots<i_r}}^{n_2} \frac{1}{\emptyset^{n_2+n_1}} \left(-\frac{1}{2}\right)^{r} \exp\left(\frac{-S}{\emptyset}\right) \exp\left[-\frac{\lambda}{\phi}\left(S_{i_1\ldots\ldots\ldots i_r}\right)\right] \quad (2.7)$$

2.3 Posterior Analysis

2 3.1Prior distribution

For the parameters ϕ and λ we assume the following prior distributions

The prior distribution for $\emptyset$ is inverted gamma distribution with pdf

$$,h_1(\phi, \alpha_1, \beta_1) = \frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1} \phi^{-\alpha_1-1} exp\left(-\frac{\beta_1}{\phi}\right), \phi > 0 \quad (3.1)$$

Further the prior distribution for λ is gamma distribution with pdf

$$h_2(\lambda, \alpha_2, \beta_2) = \frac{1}{\Gamma\alpha_2 {\beta_2}^{\alpha_2}} \lambda^{\alpha_2-1} exp\left(-\frac{\lambda}{\beta_2}\right) \ \lambda > 0 \quad (3.2)$$

3.2 Posterior distribution

Theorem1: For the posterior analysis of skew Laplace distribution the posterior distribution of λ under the prior distribution given in equation (2.3.1) for the λ and (2.3.2) is given by

$$f(\lambda|x) = \frac{\sum_{r=0}^{n_1} \sum_{\substack{i_1,\ldots\ldots,i_r=n_1+1 \\ i_1<i_2<\cdots\ldots<i_r}}^{n_2} \frac{\lambda^{\alpha_2-1} e^{-\frac{\lambda}{\beta_2}}}{\left(S+\lambda S_{i_1\ldots\ldots\ldots i_r}+\beta_1\right)^{n+\alpha_1} \left(S_{i_1,i_2,\cdots i_r}\right)^{(n+\alpha_1)}}}{\sum_{r=0}^{n_1} \sum_{\substack{i_1,\ldots\ldots,i_r=n_1+1 \\ i_1<i_2<\cdots\ldots<i_r}}^{n_2+r-1} \left(\frac{1}{\beta_2}\right)^{\alpha_2-1} \left(S_{i_1,i_2,\cdots i_r}\right)^{(n+\alpha_1)} {}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]} \quad (3.3)$$

Proof

Combining the likelihood function (2.2.7) with the prior distribution (2.3.2) we obtain the posterior distribution of λ as

$$f(\lambda|x) = \frac{\int f(x,\phi,\lambda) h_1(\phi,\alpha_1,\beta_1) h_2(\lambda,\alpha_2,\beta_2) d\phi}{\int \int_{-\infty}^{\infty} f(x,\phi,\lambda) h_1(\phi,\alpha_1,\beta_1) h_2(\lambda,\alpha_2,\beta_2) d\phi d\lambda} \quad (3.4)$$

Let,

$$\int_{-\infty}^{\infty} f(x_i, \lambda, \phi) h_1(\phi_i, \alpha_1, \beta_1)\, h_2(\lambda, \alpha_2, \beta_2) d\phi$$

$$= \int_0^{\infty} \sum_{r=0}^{n} \sum_{\substack{i_1,\ldots\ldots,i_r=n_1+1 \\ i_1<i_2<\cdots\ldots<i_r}}^{n_2} \left(-\frac{1}{2}\right)^{r} \frac{1}{\emptyset^{n_2+n_1}} \exp\left(\frac{-S}{\phi}\right) \exp\left[-\frac{\lambda}{\phi}\left(S_{i_1,i_2,\ldots i_r}\right)\right]$$

$$\frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1} \phi^{-\alpha_1-1} exp\left(-\frac{\beta_1}{\phi}\right) \frac{1}{\Gamma\alpha_2 {\beta_2}^{\alpha_2}} \lambda^{\alpha_2-1} exp\left(-\frac{\lambda}{\beta_2}\right) d\phi$$

$$= \sum_{r=0}^{n} \sum_{\substack{i_1,i_2,\ldots,i_r=n_1+1 \\ i_1<i_2<\cdots\ldots<i_r}}^{n_2} \left(-\frac{1}{2}\right)^{r} \left\{\frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1 \Gamma\alpha_2 {\beta_2}^{\alpha_2}} \lambda^{\alpha_2-1} e^{\left(-\frac{\lambda}{\beta_2}\right)}\right\} \int_0^{\infty} \frac{e^{\left(-\left\{\frac{S}{\phi}+\frac{\lambda}{\phi}\left(S_{i_1,i_2\ldots i_r}\right)+\frac{\beta_1}{\phi}\right\}\right)} d\phi}{\phi^{n+\alpha_1+1}} \quad (3.5)$$

