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On estimation of some Univariate Bayesian frailty models

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Abstract
Bayesian estimation of some univariate distributions is carried out by many authors but none has used frailty model except Parekh *et al*. (2016). In this paper we extend the results of Parekh *et al*. (2016) for some other univariate Bayesian frailty models and Bayesian estimation is carried out for all those models, the list of which is given in the introduction.

Keywords: Frailty distribution, Bayesian frailty estimation, prior distribution, posterior distribution, squared error loss function

1. Introduction
Many authors such as Clayton (1978) [2], Vaupel *et al* (1979) [14] introduced several survival models, amongst which Cox (1972) [3] Proportional Hazard regression (PH) model is well-known. Hanagal (2011) [5] used Weibull distribution when hazard function is linear function of frailty parameter. Parekh *et al*. (2015) [10] have estimated the Survival function with the use of linear hazard function and exponential base line distribution. The classical Bayesian approach for estimation is dealt by many authors, amongst which some of them are Akaike (1983) [1], Le Cam (1990) [9], Joshi (1990) [7], Geyer and Thompson (1992) [4] etc.
Some of the authors who have used Bayesian approach for the frailty models are Ibrahim *et al*. (2001) [6], Santos *et al*. (2010) [13] and they have used Weibull as base line distribution and Gamma, Log normal as frailty distribution. The Inverse Gaussian frailty model is used by Kheiri *et al*. (2007) [8]. Sahu *et al*. (1997) [12] have used prior distribution similar to the Normal distribution with mean 0 and large variance.
In this paper we establish some univariate Bayesian frailty models for the following baseline distributions and Bayesian prior distribution as frailty distribution in sections 2 to 6.

Dealing Section	Baseline distribution	Bayesian prior distribution as frailty distribution.
2	Negative Binomial	Beta
3	Poisson	Gamma
4	Gamma	Gamma
5	Univariate Normal	Gamma
6	Inverse Gaussian	Uniform

Throughout this article Y represents $logT$ where T is lifetime variable. In the special analysis we consider some of the baseline distributions and frailty prior distributions

2. Bayesian Frailty estimation of Negative Binomial distribution with Beta as frailty prior distribution
We consider the following theorem for the baseline distribution as Negative Binomial distribution with prior frailty distribution as Beta distribution.

Theorem 2.1
Let $y_1, y_2, \dots y_n$ be n observations from negative binomial baseline distribution with parameters $m, \theta;\ Neg(m, \theta)$ with known mean m and variance θ and the frailty prior

distribution, $\pi(\theta)$ of θ be $\theta \sim Be(\alpha, \beta)$where α, β be known. Then the Bayesian frailty estimate, $\delta^{\pi}(y)$ of θ is

$$\delta^{\pi}(y) = \frac{mn + \alpha}{mn + \alpha + \sum y_i + \beta}$$

Proof: As $y \sim Neg(m, \theta)$, the p.d.f. of $y \mid \theta$ is

$$f(y_i \mid \theta) = \binom{m + y_i + 1}{y_i} \theta^m (1-\theta)^x$$

and as $\theta \sim Be(\alpha, \beta)$, the p.d.f. of θ is

$$\pi(\theta; \alpha, \beta) = \frac{1}{B(\alpha,\beta)} \theta^{\alpha-1}(1-\theta)^{\beta-1},\ B(\alpha, \beta) = \frac{\Gamma\alpha\Gamma\beta}{\Gamma(\alpha+\beta)}$$

so that the joint p.d.f. of $y_1, y_2, \ldots y_n$ and θ, $h(y;\theta) = g(\theta)\prod_{i=1}^{n} f(y_i/\theta)$ will be

$$h(y_1, y_2, \ldots y_n; \theta) = \binom{m+\sum y_i+1}{\sum y_i} \frac{\theta^{nm}(1-\theta)^{\sum y_i}\theta^{\alpha-1}(1-\theta)^{\beta-1}}{B(\alpha,\beta)}$$

