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Generalized contractive conditions and single valued mapping in complete metric space

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Abstract
There are great number of generalizations of well-known Banach contraction mapping principle, M. Edelstein [01] was extended and defined contractive mapping. A contractive mapping is always continuous and which has a unique fixed point. M. Edelstein [01] proved that if T is a contractive mapping on a compact metric space (X,d) to itself then there exists a unique fixed point of T.

Keywords: Contraction mapping, contractive mapping, cauchy sequence and single-valued mapping

1. Introduction

In functional analysis the fixed point theory having incredible research field in applied mathematics. Also it has various applications to non-linear Sciences, Stefan Banach had proved one of the most famous result of fixed point theorem, which is the initial path in this direction of metric fixed point theory. A common fixed point theorem in metric space generally involves conditions of continuity, commutativity and contraction conditions with completeness. In 1976 G. Jungck [10] was the first mathematician who generalized the Banach contraction theorem by using commuting mappings and it has open problem that a pair of commuting and continuous self mapping in the interval *[0,1]* which has not a common fixed point.

There are great numbers of generalizations of well-known Banach contraction mapping principle. M. Edelstein [01] extended and defined contractive mapping such as "A mapping T of a metric space (X,d) into itself is said to be contractive if

$$d(T(x),T(y)) < d(x,y), \text{ for } x \neq y \text{ and } x,y \in X.$$

A contraction mapping is always continuous and which has a unique fixed point. M. Edelstein [01] proved that if T is a contractive mapping on a compact metric space (X,d) to itself then there exists a unique fixed point of T.

Now we consider some important generalization of Banach contraction mapping principle. In 1969 D. W. Boyd and J. S. W. Wong [02] obtain the following generalization of contraction mapping theorem.

Definition 1.1

A function $\delta : R_+ \to R_+$ is said to be upper semi continuous from the right if $r_n \downarrow r \geq 0$, $\therefore \limsup_{n\to\infty} \delta(r_n) \leq \delta(r)$.

Theorem 1.1

In a complete metric space (X,d) if $T: X \to X$ satisfied

$$d(T(x),T(y)) \leq \delta[d(x,y)], \text{ for all } x,y \in X.$$

If $\delta : R \to [0,\infty)$ be upper semi-continuous from the right such that $\delta(t) \in [0,t)$, $t > 0$ then T has a unique fixed point in X and $\{T^n(x)\}$ converges to fixed point for all $x \in X$.

Proof: For any fixed point i.e. $x \in X$, let $x_n = T^n(x)$ for any $n= 1,2,3……\infty$ and $a_n = d(x_n, x_{n+1}) = d(T^n(x), T^{n+1}(x))$. Here we show that a_n is convergent. Assume that $a_n > 0$ for all $n > 0$ then for all $n > 1$.

$$\therefore \; a_n = d[T^n(x), T^{n+1}(x)] = d[T(x_{n-1}), T(x_n)]$$
$$\leq \delta\left[d(x_{n-1}, x_n)\right] = \delta(a_{n-1})$$
$$< a_{n-1}.$$

Hence the sequence $\{a_n\}$ is monotonically decreasing and bounded below, so it is convergent.

Let $\lim_{n\to\infty} a_n = a$ we show that $a=0$, if $a > 0$ then $a_{n+1} \leq \delta(a_n)$.

Then by the upper semi continuity from the right of the function δ we get $a \leq \delta(a)$ which is a contradiction with the property of δ . Thus $a = 0$ and $a_n \to 0$ as $n \to \infty$.

We say that $\{x_n\}$ is a Cauchy sequence but assume that the sequence $\{x_n\}$ is not a Cauchy sequence then there exist $\alpha > 0$ such that for any $k \in N$ there exist $m_k > n_k \geq k$ such that

$$d(x_{mk}, x_{nk}) \geq \alpha \qquad …..(1)$$

Let us assume that for each k, the smallest number $m_k > n_k$ for each equation (1) holds $a_k = d(x_{mk}, x_{nk})$.