writing,

$$\zeta = \frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1 \Gamma\alpha_2 {\beta_2}^{\alpha_2}} \quad (3.6)$$

and

$$S^* = S + \beta_1 + \lambda S_{i_1,i_2\ldots i_r} \quad (3.7)$$

From equation (2.3.4), (2.3.5) and equation (2.3.6) we get

$$= \sum_{r=0}^{n} \sum_{\substack{i_1,i_2,\ldots,i_r=n_1+1 \\ i_1<i_2<\cdots\ldots<i_r}}^{n_2} \left(-\frac{1}{2}\right)^{r} \left\{\zeta \lambda^{\alpha_2-1} e^{\left(-\frac{\lambda}{\beta_2}\right)}\right\} \int_0^{\infty} \frac{e^{-\frac{S^*}{\phi}}}{\phi^{n+\alpha_1+1}} \ d\phi \quad (3.8)$$

writing

$$\frac{S^*}{\phi} = t$$

$$\frac{1}{\phi} = \frac{t}{S^*} \quad (3.9)$$

$$-\frac{s^* d\phi}{\phi^2} = dt \quad (3.10)$$

Put the value of ϕ and $d\phi$ in equation (3.8) from equation (3.9) and (3.10) we get

$$=\sum_{r=0}^{n}\sum_{\substack{i_1,i_2,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-\frac{1}{2})^r\left\{\zeta\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}\right\}\int_0^\infty\frac{e^{-\frac{S^*}{\phi}}}{\phi^{n+\alpha_1+1}}\,d\phi$$

$$=\frac{\sum_{r=0}^{n}\sum_{\substack{i_1,i_2,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-\frac{1}{2})^r\left\{\zeta\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}\left(-\frac{\lambda}{\beta_2}\right)\right\}}{S^{*n+\alpha_1}}\int_0^\infty t^{n+\alpha_1-1}e^{-t}dt$$

$$=\frac{\sum_{r=0}^{n}\sum_{\substack{i_1,i_2,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-\frac{1}{2})^r}{S^{*(n+\alpha_1)}}\zeta\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}\Gamma(n+\alpha_1) \tag{3.11}$$

Let us find out the value of

$$\int_0^\infty\int_0^\infty f(x_i,\lambda,\phi)h_1(\phi_i,\alpha_1,\beta_1)\,h_2(\lambda,\alpha_2,\beta_2)d\phi\,d\lambda \tag{3.12}$$

Further we have

$$=\int_0^\infty\int_0^\infty\sum_{r=0}^{n}\sum_{\substack{i_1,\dots\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\left(-\frac{1}{2}\right)^r\frac{1}{\phi^{n_2+n_1}}\exp\left(\frac{-S}{\phi}\right)e^{\left[-\frac{\lambda}{\phi}\left(S_{i_1,i_2,\dots i_r}\right)\right]}*$$

$$\frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1}\phi^{-\alpha_1-1}e^{\left(-\frac{\beta_1}{\phi}\right)}\frac{1}{\Gamma\alpha_2{\beta_2}^{\alpha_2}}\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}d\phi$$

$$=\sum_{r=0}^{n}\sum_{\substack{i_1,i_2,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-\frac{1}{2})^r\left\{\frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1\Gamma\alpha_2{\beta_2}^{\alpha_2}}\right\}\int_0^\infty\int_0^\infty\frac{e^{\left(-\left\{\frac{S}{\phi}+\frac{\lambda}{\phi}\left(S_{i_1,i_2\dots i_r}\right)+\frac{\beta_1}{\phi}\right\}\right)}\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}}{\phi^{n+\alpha_1+1}}\,d\phi d\lambda$$

$$=\sum_{r=0}^{n}\sum_{\substack{i_1,i_2,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-\frac{1}{2})^r\,\zeta\int_0^\infty\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}\left[\int_0^\infty\frac{e^{-\frac{S^*}{\phi}}}{\phi^{n+\alpha_1+1}}\,d\phi\right]d\lambda$$

$$=\sum_{r=0}^{n}\sum_{\substack{i_1,i_2,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-\frac{1}{2})^r\,\zeta\frac{\Gamma(n+\alpha_1)}{S^{*(n+\alpha_1)}}\int_0^\infty\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}d\lambda$$

$$=\sum_{r=0}^{n}\sum_{\substack{i_1,i_2,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-\frac{1}{2})^r\,\zeta\Gamma(n+\alpha_1)\int_0^\infty\frac{\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}}{S^{*(n+\alpha_1)}}d\lambda$$

$$=\sum_{r=0}^{n}\sum_{\substack{i_1,i_2\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\left(-\frac{1}{2}\right)^r\Gamma(n+\alpha_1)\int_0^\infty\frac{\lambda^{\alpha_2-1}e^{-\frac{\lambda}{\beta_2}}}{\left(S+\lambda S_{i_1,i_2,\cdots i_r}+\beta_1\right)^{(n+\alpha_1)}}d\lambda$$