$$= \binom{m+\sum y_i+1}{\sum y_i} \frac{\theta^{nm+\alpha-1}(1-\theta)^{\sum y_i+\beta-1}}{B(\alpha,\beta)} \tag{2.1}$$

and the marginal distribution of $y, m(y)$ is

$$m(y) = \binom{m+\sum y_i+1}{\sum y_i} \frac{1}{B(\alpha,\beta)} \int_0^1 \theta^{nm+\alpha-1}(1-\theta)^{\sum y_i+\beta-1} d\theta$$

$$= \binom{m+\sum y_i+1}{\sum y_i} \frac{B(nm+\alpha, \sum y_i+\beta)}{B(\alpha,\beta)} \tag{2.2}$$

We get the posterior frailty distribution of θ from (2.1) and (2.2) as conditional distribution of θ given $y, \pi(\theta \mid y)$

$$\pi(\theta \mid y_1, y_2, \ldots y_n) = \frac{\theta^{nm+\alpha-1}(1-\theta)^{\sum y_i+\beta-1}}{B(nm+\alpha, \sum y_i+\beta)} \sim Be(nm+\alpha, \sum y_i + \beta)$$

Taking squared error loss function, the Bayesian frailty estimate, $\delta^{\pi}(y)$ of θ will be

$$\delta^{\pi}(y) = \frac{mn + \alpha}{mn + \alpha + \sum y_i + \beta}$$

Remark 2.1

Considering loss function $L(\theta, \delta) = (\delta - \frac{1}{\theta})^2$ the Bayesian frailty estimate, $\delta_1^{\pi}(y)$ of $\frac{1}{\theta}$ is then

$$\delta_1^{\pi}(y) = \binom{m+\sum y_i+1}{\sum y_i} \frac{1}{B(\alpha,\beta)} \int_0^1 \frac{1}{\theta} \theta^{nm+\alpha-1}(1-\theta)^{\sum y_i+\beta-1} d\theta = \frac{nm+\alpha-1}{mn+\alpha+\sum y_i+\beta}$$

3. Bayesian Frailty estimation of Poisson distribution with Gamma as frailty prior distribution.

The following theorem discusses the estimation of Poisson distribution with Gamma frailty distribution.

Theorem 3.1

Let Y have Poisson baseline distribution, $P(\theta)$ with parameters θ. The frailty prior distribution, $\pi(\theta)$ of θ be $\theta \sim G(\alpha, \beta)$ where α, β be known then the Bayesian frailty estimate, $\delta^{\pi}(y)$ of θ is

$$\delta^{\pi}(y) = \frac{y + \alpha}{\beta + 1}$$

Proof: As, the p.d.f. of $y \mid \theta$ is

$f(y \mid \theta,) = \frac{e^{-\theta}\theta^y}{y!}$, $y > 0, \theta > 0$ and as $\theta \sim G(\alpha, \beta)$, the p.d.f. of θ is

$$\pi(\theta; \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma\alpha} \theta^{\alpha-1} e^{-\beta\theta},\ 0 \leq y < \infty$$

so that joint probability distribution $y, h(y;\theta)$ given θ will be

$$h(y, \theta) = \frac{\beta^{\alpha}}{\Gamma\alpha y!} \theta^{y+\alpha-1} e^{-(\beta+1)\theta} \tag{3.1}$$

and the marginal distribution of $y, m(y)$ is

$$m(y) = \frac{\beta^{\alpha}}{\Gamma\alpha\, y!} \int_0^{\infty} \theta^{y+\alpha-1} e^{-(\beta+1)\theta} d\,\theta = \frac{\beta^{\alpha}}{\Gamma\alpha\, y!} \frac{\Gamma(y+\alpha)}{(\beta+1)^{y+\alpha}} \tag{3.2}$$

We get the posterior frailty distribution of θ from (3.1) and (3.2) as conditional distribution of θ given $y, \pi(\theta \mid y)$

$$\pi(\theta \mid y) = \frac{\theta^{y+\alpha-1} e^{-(\beta+1)\theta} (\beta+1)^{y+\alpha}}{\Gamma(y+\alpha)} = G(y+\alpha, \beta+1)$$