Since $\lim_{n\to\infty} d(x_n, x_{n+1}) = \lim_{n\to\infty} a_n = 0$ there exist k_0 such that $d(x_k, x_{k+1}) \leq \alpha$ for all $k \geq k_0$ for each k we have

$$\alpha \leq d(x_{mk}, x_{nk}) \leq d(x_{mk}, x_{mk-1}) + d(x_{mk-1}, x_{nk})$$
$$\leq d(x_{mk}, x_{mk-1}) + \alpha \leq d(x_k, x_{k-1}) + \alpha .$$

It proves that $\lim_{k\to\infty} d(x_{mk}, x_{nk}) = \lim_{k\to\infty} a_k = \alpha$.

On the other hand we have,

$$d(x_{mk}, x_{nk}) \leq d(x_{mk}, x_{mk+1}) + d(x_{mk+1}, x_{nk+1}) + d(x_{nk+1}, x_{nk})$$
$$\leq a_{mk} + \delta(d(x_{mk}, x_{nk})) + a_{nk}$$
$$\leq 2a_k + \delta(d(x_{mk}, x_{nk})).$$

As $k \to \infty$ we obtain the following condition

$$\alpha = \lim_{n\to\infty} d(x_{mk}, x_{nk}) \leq \lim_{n\to\infty}(2a_k + \delta(x_{mk-1}, x_{nk}) = \delta(\alpha) .$$

Thus $\alpha \leq \delta(\alpha)$ this is contradiction. Hence $\{T^n(x)\} = \{x_n\}$ is a Cauchy sequence. Since $\{T^n(x)\}$ is a Cauchy sequence and X is complete.

Therefore $\lim_{n\to\infty} T^n(x) = x$ for $x \in X$. Since T is continuous hence $T(x) = x$.

Remark 1.1 In above theorem 1.1 if we replace the condition $\delta(t) < t$ by the condition $\delta(t_0) < t_0$ for at least one value to t_0 then *T* may not have a fixed point.

Example 1.1 Let $X = (-\infty, -1] \cup [1, \infty)$ be a metric space with the metric *X* and let

$$T_1(x) = \begin{cases} \frac{1}{2}(x+1), if.x \geq 1 \\ \frac{1}{2}(x-1), if.x \leq -1 \end{cases}$$ and $T_2(x) = -T_1(x)$, for all $x \in X$.

Hence T_1 and T_2 satisfies the equation (1)

$$\therefore \quad \delta(t) = \begin{cases} \frac{1}{2}t, if.t < 2 \\ \frac{1}{2}(t+1), if.t \geq 2 \end{cases}.$$

Hence the function δ satisfies all the conditions in above theorem except $\delta(2) = 2$. And we observe that T_1 has two fixed points -1 and 1, while T_2 has no fixed points.

In the following result the continuity condition on δ is replaced with another suitable condition.

Theorem 1.2

In a complete metric space (X, d) let $T : X \to X$ be the mapping has satisfies

$$d(T(x), T(y)) \leq \delta[d(x, y)], \text{ for all } x, y \in X .$$

where $\delta : (0, \infty) \to (0, \infty)$ be monotone non-decreasing function and satisfies $\lim_{n\to\infty} \delta^n(t) = 0$ for all $t > 0$ then it has a unique fixed point *x* and

$$\lim_{n\to\infty} d(T^n(x), x) = 0 \quad \text{for all } x \in X .$$

Proof. Let $x_n = T^n(x)$ for $n = 1, 2, 3.......\infty$ for any $x \in X$ then $x_1 = T(x) \neq x$ otherwise *x* would be a fixed point of *T* then

$$\begin{aligned} d(T^n(x), T^{n+1}(x)) &\leq \delta(T^{n-1}(x), T^n(x)) \\ &\leq \delta^2(d(T^{n-2}(x), T^{n-1}(x))) \\ &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ &\leq \delta^n(d(x), T(x)) = \delta^n(d(x, x_1)). \end{aligned}$$

Hence

$$0 \leq \lim_{n\to\infty} d(x_n, x_{n+1}) = \lim_{n\to\infty} d(T^n(x), T^{n+1}(x))$$

$$\leq \lim_{n\to\infty} \delta^n[d(x, x_1)] = 0$$

Thus $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0.$

We show that $\{x_n\}$ is a Cauchy sequence. Since $\delta^n(t) \to 0$ for all $t > 0$, $\delta(\alpha) < \alpha$ for any $\alpha > 0$.