$$=\sum_{r=0}^{n}\sum_{\substack{i_1,i_2\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\frac{(-1)^r\beta_1^{\alpha_1}}{\Gamma\alpha_1\Gamma\alpha_2{\beta_2}^{\alpha_2}}\frac{1\Gamma(n+\alpha_1)}{2^r}\int_0^\infty\frac{\lambda^{\alpha_2-1}e^{-\frac{\lambda}{\beta_2}}}{\left[(S+\beta_1)+\lambda S_{i_1,i_2,\cdots i_r}\right]^{(n+\alpha_1)}}d\lambda \tag{3.13}$$

writing,

$$\xi=\frac{(S+\beta_1)}{S_{i_1,i_2,\cdots i_r}} \tag{3.14}$$

$$\zeta=\frac{{\beta_1}^{\alpha_1}}{\Gamma\alpha_1\Gamma\alpha_2{\beta_2}^{\alpha_2}} \tag{3.15}$$

Put the value of ξ and ζ from equation (2.3.12) and(2.3.13) in equation (2.3.10) we get

$$=\sum_{r=0}^{n_1}\sum_{\substack{i_1,i_2\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\frac{(-1)^r}{2^r}\zeta\frac{\Gamma(n+\alpha_1)}{\left(S_{i_1,i_2,\cdots i_r}\right)^{(n+\alpha_1)}}\int_0^\infty(\xi+\lambda)^{-(n+\alpha_1)}\lambda^{\alpha_2-1}e^{-\frac{\lambda}{\beta_2}}d\lambda$$

$$=\sum_{r=0}^{n_1}\sum_{\substack{i_1,i_2\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-)^r\frac{1}{2^r}\frac{\zeta\Gamma(n+\alpha_1)}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}}\left(\frac{1}{\beta_2}\right)^{\alpha_2-1}\;{}_2F_0[(\alpha_2 1)-;(\alpha_2+1+n+\alpha_1);\beta_2] \tag{3.16}$$

and we have utilized the result

$$\left[\int_0^\infty(\xi+\lambda)^{-m}\lambda^{r-1}e^{-\frac{\lambda}{\beta_2}}d\lambda=\frac{1}{{\beta_2}^{r-1}}\;{}_2F_0[r-1;r+m+1;\beta_2]\right]$$

Substituting (2.3.11) and (2.3.16) in (2.3.4) we get

$$f(\lambda|x) = \frac{\int f(x,\phi,\lambda)h_1(\phi,\alpha_1,\beta_1)\,h_2(\lambda,\alpha_2,\beta_2)d\phi}{\int\int_{-\infty}^{\infty} f(x,\phi,\lambda)\,h_{1_1}(\phi,\alpha_1,\beta_1)\,h_2(\lambda,\alpha_2,\beta_2)d\phi d\lambda}$$

$$= \frac{\sum_{r=0}^{n}\sum_{\substack{i_1,i_2,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-\frac{1}{2})^r\zeta\lambda^{\alpha_2-1}e^{\left(-\frac{\lambda}{\beta_2}\right)}\frac{\Gamma(n+\alpha_1)}{s^{*(n+\alpha_1)}}}{\sum_{r=0}^{n_1}\sum_{\substack{i_1,i_2\dots i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-)^r\frac{1}{2^r}\frac{\zeta\Gamma(n+\alpha_1)}{(S_{i_1,i_2\cdots i_r})^{(n+\alpha_1)}}\left(\frac{1}{\beta_2}\right)^{\alpha_2-1}{}_2F_0[(\alpha_2 1)-;(\alpha_2+1+n+\alpha_1);\beta_2]}$$

$$= \frac{\sum_{r=0}^{n_1}\sum_{\substack{i_1,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\frac{\lambda^{\alpha_2-1}e^{-\frac{\lambda}{\beta_2}}}{(S+\lambda S_{i_1,i_2\dots i_r}+\beta_1)^{n+\alpha_1}(S_{i_1,i_2\cdots i_r})^{(n+\alpha_1)}}}{\sum_{r=0}^{n_1}\sum_{\substack{i_1,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\left(\frac{1}{\beta_2}\right)^{\alpha_2-1}(S_{i_1,i_2\cdots i_r})^{(n+\alpha_1)}\;{}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]}$$

which is the required result ■

Theorem 2: The posterior distribution of ϕ under the prior distributions (3.1) and (3.2) for the ϕ and λ is given by

$$f(\phi|x) = \frac{\sum_{r=0}^{n_1}\sum_{\substack{i_1,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\frac{1}{\phi^{n+\alpha_1+1}}exp-\left(\frac{S+\beta_1}{\phi}\right)\frac{\Gamma\alpha_2}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)^{\alpha_2}}}{\sum_{r=0}^{n_1}\sum_{\substack{i_1,i_2\dots i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-)^r\frac{1}{2^r}\frac{1}{(S_{i_1,i_2\cdots i_r})^{(n+\alpha_1)}}\Gamma(n+\alpha_1)\left(\frac{1}{\beta_2}\right)^{\alpha_2-1}{}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]} \tag{3.16}$$