Taking squared error loss function, the Bayesian frailty estimate, $\delta^{\pi}(y)$ of θ will be

$$\delta^{\pi}(y) = \frac{y+\alpha}{\beta+1}$$

Remark 3.1
Considering loss function

$$L(\theta, \delta) = (\delta - \frac{1}{\theta})^2$$

The Bayesian frailty estimate, $\delta_1^{\pi}(y)$ of $\frac{1}{\theta}$ is then

$$\delta_1^{\pi}(y) = \frac{(\beta+1)^{y+\alpha}}{\Gamma(y+\alpha)} \int_0^{\infty} \frac{1}{\theta} \theta^{y+\alpha-1} e^{-(\beta+1)\theta} d\,\theta = \frac{\beta+1}{y+\alpha-1}$$

Remark 3.2
If $y_1, y_2, \ldots, y_n$ are n observations from Poisson baseline distribution, $P(\theta)$ with parameter θ , and if θ has Gamma distribution $G(\alpha, \beta)$ then the Bayesian frailty estimates of θ, $\delta_2^{\pi}(y)$ and the Bayesian frailty estimates of $\frac{1}{\theta}$, $\delta_3^{\pi}(y)$ are

$\delta_2^{\pi}(y) = \frac{\sum y_i+\alpha}{\beta+1}$ and $\delta_3^{\pi}(y) = \frac{\beta+1}{\sum y_i+\alpha-1}$ respectively.

4. Bayesian Frailty estimation of Gamma distribution with Gamma as frailty prior distribution.

Taking baseline distribution as Gamma distribution with prior frailty distribution as Gamma distribution, we obtain the Bayesian frailty estimation of shape parameter in the following theorem 4.1.

Theorem 4.1
Let Y have Gamma baseline distribution with parameters, θ; $G(\nu, \theta)$, where the shape parameter ν is known and the frailty prior distribution, $\pi(\theta)$ of θ be $\theta \sim$G(α, β) where α, β be known then the Bayesian frailty estimate $\delta^{\pi}(y)$ of θ is

$$\delta^{\pi}(y) = \frac{\alpha+\nu}{y+\beta}$$

Proof: As $y \sim G(\nu, \theta)$, the p.d.f. of $y \mid \theta$ is

$$f(y \mid \theta, \nu) = \frac{\theta^{\nu}}{\Gamma\nu} y^{\nu-1} e^{-\theta y} \text{ , } 0 \leq y < \infty$$

and as $\theta \sim G(\alpha, \beta)$, the p.d.f. of θ is

$$\pi(\theta; \alpha, \beta) = \frac{\beta^{\alpha}}{\Gamma\alpha} \theta^{\alpha-1} e^{-\beta\theta} \text{ , } 0 \leq y < \infty$$

so that joint probability distribution y, $h(y; \theta)$ given θ will be

$$h(y, \theta) = \frac{\beta^{\alpha} y^{\nu-1}}{\Gamma\nu\, \Gamma\beta} \theta^{\alpha+\nu-1} e^{-(y+\beta)\theta} \tag{4.1}$$

and the marginal distribution of y, $m(y)$ is

$$m(y) = \frac{\beta^{\alpha} y^{\nu-1}}{\Gamma\nu\, \Gamma\beta} \int_0^{\infty} \theta^{\alpha+\nu-1} e^{-(y+\beta)\theta}\, d\theta$$

$$= \frac{\beta^{\alpha} y^{\nu-1}}{\Gamma\nu\, \Gamma\beta} \frac{\Gamma(\alpha+\nu)}{(y+\beta)^{\alpha+\nu}} \tag{4.2}$$

We get the posterior frailty distribution of θ from (4.1) and (4.2) as conditional distribution of θ given y, $\pi(\theta \mid y)$

$$\pi(\theta \mid y) = \frac{(y+\beta)^{\alpha+\nu}}{\Gamma(\alpha+\nu)} .\, \theta^{\alpha+\nu-1} e^{-(y+\beta)\theta}$$