Since $\lim_{n\to\infty} d(x_n, x_{n+1}) = 0$ for any $\alpha > 0$, then we choose *n* such that

$$d(x_{n+1}, x_n) \leq \alpha - \delta(\alpha).$$

Let $P_\alpha(x_n) = \{x \in X : d(x, x_n) \leq \alpha\}$ if $z \in P_\alpha(x_n)$ then $d(z, x_n) \leq \alpha$ and

$$d(T(z), x_n) \leq d(T(z), T(x_n)) + d(T(x_n), x_n)$$
$$\leq \delta(d(z, x_n)) + d(x_{n+1}, x_n) \text{ as } T(x_n) = x_{n+1}$$
$$\leq \delta(\alpha) + (\alpha - \psi(\alpha)) = \alpha.$$

Therefore $T(z) \in P_\alpha(x_n)$ and $T : P_\alpha(x_n) \to P_\alpha(x_n)$.

It follows that $d(x_m, x_n) \leq \alpha$ for all $m \geq n$ and hence $\{x_n\}$ is a Cauchy sequence. Which is the conclusion of our proof follows as in above theorem 1.4.1.

Now in the following theorem we present a different kind of principle in which the contractive condition is imposed only at the first step.

Theorem 1.3

Let (X, d) is a complete metric space and $T : X \to X$ be a continuous mapping such that for some function $\gamma : X \to R$ the following condition holds

$$d(x, T(x)) \leq \gamma(x) - \gamma(T(x)), \text{ for } x \in X. \quad \text{.....(I)}$$

then $\{T^n(x)\}$ converges to a fixed point of *T* for all $x \in X$.

Proof. For any $x \in X$ let $x_n = T^n(x)$ for $n = 1, 2, 3.......\infty$ then by the inequality (I) we have $0 \leq \gamma(x) - \gamma(T(x))$ if and only if $\gamma(T(x) \leq \gamma(x)$ for all $x \in X$.

$$\therefore \gamma(x_{n+1}) = \gamma(T^{n+1}(x)) = \gamma(T(T^n(x))),$$
$$= \gamma(T^n(x) \leq \gamma(x_n)).$$

Thus $\{\gamma(T^n(x)\} = \{\gamma(x_n)\}$ is monotonically decreasing and bounded below.

Hence $\lim_{n\to\infty} \gamma(T^n(x)) = r \geq 0$, by the triangle inequality if $m, n \in N \;\&\; m > n$ then

$$d(T^n(x), T^m(x)) \leq d(T^n(x), T^{n+1}(x)) + d(T^{n+1}(x), T^{n+2}(x)) + ... + d(T^{m-1}(x), T^m(x))$$
$$\leq \gamma(T^n(x)) - \gamma(T^{n+1}(x)) + \gamma(T^{n+1}(x)) - \gamma(T^{n+2}(x)) + ... + \gamma(T^{m-1}(x)) - \gamma(T^m(x))$$
$$\leq \gamma(T^n(x)) - \gamma(T^m(x)).$$

Hence $\lim_{m,n\to\infty} d(T^n(x), T^m(x)) = 0.$

It follows that $\{T^n(x)\} = \{x_n\}$ is a Cauchy sequence in *X*.

Since *X* is complete there exist $x \in X$ such that $\lim_{n\to\infty} T^n(x) = x$ and by continuity of T we get $T(x) = x$.

Remark 1.2 In above theorem 1.3 we can obtain an estimate on the rate of convergence of $\{T^n(x)\}$ by referring back to the inequality

$$\sum_{i=n}^{m-1} d(T^i(x), d(T^{i+1}(x)) \leq \gamma(T^n(x)) - \gamma(T^m(x))$$

This yield $d(T^n(x)), d(T^m(x)) \leq \gamma(T^n(x)) - \gamma(T^m(x)) \leq \gamma(T^n(x))$

and if $T(x) = x$ upon letting $m \to \infty$ we get $d(T^n(x), x) \le \gamma(T^n(x))$.