Proof: We observe that

$$\int_{-\infty}^{\infty} f(x,\phi,\lambda)\,h_{1_1}(\phi,\alpha_1,\beta_1)h_2(\lambda,\alpha_2,\beta_2)d\lambda$$

$$= \sum_{r=0}^{n_1}\sum_{\substack{i_1,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1\Gamma\alpha_2\beta_2{}^{\alpha_2}}\left(-\frac{1}{2}\right)^r\frac{1}{\emptyset^{n+\alpha_1+1}}exp\left(\frac{S+\beta_1}{\phi}\right)\int_0^\infty\lambda^{\alpha_2-1}exp\left\{-\lambda\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)\right\}d\lambda \tag{3.13}$$

using equation (3.11) we get equation (3.13)

$$= \sum_{r=0}^{n_1}\sum_{\substack{i_1,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\zeta\left(-\frac{1}{2}\right)^r\frac{1}{\emptyset^{n+\alpha_1+1}}exp-\left(\frac{S+\beta_1}{\phi}\right)\frac{\Gamma\alpha_2}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)^{\alpha_2}} \tag{3.17}$$

using equation (3.11) and equation (3.17) we get

$$f(\phi|x) = \frac{\int f(x,\phi,\lambda)h_1(\phi,\alpha_1,\beta_1)\,h_2(\lambda,\alpha_2,\beta_2)d\lambda}{\int\int_{-\infty}^{\infty} f(x,\phi,\lambda)\,h_{1_1}(\phi,\alpha_1,\beta_1)\,h_2(\lambda,\alpha_2,\beta_2)d\phi d\lambda}$$

$$= \frac{\sum_{r=0}^{n_1}\sum_{\substack{i_1,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\zeta\left(-\frac{1}{2}\right)^r\frac{1}{\emptyset^{n+\alpha_1+1}}exp-\left(\frac{S+\beta_1}{\phi}\right)\frac{\Gamma\alpha_2}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)^{\alpha_2}}}{\sum_{r=0}^{n_1}\sum_{\substack{i_1,i_2\dots i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-)^r\frac{1}{2^r}\frac{\zeta\Gamma(n+\alpha_1)}{(S_{i_1,i_2\cdots i_r})^{(n+\alpha_1)}}\left(\frac{1}{\beta_2}\right)^{\alpha_2-1}{}_2F_0[(\alpha_2 1);(\alpha_2+1+n+\alpha_1);\beta_2]}$$

$$= \frac{\sum_{r=0}^{n_1}\sum_{\substack{i_1,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}\frac{1}{\phi^{n+\alpha_1+1}}exp-\left(\frac{S+\beta_1}{\phi}\right)\frac{\Gamma\alpha_2}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)^{\alpha_2}}}{\sum_{r=0}^{n_1}\sum_{\substack{i_1,i_2\dots i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2}(-)^r\frac{1}{2^r}\frac{1}{(S_{i_1,i_2\cdots i_r})^{(n+\alpha_1)}}\Gamma(n+\alpha_1)\left(\frac{1}{\beta_2}\right)^{\alpha_2-1}{}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]}$$

which is the required result ■

Theorem 3: The posterior mean of parameter λ is given by

$$E(\lambda|x) = \frac{\sum_{r=0}^{n_2}\sum_{\substack{i_1,i_2\cdots i_r=n_1+1\\ i_1<i_2<\cdots i_r}}^{n_2}\frac{1}{2^{\alpha_2-1}}\frac{1}{(\beta_2)^{\alpha_2}}2F_0\left[(\alpha_2;1+\alpha_2-(n+\alpha_1));-\left(\frac{1}{\beta_2}\right)^{-1}\right]}{\sum_{r=0}^{n_1}\sum_{\substack{i_1,\dots,i_r=n_1+1\\ i_1<i_2<\cdots<i_r}}^{n_2+r-1}(-)^r\frac{1}{2^r}\frac{\beta_1^{\alpha_1}}{\Gamma\alpha_1\Gamma\alpha_2\beta_2{}^{\alpha_2}}\Gamma n+\alpha_1\left(\frac{1}{\beta_2}\right)^{\alpha_2-1}2F_0\left[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\left(\frac{1}{\beta_2}\right)^{-1}\right]}\frac{1}{(S_{i_1,i_2\cdots i_r})^{(n+\alpha_1)}} \tag{3.18}$$