Taking squared error loss function, the Bayesian frailty estimate, $\delta^{\pi}(y)$ of θ will be

$$\delta^{\pi}(y) = \frac{\alpha+\nu}{\beta+y}$$

Remark 4.1

Considering loss function

$$L(\theta,\delta) = (\delta - \frac{1}{\theta})^2$$

The Bayesian frailty estimate, $\delta_1^\pi(y)$ of $\frac{1}{\theta}$ is then

$$\delta_1^\pi(y) = \frac{(y+\beta)^{\alpha+\nu}}{\Gamma(\alpha+\nu)} \int_0^\infty \frac{1}{\theta}\theta^{\alpha+\nu-1} e^{-(y+\beta)\theta}\, d\theta$$

$$= \frac{\beta+y}{\alpha+\nu-1}$$

Remark 4.2

If $y_1, y_2, \dots, y_n$ are n observations from Gamma distribution, $G(\nu,\theta)$ and if θ has Gamma distribution $G(\alpha,\beta)$ then the Bayesian frailty estimates of θ, $\delta_2^\pi(y)$ and the Bayesian frailty estimates of $\frac{1}{\theta}$, $\delta_3^\pi(y)$ are

$$\delta_2^\pi(y) = \frac{\alpha+\nu}{\beta + \sum y_i}$$

and

$\delta_3^\pi(y) = \frac{\beta+\sum y_i}{\alpha+\nu-1}$ respectively.

5. Bayesian Frailty estimation of Univariate Normal distribution with Gamma as frailty prior distribution.

Considering Univariate Normal distribution, $N(\mu,\frac{1}{\theta})$ as Baseline distribution and prior distribution of variance, $\frac{1}{\theta}$ is $G(\frac{\alpha}{2},\frac{\beta}{2})$, we have obtained Bayes estimator (as frailty parameter) by taking squared error loss function in the following theorem 5.1 and also discussed two important remarks.

Theorem 5.1

Let Y have Normal baseline distribution with parameters $\mu, \frac{1}{\theta}$ as $N(\mu,\frac{1}{\theta})$, with known mean μ and variance $\frac{1}{\theta}$ and the frailty prior distribution, $\pi(\theta)$ of θ be $\theta \sim G(\frac{\alpha}{2},\frac{\beta}{2})$ where α, β be known then the Bayesian frailty estimate, $\delta^\pi(y)$ of θ is

$$\delta^\pi(y) = \frac{\alpha+1}{(y-\mu)^2+\beta}$$

Proof: As $y \sim N(\mu,\frac{1}{\theta})$, the p.d.f. of $y \mid \theta$ is

$$f(y \mid \theta) = \frac{\sqrt{\theta}}{\sqrt{2\pi}} e^{-\frac{\theta}{2}(y-\mu)^2}$$

and as $\theta \sim G(\frac{\alpha}{2},\frac{\beta}{2})$, the p.d.f. of θ is

$$\pi\left(\theta; \frac{\alpha}{2},\frac{\beta}{2}\right) = \frac{\left(\frac{\beta}{2}\right)^{\frac{\alpha}{2}}}{\Gamma\left(\frac{\alpha}{2}\right)} \theta^{\frac{\alpha}{2}-1} e^{-\frac{\beta}{2}\theta} ,$$

so that the joint p.d.f. of y and θ, $h(y;\theta)$ will be

$$h(y,\theta) = \frac{\sqrt{\theta}}{\sqrt{2\pi}} e^{-\frac{\theta}{2}(y-\mu)^2} \frac{\left(\frac{\beta}{2}\right)^{\frac{\alpha}{2}}}{\Gamma\left(\frac{\alpha}{2}\right)} \theta^{\frac{\alpha}{2}-1} e^{-\frac{\beta}{2}\theta}$$

$$= \frac{\left(\frac{\beta}{2}\right)^{\frac{\alpha}{2}}}{\Gamma\left(\frac{\alpha}{2}\right)\sqrt{2\pi}} \theta^{\frac{\alpha}{2}+\frac{1}{2}-1} e^{-\frac{\theta}{2}[(y-\mu)^2+\beta]} \tag{5.1}$$