Remark 1.3 If $T: X \to X$ is a contraction mapping then it is continuous and satisfies equation (I) inequality in above theorem 1.3.
Since *T* is a contraction mapping then

$$d(T(x), T^2(x)) \le \alpha\, d(x, T(x)), \text{ for all } x \in X.$$

Adding $d(x, T(x))$ to both the sides of the above inequality yields

$$\therefore\ d(x, T(x)) + d(T(x), T^2(x)) \le d(x, T(x)) + \alpha\, d(x, T(x))$$

is equivalent to

$$d(x, T(x)) - \alpha\, d(x, T(x)) \le d(x, T(x)) - d(T(x), T^2(x)).$$

Then $$d(x, T(x)) \le \frac{1}{1-\alpha}[d(x, T(x)) - d(T(x), T^2(x)].$$

Hence define the function $\gamma: X \to R$ by

$$\gamma(x) \le \frac{1}{1-\alpha} d(x, T(x), \text{ for all } x \in X.$$

This gives us the basic inequality

$$d(x, T(x)) \le \gamma(x) - \gamma(T(x)), \text{ for all } x \in X.$$

2. Single-valued Mapping

In this paper *S* be the complete metric space with the metric *d*, let *R* be the set of all real numbers, *N* be the set of positive integer and *B(S)* be the set of all nonempty bounded subset of *S*, *CB(S)* be the set of all nonempty bounded closed subset of *S*, *CL(S)* is the class of nonempty closed subset of *S* and *K(S)* be the set of all nonempty compact subset of *S* respectively.
For any *P*, *Q* belongs to *CB(S)* then

$$\delta(P, Q) = \sup\{d(x, Q) : x \in P\} \text{ and } D(P, Q) = \inf\{d(x, Q) : x \in P\}$$

A single point *x* belongs to *P* we can write $\delta(P, Q) = \delta(x, Q)$, and if *P={x}* and *Q= {y}* then we write $\delta(P, Q) = d(x, y)$.
Let *CB(S)* be the class of all nonempty bounded closed subset of *S* and *H* is the Hausdorff metric with respect to *δ* then

$$H(P, Q) = \max\left\{\sup_{m \in P} \delta(m, P), \sup_{n \in Q} \delta(n, Q)\right\} \text{ where } \delta(m, P) = \inf_{n \in P} \delta(m, n).$$

Then the function *H* is a metric on *CB(S)* and is called Hausdorff metric. And the pair *(CB(S), H)* is called generalized Hausdroff distance induced by *d*.

Example 2.1 Let *P = (1,2)* and *Q = (2,3)* where *S = R* be the set of all real numbers then

$$\delta(P, Q) = \sup_{m \in Q} \delta(m, P) = 1$$

$$\delta(Q, P) = \sup_{n \in P} \delta(n, Q) = 1$$

$$H(P, Q) = \max\{\delta(P, Q), \delta(Q, P)\} = 1.$$

Where the set distance *δ* is not symmetric.

In 1989 Kaneko and Sessa [12] introduced the concept on compatible mapping of single valued and multi-valued mapping.

Definition 2.1 [04] Two mappings $f: S \to S$ and $T: S \to CB(S)$ in metric space (S, d) are said to be compatible if fT_x belongs to *CB(S)* for all $x \in S$, and $\lim_{n\to\infty} H(Tfx_n, fTx_n) = 0$ and $\lim_{n\to\infty} Tx_n = P$, for some $P \in CB(S)$, where $\{X_n\}$ is a sequence in *S* and $\lim_{n\to\infty} fx_n = l$ for some $l \in S$.

Definition 2.2 [04] A single valued mapping $f: S \to S$ and a multi-valued mapping $T: S \to CB(S)$ in metric space (S, d) are said to be weakly compatible if they commute at their coincidence points i.e. $fT_x = Tf_x$ where $f_x \in T_s$, we know that compatible mappings are weakly compatible but converse is not true.

Example 2.2 Let the two single valued mappings $f, g: S \to S$ in the set $S = [1, \infty)$ defined by $f_x = \frac{x}{7}$ and $g_x = 7x$ for all $x \in S$. And let the sequence $\{X_n\}$ in *S* is defined by $x_n = \frac{1}{n}$ for each $n \geq 1$ then $\lim_{n\to\infty} fx_n = \lim_{n\to\infty} gx_n = f(0)$. Hence the mapping *f* and *g* has satisfies the common limit in the range with *g*.

Definition 2.3 In a metric space (S, d) two mappings $f, g: S \to S$ are said to be occasionally weakly compatible *(OWC)* if there exist a point *t* in *S* such that $ft = gt$ and $fgt = gft$.