Proof: We have

$$E(\lambda|x) = \int_0^\infty \lambda\; f(\lambda|x)\,d\lambda$$

$$= \int_0^\infty \lambda \frac{\sum_{r=0}^{n} \sum_{\substack{i_1,i_2,\dots,i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2} (-\frac{1}{2})^r \zeta \lambda^{\alpha_2-1} e^{\left(-\frac{\lambda}{\beta_2}\right)} \frac{\Gamma(n+\alpha_1)}{s^{*(n+\alpha_1)}}}{\sum_{r=0}^{n_1} \sum_{\substack{i_1,i_2\dots i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2} (-)^r \frac{1}{2^r} \frac{1}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}} \Gamma(n+\alpha_1) \left(\frac{1}{\beta_2}\right)^{\alpha_2-1} {}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]} d\lambda$$

$$= \frac{\sum_{\substack{i_1,\dots i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2} \int_0^\infty \frac{\lambda^{\alpha_2} e^{-\frac{\lambda}{\beta_2}}}{\left(S+\lambda S_{i_1,i_2\cdots i_r}+\beta_1\right)^{n+\alpha_1}} d\lambda}{\sum_{r=0}^{n_1} \sum_{\substack{i_1,i_2\dots i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2} (-)^r \frac{1}{2^r} \frac{1}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}} \Gamma(n+\alpha_1) \left(\frac{1}{\beta_2}\right)^{\alpha_2-1} {}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]} \quad (3.19)$$

Now we integrate the term

$$= \int_0^\infty \frac{\lambda^{\alpha_2} e^{-\frac{\lambda}{\beta_2}}}{\left(S+\lambda S_{i_1,i_2\cdots i_r}+\beta_1\right)} d\lambda \quad (3.20)$$

writing the term

$$\frac{(S+\beta_1)}{S_{i_1,i_2\cdots i_r}} = \delta \quad (3.21)$$

Put the value of δ from equation (3.15) in equation (3.14) we get

$$= \int_0^\infty \frac{\lambda^{\alpha_2} e^{-\frac{\lambda}{\beta_2}}}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)} (\delta+\lambda)^{n+\alpha_1}} d\lambda$$

$$= \frac{1}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}} \frac{1}{(\beta_2)^{-\alpha_2}} \; {}_2F_0\left[\left(\alpha_2; 1+\alpha_2-(n+\alpha_1)\right); \beta_2\right] \quad (3.22)$$

Substituting the value of integral (3.22) in equation (3.19) we get

$$E(\lambda|x) = \frac{\sum_{\substack{i_1,\dots i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2+r-1} \frac{1}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}} \frac{1}{(\beta_2)^{-\alpha_2}} \; {}_2F_0[(\alpha_2;1+\alpha_2-(n+\alpha_1));\beta_2]}{\sum_{r=0}^{n_1} \sum_{\substack{i_1,i_2\dots i_r \\ i_1<i_2<\dots<i_r}}^{n_2} (-)^r \frac{1}{2^r} \frac{\beta_1^{\alpha_1}}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}} \Gamma(n+\alpha_1) \left(\frac{1}{\beta_2}\right)^{\alpha_2-1} {}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]}$$

Theorem 4: The posterior mean of parameter $\emptyset$ is given by

$$E(\phi|x) = \frac{\sum_{r=0}^{n_1} \sum_{\substack{i_1,\dots i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2} \frac{1}{\phi^{n+\alpha_1+1}} exp-\left(\frac{S+\beta_1}{\phi}\right) \frac{\Gamma\alpha_2}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)^{\alpha_2}}}{\sum_{r=0}^{n_1} \sum_{\substack{i_1,i_2\dots i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2} (-)^r \frac{1}{2^r} \frac{\beta_1^{\alpha_1}}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}} \Gamma(n+\alpha_1) \left(\frac{1}{\beta_2}\right)^{\alpha_2-1} {}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]} \quad (3.23)$$

Proof: We have

$$E(\phi|x) = \int_{\mathbf{0}}^\infty \emptyset f(\phi|x)\, d\emptyset$$

$$= \frac{\int_0^\infty \phi \sum_{r=0}^{n_1} \sum_{\substack{i_1,i_2\dots i_r \\ i_1<i_2<\dots<i_r}}^{n_2} \frac{1}{\emptyset^{n+\alpha_1+1}} exp-\left(\frac{S+\beta}{\phi}\right) \frac{\Gamma\alpha_2}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\dots i_r}}{\phi}\right)^{\alpha_2}} d\phi}{\sum_{r=0}^{n_1} \sum_{\substack{i_1,i_2\dots i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2} (-)^r \frac{1}{2^r} \frac{\beta_1^{\alpha_1}}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}} \Gamma(n+\alpha_1) \left(\frac{1}{\beta_2}\right)^{\alpha_2-1} {}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]}$$