and the marginal distribution of $y, m(y)$ is

$$m(y) = \frac{\left(\frac{\beta}{2}\right)^{\frac{\alpha}{2}}\Gamma\left(\frac{\alpha}{2}+\frac{1}{2}\right)}{\Gamma\left(\frac{\alpha}{2}\right)\sqrt{2\pi}\left[\frac{(y-\mu)^2+\beta}{2}\right]^{\frac{\alpha}{2}+\frac{1}{2}}} \tag{5.2}$$

We get the posterior frailty distribution of θ from (5.1) and (5.2) as conditional distribution of θ given $y, \pi(\theta \mid y)$

$$\pi(\theta \mid y) = \frac{\left[\frac{(y-\mu)^2+\beta}{2}\right]^{\frac{\alpha}{2}+\frac{1}{2}}}{\Gamma\left(\frac{\alpha+1}{2}\right)}\theta^{\frac{\alpha}{2}+\frac{1}{2}-1}e^{-\frac{\theta}{2}[(y-\mu)^2+\beta]} \sim G\left(\frac{\alpha+1}{2}, \frac{(y-\mu)^2+\beta}{2}\right)$$

Taking squared error loss function, the Bayesian frailty estimate, $\delta^{\pi}(y)$ of θ will be

$$\delta^{\pi}(y) = \frac{\alpha+1}{(y-\mu)^2+\beta}$$

Remark 5.1

Considering loss function

$$L(\theta, \delta) = (\delta - \frac{1}{\theta})^2$$

The Bayesian frailty estimate, $\delta_1^{\pi}(y)$ of $\frac{1}{\theta}$ is then

$$\delta_1^{\pi}(y) = \frac{\left[\frac{(y-\mu)^2+\beta}{2}\right]^{\frac{\alpha+1}{2}}}{\Gamma\left(\frac{\alpha+1}{2}\right)}\int_0^{\infty}\frac{1}{\theta}\,\theta^{\frac{\alpha+1}{2}-1}e^{-\frac{\theta}{2}[(y-\mu)^2+\beta]}$$

$$= \frac{(y-\mu)^2+\beta}{\alpha-1}$$

Remark 5.2

If $y_1, y_2, \dots, y_n$ are n observations from Normal distribution, $N(\mu, \frac{1}{\theta})$, and if θ has Gamma distribution, $G(\frac{\alpha}{2}, \frac{\beta}{2})$ then the Bayesian frailty estimates of θ, $\delta_2^{\pi}(y)$ and the Bayesian frailty estimates of $\frac{1}{\theta}$, $\delta_3^{\pi}(y)$ are

$\delta_2^{\pi}(y) = \frac{\alpha+1}{(\sum y_i-\mu)^2+\beta}$ and $\delta_3^{\pi}(y) = \frac{(\sum y_i-\mu)^2+\beta}{\alpha-1}$ respectively.

6. Bayesian Frailty estimation of Inverse Gaussian distribution with Uniform as frailty prior distribution.

We consider the following theorem for the baseline distribution as Inverse Gaussian distribution with prior frailty distribution as Uniform distribution.

Theorem 6.1

Let Y have Inverse Gaussian baseline distribution, $IG(y \mid \theta)$ with p.d.f.

$$f(y \mid \theta) = \left(\frac{\theta}{2\pi y^3}\right)^{\frac{1}{2}} exp\left\{-\frac{\theta(y-1)^2}{2y}\right\}; \text{y} > 0, \theta > 0 \tag{6.1}$$

and let frailty prior distribution, $\pi(\theta)$ of θ be proportional to $\frac{1}{\theta}$.

i.e. $\pi(\theta) = \frac{k}{\theta}$, where k is a known constant.