Definition 2.4 [07] A single-valued mapping $f: S \to S$ and a multi-valued mapping $T: S \to CB(S)$ are said to be occasionally weakly compatible if $fTx \subset Tfx$ for some *x* in *S* and *fx* belongs *Tx*.

Definition 2.5 Let $f, g: S \to S$ be a single-valued mapping and $T, U: S \to CB(S)$ be multi-valued mappings then

1. A point *x* in *S* is said to be coincidence point of *f* and *T* if *fx* belongs to *Tx.*
2. A point *x* in *S* is called common fixed point of *f, g, T* and *U* if $x = fx = gx$ belongs Tx and $x = fx = gx$ belongs to Ux.

Theorem 2.1 Let $f, g: S \to S$ be a single-valued mapping and $T, U: S \to CB(S)$ be the multi-valued mappings satisfying the following conditions

i) $$\delta^r(Tx, Uy) \leq \xi\left[\max\left\{d^r(fx, gy), \frac{d^r(fx, Tx)d^r(gy, Uy)}{1 + d^r(fx, gy)}, \frac{d^r(fx, Uy)d^r(gy, Tx)}{1 + d^r(fx, gy)}\right\}\right]$$

for all $x, y \in S$, $r \geq 1$ and $\xi: [0, \infty) \to [0, \infty)$, is a function such that and $\xi(0) = 0$ and $\xi(t) < t$ for all $t > 0$.
ii) The pairs *(T, f)* and *(U, g)* are occasionally weakly compatible then *f, g, t* and *U* have a unique common fixed point in *S*.

Proof. Let $x, y \in S$ and the pairs *(T, f)* and *(U, g)* satisfy occasionally weakly compatible (*OWC*) property such that $f_x \in Tx$, $fTx \subset Tfx$ and $gUy \subset Ugy$, which implies that $ffx \in Tfx$ and $ggx \in Ugx$.

Then we have to prove that $fx = gy$. Now if $fx \neq gy$ then by using (i) condition we have

$$\delta^r(Tx, Uy) \leq \xi\left[\max\left\{d^r(fx, gy), \frac{d^r(fx, Tx)d^r(gy, Uy)}{1 + d^r(fx, gy)}, \frac{d^r(fx, Uy)d^r(gy, Tx)}{1 + d^r(fx, gy)}\right\}\right]$$

$$= \xi\left[\max\left\{d^r(fx, gy), \frac{d^r(fx, Uy)d^r(gy, Tx)}{1 + d^r(fx, gy)}\right\}\right].$$

Since $f_x \in Tx$ and $gy \in Uy$ then we have

$$\frac{d^r(fx,Uy)d^r(gy,Tx)}{1+d^r(fx,gy)} \le \frac{d^r(fx,gy)d^r(gy,fx)}{1+d^r(fx,gy)} < d^r(fx,gy)$$

and $\delta^r(Tx,Uy) \le \xi\left(d^r(fx,gy)\right)$.

Hence by the property of ξ that we have

$$d^r(fx,gy) \le \delta^r(Tx,Uy) \le \xi\left(d^r(fx,gy)\right) < d^r(fx,gy).$$

Which is contradiction to our assumption and hence *fx* = *gy*. Then we have to prove *fx* is a fixed point of *f*.
Assume that $ffx \ne fx$ by using (i) condition, we have

$$d^r(ffx,fx) = d^r(ffx,gy) \le \delta^r(Tfx,Uy)$$
$$\le \xi\left[\max\left\{d^r(ffx,gy), \frac{d^r(ffx,Tfx)d^r(gy,Uy)}{1+d^r(fTx,gy)}, \frac{d^r(ffx,Uy)d^r(gy,Tfx)}{1+d^r(fTx,gy)}\right\}\right].$$

Since $ffx \in Tfx$ and $gy \in Ty$ then

$$\frac{d^r(ffx,Uy)d^r(gy,Tfx)}{1+d^r(fTx,gy)} \le d^r(ffx,Uy) < d^r(ffx,gy)$$
$$\therefore \delta^r(Tfx,Uy) \le \xi\left(d^r(ffx,gy)\right).$$

Then from the property of ξ that

$$d^r(ffx,fx) = d^r(ffx,gy) \le \delta^r(Tfx,Uy)$$
$$\le \xi\left(d^r(ffx,gy)\right) < d^r(ffx,gy)$$
$$= d^r(ffx,fx).$$