$$= \frac{\sum_{r=0}^{n_1} \sum_{\substack{i_1,i_2\dots i_r \\ i_1<i_2<\dots<i_r}}^{n_2} \int_0^\infty \frac{1}{\phi^{n+\alpha_1+1}} exp-\left(\frac{S+\beta}{\phi}\right) \frac{\Gamma\alpha_2}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)^{\alpha_2}} d\phi}{\sum_{r=0}^{n_1} \sum_{\substack{i_1,i_2\dots i_r=n_1+1 \\ i_1<i_2<\dots<i_r}}^{n_2} (-)^r \frac{1}{2^r} \frac{\beta_1^{\alpha_1}}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}} \Gamma(n+\alpha_1) \left(\frac{1}{\beta_2}\right)^{\alpha_2-1} {}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]} \quad (3.24)$$

Let,

$$= \int_0^\infty \emptyset \frac{1}{\emptyset^{n+\alpha_1+1}} \frac{exp-\left(\frac{S+\beta_1}{\phi}\right)}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)^{\alpha_2}} d\emptyset$$

$$= \int_0^\infty \frac{1}{\emptyset^{n+\alpha_1}} \frac{exp-\left(\frac{S+\beta_1}{\phi}\right)}{\left(\frac{1}{\beta_2}+\frac{S_{i_1,i_2\cdots i_r}}{\phi}\right)^{\alpha_2}} d\emptyset \quad (3.25)$$

Let

$\frac{1}{\emptyset} = u$ (3.26)

$-\frac{d\emptyset}{\emptyset^2} = du$ (3.27)

Put the value of $\emptyset$ and $d\emptyset$ from equation (3.18) and (3.19) in equation (3.17) we get

$$= \int_0^\infty \frac{u^{(n+\alpha_1-2)} e^{-u(S+\beta_1)}}{\left(S_{i_1,i_2\ldots i_r}\right)^{\alpha_2} \left(\frac{1}{\beta_2 S_{i_1,i_2\cdots i_r}} + u\right)^{\alpha_2}} du$$

$$= \frac{1}{\left(S_{i_1,i_2\cdots i_r}\right)^{\alpha_2}} \left(\frac{\mathbf{1}}{(S+\beta_1)}\right)^{-(n+\alpha_1-1)} {}_2F_0\left[(n+\alpha_1-1); (n+\alpha_1+\alpha_2); -\left(\frac{1}{(S+\beta_1)}\right)^{-1}\right]$$

(3.28)

put the value of equation (3.28) in equation (3.24) we get

$$= \frac{\frac{1}{\left(S_{i_1,i_2\cdots i_r}\right)^{\alpha_2}}\left(\frac{\mathbf{1}}{(S+\beta_1)}\right)^{-(n+\alpha_1-1)} {}_2F_0\left[(n+\alpha_1-1);(n+\alpha_1+\alpha_2);-\left(\frac{1}{(S+\beta_1)}\right)^{-1}\right]}{\sum_{r=0}^{n_1}\sum_{\substack{i_1,i_2\ldots.i_r=n_1+1\\ i_1<i_2<\cdots\ldots<i_r}}^{n_2}(-)^r\frac{1}{2^r}\frac{\beta_1^{\alpha_1}}{\left(S_{i_1,i_2\cdots i_r}\right)^{(n+\alpha_1)}}\Gamma(n+\alpha_1)\left(\frac{1}{\beta_2}\right)^{\alpha_2-1} {}_2F_0[(\alpha_2-1);(\alpha_2+1+n+\alpha_1);\beta_2]}$$

2.4 Survival Analysis

The survival function for Truncated Skew Laplace distribution is given by

$$S(t) = P(X > t)$$

$$S(t) = 1 - F(t)$$

$$S(t) = 1 - \left[1 + \frac{e^{-\frac{(1+\lambda)t}{\phi}} - 2(1+\lambda)e^{-\frac{t}{\phi}}}{(2\lambda+1)}\right]$$

$$= \frac{2(1+\lambda)}{2\lambda+1} e^{-t\phi}[1 - \frac{1}{2(1+\lambda)} e^{-\frac{\lambda t}{\phi}}]$$

$$= \frac{2(1+\lambda)}{2\lambda+1} e^{-t\phi} - \frac{1}{2(1+\lambda)} e^{-\frac{\lambda t}{\phi}} e^{-t\phi}$$

$$= \frac{2(1+\lambda)}{2\lambda+1} e^{-t\phi} - \frac{1}{2(1+\lambda)} e^{-\frac{t}{\emptyset}(1+\lambda)}$$