Then Bayesian frailty estimate, $\delta^{\pi}(y)$ of θ is

$$\delta^{\pi}(y) = \frac{y}{(y-1)^2}$$

Proof: Since, $Y \sim IG(y, \theta)$, the p.d.f. of $y \mid \theta$ is

$$f(y \mid \theta) = \left(\frac{\theta}{2\pi y^3}\right)^{\frac{1}{2}} exp\left\{-\frac{\theta(y-1)^2}{2y}\right\}; \text{y} > 0, \theta > 0$$

and prior distribution of θ is

$\pi(\theta) = \frac{k}{\theta}$, where k is a known constant.

So that the joint probability distribution of y and θ, $h(y,\theta)$ is

$$h(y,\theta) = k\left(\frac{\theta}{2\pi\lambda y^3}\right)^{\frac{1}{2}} exp\left\{-\frac{\theta(y-1)^2}{2y}\right\} \quad (6.2)$$

and the marginal distribution of $y, m(y)$ is

$$m(y) = \frac{k}{\sqrt{2\pi y^3}}\int_0^\infty \frac{\exp\left\{-\frac{\theta(y-1)^2}{2y}\right\}}{\sqrt{\theta}}d\theta$$

$$= \frac{k}{\sqrt{2\pi y^3}}\int_0^\infty \theta^{\frac{1}{2}-1}e^{-a\theta}d\theta \text{ , where } a = \frac{(y-1)^2}{2y}$$

$$= \frac{k}{\sqrt{2\pi y^3}}\frac{\Gamma(\frac{1}{2})}{a^{\frac{1}{2}}}$$

$$= \frac{k\sqrt{2y}}{\sqrt{2\pi y^3}(y-1)}$$

$$= \frac{k}{y(y-1)}. \quad (6.3)$$

We get the posterior frailty distribution of θ from (6.2) and (6.3) as conditional distribution of θ given $y, \pi(\theta \mid y)$

$$\pi(\theta \mid y) = \frac{(y-1)\exp\left\{-\frac{\theta(y-1)^2}{2y}\right\}}{\sqrt{2\pi y\theta}}$$

$$= \sqrt{\frac{a}{\pi}}\theta^{\frac{1}{2}-1}e^{-a\theta}, \theta > 0$$

which is proportional to Gamma distribution with shape parameter $\frac{1}{2}$ and scale parameter $\frac{(y-1)^2}{2y}$.

Taking squared error loss function the Bayesian frailty estimate, $\delta^\pi(y)$ of θ will be

$$\delta^\pi(y) = \frac{y}{(y-1)^2}$$

Remark 6.1

If $y_1, y_2, \dots, y_n$ are observations from Gaussian distribution defined in (6.1) then the posterior distribution of θ given $y_1, y_2, \dots, y_n$ will be proportional to Gamma distribution with shape parameter $\frac{n}{2}$ and scale parameter $\frac{(\sum y_i-1)^2}{2\sum y_i}$ and hence Bayesian frailty estimate of θ is

$$\delta_1^\pi(y) = \frac{n\sum y_i}{(\sum y_i-1)^2}, \sum y_i \neq 1$$

Remark 6.2

Considering n observations $y_1, y_2, \dots, y_n$ from the Gaussian distribution defined in (6.1), the Bayesian frailty estimate, $\delta_2^\pi(y)$ of $\frac{1}{\theta}$ is

$$\delta_2^\pi(y) = \frac{a_1^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}\int_0^\infty \frac{1}{\theta}\theta^{\frac{n}{2}-1}e^{-a_1\theta}, \qquad \text{where, } a_1 = \frac{(\sum y_i-1)^2}{2\sum y_i}$$

$$= \frac{a_1^{\frac{n}{2}}}{\Gamma(\frac{n}{2})}\frac{\Gamma(\frac{n}{2}-1)}{a_1^{\frac{n}{2}-1}}$$

$$= \frac{(\sum y_i-1)^2}{(n-2)\sum y_i}, n>2$$

7. Conclusion: In this article we have discussed some continuous models with prior distributions as frailty and using the conditional distribution of the frailty parameter of the prior, we obtained frailty posterior mean which is the Bayesian frailty estimator when loss function is squared error.

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