Which is a contradiction, hence $ffx = fx$.
Similarly we can prove $fx = gfx = ffx$ then we have $fx = ffx \in Tfx$ and $fx = gfx = ggy \in Ugy = Ufx$. Therefore *fx* is a common fixed point of *f, g, T* and *U* moreover by the (i) condition we get

$$\delta^r(Tfx,Ufx) \le \xi\left[\max\left\{d^r(ffx,gfx), \frac{d^r(ffx,Tfx)d^r(gfx,Ufx)}{1+d^r(ffx,gfx)}, \frac{d^r(ffx,Ufx)d^r(gfx,Tfx)}{1+d^r(ffx,gfx)}\right\}\right]$$
$$= \xi\left[\max\{0,0,0\}\right] = 0$$

$$\therefore Tfx = Ufx = \{fx\}.$$

then assume that $l \ne m$ is another common fixed point of *f, g, T and U*, hence from condition (i) we get

$$d^r(m,l) = \delta^r(Tm,Ul) \le \xi\left(\max\left\{d^r(fm,gl), \frac{d^r(fm,Tm)d^r(gl,Ul)}{1+d^r(fm,gl)}, \frac{d^r(fm,Ul)d^r(gl,Tm)}{1+d^r(fm,gl)}\right\}\right)$$
$$= \xi\left(\max\left\{d^r(m,l), 0, \frac{d^r(m,l)d^r(l,m)}{1+d^r(m,l)}\right\}\right),$$
$$= \xi\left(d^r(m,l)\right) < d^r(m,l).$$

Which is a contradiction, hence the common fixed point *m* is unique.

Corollary 2.1 Let $f : S \to S$ be a single-valued mapping and $T : S \to CB(S)$ be a multi-valued mapping in a metric space (S, d) satisfying the following conditions

i) $$\delta^r(Tx, Ty) \le \xi\left[\max\left\{d^r(fx, fy), \frac{d^r(fx, Tx)d^r(fy, Ty)}{1 + d^r(fx, fy)}, \frac{d^r(fx, Ty)d^r(fy, Sx)}{1 + d^r(fx, fy)}\right\}\right]$$

for all $x, y \in S$ where $r \geq 1$ and $\xi : [0, \infty) \to [0, \infty)$ is a function such that $\xi(0) = 0$ and $\xi(t) < t$ for all t > 0.

ii) The pair *(T, f)* satisfies *OWC* property then *f* and *T* have a unique common fixed point in *S*.

If *T* is a single-valued mapping then above corollary becomes as follows.

Corollary 2.2 Let $f, T : S \to S$ be two single-valued mapping in metric space (S, d) satisfying the following conditions

i) $$d^r(Tx, Ty) \le \xi\left[\max\left\{d^r(fx, fy), \frac{d^r(fx, Tx)d^r(fy, Ty)}{1 + d^r(fx, fy)}, \frac{d^r(fx, Ty)d^r(fy, Tx)}{1 + d^r(fx, fy)}\right\}\right]$$

for all $x, y \in S$ where $\xi \geq 1$ and $\xi : [0, \infty) \to [0, \infty)$ is a function such that $\xi(0) = 0$ and $\xi(t) < t$, for all t >0.

ii) The pair *(T, f)* satisfies the *OWC* property then *f* and *T* have a unique common fixed point in *S*.

Example 2.3 Let *S= [0, ∞)* be the set of real numbers with the usual metric $d(x, y) = |x - y|$ for all $x, y \in S$.

Define two single-valued mapping $f, T : S \to S$ by

$$Tx = \begin{cases} \{4, 0 \le x < 1 \\ x^4, 1 \le x < \infty \end{cases} \quad \text{And} \quad fx = \begin{cases} 3, 0 \le x < 1 \\ 1 - \frac{1}{x^4}x, 1 \le x < \infty \end{cases}.$$

Then *f(1) =T(1) =1* and *fT(1) =1 =Tf(1)* and so the pair *(T, f)* satisfies *OWC* property.

And for some *J* belongs to *[0,1)* if we define a function $\xi(t) = Jt$ for all $t \in [0, \infty)$ then all conditions in above corollary are satisfied and further the point *1* is a unique common fixed point of *T* and *f*.

3. Conclusion

In this paper we generalized contractictive condition and single valued mapping theorem in complete metric space.

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