Hence under the squared error loss function, the Baye's estimator of survival function of parameter ϕ and λ is given by

$$\hat{S}(t) = \int_0^\infty \int_0^\infty s(t)\, \pi f(x_i, \lambda, \phi) h_1(\lambda) h_2(\phi)\, d\phi\, d\lambda$$

$$= \int_0^\infty \int_0^\infty \frac{2(1+\lambda)}{2\lambda+1} e^{-t\phi} \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r \frac{2^{n-r}(1+\lambda)^{n-r}}{(2\lambda+1)^n \phi^{n_1}} e^{-\frac{1}{\phi}(\sum x_i + n_2 t)} \;*$$

$$e^{-\frac{\lambda}{\phi}(s_{i_1,i_2\cdots i_r})} \frac{{\beta_1}^{\alpha_1}}{\Gamma\alpha_1} \phi^{-\alpha_1-1} e^{-\frac{\beta_1}{\phi}} \frac{1}{\Gamma\alpha_2 {\beta_2}^{\alpha_2}} \lambda^{\alpha_2} e^{-\left(\frac{\lambda}{\beta_2}\right)}\, d\phi\, d\lambda$$

$$= \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r \int_0^\infty \int_0^\infty \frac{2^{n-r+1}(1+\lambda)^{n-r+1}}{(2\lambda+1)} \frac{1}{\phi^{n_1+\alpha_1}} e^{-\frac{1}{\phi}(t+\sum x_i+n_2 t+\beta_1+\lambda(s_{i_1,i_2\cdots i_r})} * \frac{{\beta_1}^{\alpha_1}}{\Gamma\alpha_1\Gamma\alpha_2{\beta_2}^{\alpha_2}} \lambda^{\alpha_2} e^{-\frac{\lambda}{\beta_2}} d\phi\, d\lambda$$

Put the value of $\frac{{\beta_1}^{\alpha_1}}{\Gamma\alpha_1\Gamma\alpha_2{\beta_2}^{\alpha_2}}$ from equation (3.6) we ge

$$= \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \zeta \int_0^\infty \frac{\lambda^{\alpha_2}(1+\lambda)^{n-r+1}}{(2\lambda+1)^{n+1}} e^{-\frac{\lambda}{\beta_2}} \left[\int_0^\infty \frac{1}{\phi^{n_1+\alpha_1+1}} e^{-\frac{1}{\phi}(\eta+\lambda s_{i_1,i_2\cdots i_r})} d\phi\right] d\lambda \quad (4.1)$$

writing,

$\frac{1}{\phi}\left(\eta + \lambda s_{i_1,i_2\cdots i_r}\right) = u$

$\frac{1}{\emptyset} = \frac{u}{(\eta+\lambda s_{i_1,i_2\cdots i_r})}$ (4.2)

$-\frac{d\emptyset}{\emptyset^2} = \frac{du}{(\eta+\lambda s_{i_1,i_2\cdots i_r})}$ (4.3)

Put the value of $\emptyset$ and $d\emptyset$ from equation (4.2) and (4.3) in equation (4.1) we get

$$= \zeta \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \int_0^\infty \frac{\lambda^{\alpha_2}(1+\lambda)^{n-r+1}}{(2\lambda+1)^{n+1}} e^{-\frac{\lambda}{\beta_2}} \left[\int_0^\infty \frac{u^{n_1+\alpha_1-1}}{\left(\eta+\lambda s_{i_1,i_2\cdots i_r}\right)^{n_1+\alpha_1}} e^{-u} du\right] d\lambda$$

$$= \zeta \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \int_0^\infty \frac{\lambda^{\alpha_2}(1+\lambda)^{n-r+1}}{(2\lambda+1)^{n+1}} e^{-\frac{\lambda}{\beta_2}} \frac{\Gamma n_1+\alpha_1}{(\eta+\lambda s_{i_1,i_2\cdots i_r})^{n_1+\alpha_1}} d\lambda$$

$$= \zeta \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \int_0^\infty \frac{\lambda^{\alpha_2-1}(1+\lambda)^{n-r}}{(1+\lambda)^n(1+\frac{\lambda}{1+\lambda})^n} \frac{\Gamma n_1+\alpha_1}{(\eta+\lambda)^{n_1+\alpha_1}} d\lambda$$

$$= \zeta \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \int_0^\infty \frac{\lambda^{\alpha_2}(1+\lambda)^{n-r+1}}{(1+\lambda)^{n+1}(1+\frac{\lambda}{1+\lambda})^{n+1}} e^{-\frac{\lambda}{\beta_2}} \frac{\Gamma n_1+\alpha_1}{(\eta+\lambda s_{i_1,i_2\cdots i_r})^{n_1+\alpha_1}} d\lambda$$

$$=\Gamma n_1+\alpha_1 \zeta \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \int_0^\infty \frac{\lambda^{\alpha_2}(1+\lambda)^{-r}}{(\eta+\lambda s_{i_1,i_2\cdots i_r})^{n_1+\alpha_1}} (1+\frac{\lambda}{1+\lambda})^{-(n+1)} e^{-\frac{\lambda}{\beta_2}} d\lambda \qquad (4.4)$$

writing

$$\zeta^* = \Gamma n_1 + \alpha_1 \zeta \qquad (4.5)$$

Put the value of ζ^* from equation (4.5) in equation (4.4) we get

$$= \zeta^* \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \int_0^\infty \frac{\lambda^{\alpha_2}(1+\lambda)^{-r}}{(\eta+\lambda s_{i_1,i_2\cdots i_r})^{n_1+\alpha_1}} \sum_{k=0}^\infty \binom{n+k}{k} \frac{\lambda^k e^{-\frac{\lambda}{\beta_2}}}{(1+\lambda)^k} d\lambda$$

$$= \zeta^* \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \sum_{k=0}^\infty \binom{n+k}{k} \int_0^\infty \frac{\lambda^{\alpha_2+k}(1+\lambda)^{-(r+k)}}{(\eta+\lambda s_{i_1,i_2\cdots i_r})^{n_1+\alpha_1}} e^{-\frac{\lambda}{\beta_2}} d\lambda$$

$$= \zeta^* \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \sum_{k=0}^\infty \binom{n+k}{k} \frac{1}{(s_{i_1,i_2\cdots i_r})^{n_1+\alpha_1}} \int_0^\infty \frac{\lambda^{\alpha_2+k}(1+\lambda)^{-(r+k)}}{\left[\left(\frac{\eta}{s_{i_1,i_2\cdots i_r}}\right)+\lambda\right]^{n_1+\alpha_1}} e^{-\frac{\lambda}{\beta_2}} d\lambda \qquad (4.6)$$

writing,

$$\frac{\eta}{s_{i_1,i_2\cdots i_r}} = \delta \qquad (4.7)$$

From equation (4.6) and equation (4.7) we get

$$= \zeta^* \sum_{i_1,i_2\cdots i_r=1}^{n_1} \sum_{r=0}^{n_2} \binom{n_2}{r} (-1)^r 2^{n-r+1} \sum_{k=0}^\infty \binom{n+k}{k} \frac{1}{(s_{i_1,i_2\cdots i_r})^{n_1+\alpha_1}} \int_0^\infty \frac{\lambda^{\alpha_2+k}(1+\lambda)^{-(r+k)}}{[\delta+\lambda]^{n_1+\alpha_1}} e^{-\frac{\lambda}{\beta_2}} d\lambda \qquad (4.8)$$

we take the integral part from equation (4.8) we get

$$\int_0^\infty \frac{\lambda^{\alpha_2+k}(1+\lambda)^{-(r+k)}}{[\delta+\lambda]^{n_1+\alpha_1}} e^{-\frac{\lambda}{\beta_2}} d\lambda$$

$$= \int_0^\infty \frac{\lambda^{\alpha_2+k}(1+\lambda)^{-(r+k)}}{\delta^{n_1+\alpha_1}\left[1-\left(\frac{\delta-1}{\delta}\right)\frac{\lambda}{(1+\lambda)}\right]^{n_1+\alpha_1}} e^{-\frac{\lambda}{\beta_2}} d\lambda$$

$$= \int_0^\infty \frac{\left[1-\left(\frac{\delta-1}{\delta}\right)\frac{\lambda}{(1+\lambda)}\right]^{-(n_1+\alpha_1)} \lambda^{\alpha_2+k}(1+\lambda)^{-(r+k)}}{\delta^{n_1+\alpha_1}} e^{-\frac{\lambda}{\beta_2}} d\lambda$$

$$= \int_0^\infty \sum_{j=0}^\infty \binom{n_1+\alpha_1}{j} \left(\frac{\delta-1}{\delta}\right)^j \left[\frac{\lambda}{(1+\lambda)}\right]^j \frac{\lambda^{\alpha_2+k}(1+\lambda)^{-(r+k)}}{\delta^{n_1+\alpha_1}(1+\lambda)^{n_1+\alpha_1}} e^{-\frac{\lambda}{\beta_2}} d\lambda$$

$$= \sum_{j=0}^\infty \binom{n_1+\alpha_1}{j} \left(\frac{\delta-1}{\delta}\right)^j \frac{1}{\delta^{n_1+\alpha_1}} \int_0^\infty \lambda^{j+\alpha_2+k} (1+\lambda)^{-(r+k+j+n_1+\alpha_1)} e^{-\frac{\lambda}{\beta_2}} d\lambda$$

$$= \sum_{j=0}^\infty \binom{n_1+\alpha_1}{j} \left(\frac{\delta-1}{\delta}\right)^j \frac{1}{\delta^{n_1+\alpha_1}}$$

Concluding Remarks

The present paper deals with the Bayesian analysis of skewed Laplace distribution which received attention of many researchers in recent past and applied to model various survival data. The expressions for posterior distribution and Bayes estimators for parameters are derived

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