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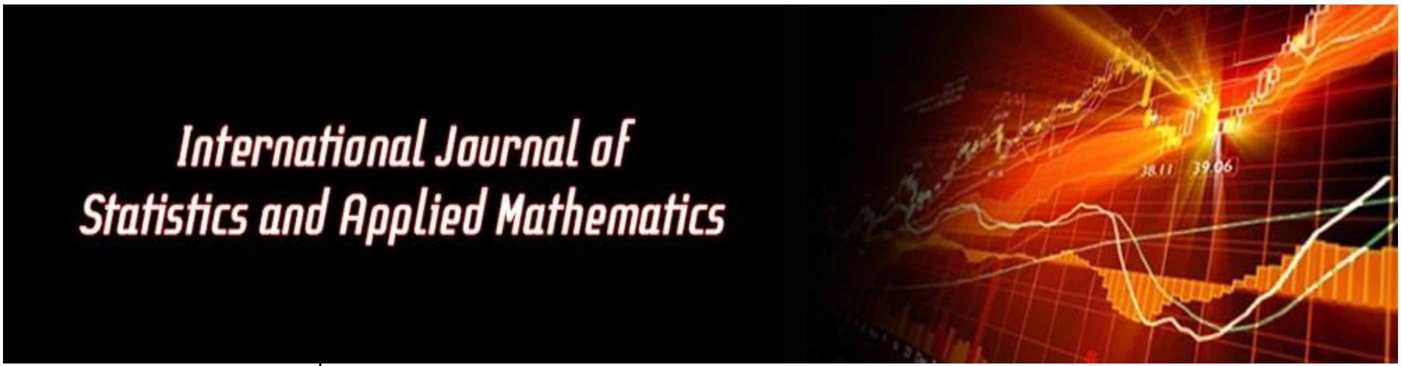

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Three dimensional Debye model on elastic waves of solids

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Abstract
This paper based on principal of three dimensional Debye‘s Model of specific heat of solids. We assumed that the energy of the elastic standing waves is quantized. Expression for total number of vibrational mode of solid is obtained. We obtained relation between the vibration and frequency of waves and find total number of vibrational modes of wave analytically by applying mathematical computation.

Keywords: Elastic standing waves, frequency, vibrational mode

1. Introduction

Debye model is a method developed by Peter Debye in 1912 for estimating the phonon contribution to the specific heat (heat capacity) in a solid. This model correctly explains the low temperature dependence of the heat capacity, which is proportional to T^3. Oganov, A.R.*et al*. (2000) gives the Comparative study of quasi-harmonic lattice dynamics, molecular dynamics and Debye model applied to MgSiO3 perovskite. Shun-Li Shang (2010) gives the First-principles thermodynamics from phonon and Debye model: Application to Ni and Ni3Al. Degueldre, C. *et al*. Specific heat capacity and Debye temperature of zirconia and its solid solution. In presented paper we assumed that the energy of the elastic standing waves is quantized. Expression for total number of vibrational mode of solid is obtained. We obtained relation between the vibration and frequency of waves and find total number of vibrational modes of wave analytically by applying mathematical competition.

2. Formulation of problem

We consider the three dimensional continuous string of Length L Which is fixed at both the ends and vibrating in a longitudinal mode. If the displacement of the string at any point (x_1, x_2, x_3) at any instant of time t is $V(x_1, x_2, x_3, t)$ then the wave equation is written as

$$\frac{\partial^2 V}{\partial x_1^2}+\frac{\partial^2 V}{\partial x_2^2}+\frac{\partial^2 V}{\partial x_3^2}=\frac{1}{\upsilon^2}\frac{\partial^2 V}{\partial t^2} \qquad (1)$$

Taking the three dimensional continuous medium in form of cube of side a with force fixed and solving standing wave equation (1) the solution is of form

$$V(x_1,x_2,x_3,t)=D\sin\left(\frac{n_{x_1}\pi x_1}{a}\right)\sin\left(\frac{n_{x_2}\pi x_2}{a}\right)\sin\left(\frac{n_{x_3}\pi x_3}{a}\right)\cos 2\pi\upsilon t \qquad (2)$$

Where $n_{x_1}, n_{x_2}, n_{x_3}$ are positive integer ≥ 1.

Differentiating E.Q. (2) twice with respect to x_1 we obtain.

$$\frac{\partial^2 V}{\partial x_1^2}=-\left(\frac{n_{x_1}\pi}{a}\right)^2 D\sin\left(\frac{n_{x_1}\pi x_1}{a}\right)\sin\left(\frac{n_{x_2}\pi x_2}{a}\right)\sin\left(\frac{n_{x_3}\pi x_3}{a}\right)\cos 2\pi\upsilon t$$

Implies $\frac{\partial^2 V}{\partial x_1^2} = -\left(\frac{n_{x_1}\pi}{a}\right)^2 V(x_1, x_2, x_3, t)$ (3)

Similarly differentiating E.Q. (2) twice with respect to x_2, x_3, t respectively we get

$$\frac{\partial^2 V}{\partial x_2^2} = -\left(\frac{n_{x_2}\pi}{a}\right)^2 V(x_1, x_2, x_3, t) \quad (4)$$

$$\frac{\partial^2 V}{\partial x_3^2} = -\left(\frac{n_{x_3}\pi}{a}\right)^2 V(x_1, x_2, x_3, t) \quad (5)$$

$$\frac{\partial^2 V}{\partial t^2} = -(2\pi\upsilon)^2 V(x_1, x_2, x_3, t) \quad (6)$$

Substituting these in equation (1) we get

$$\left(\frac{n_{x_1}\pi}{a}\right)^2 + \left(\frac{n_{x_2}\pi}{a}\right)^2 + \left(\frac{n_{x_3}\pi}{a}\right)^2 = \frac{1}{\upsilon^2} 4\pi^2 v^2 = \frac{4\pi^2}{\lambda^2} \quad (7)$$

$$n_{x_1}^2 + n_{x_2}^2 + n_{x_3}^2 = \frac{4a^2 v^2}{\upsilon^2} \quad (8)$$

This equation gives the possible mode of vibration. Three integers $n_{x_1}, n_{x_2}, n_{x_3}$ determine frequency of possible mode of vibrations.

2.1 Possible mode of vibration

To find out the possible no of modes of vibration $D(v)dv$ in the frequency range v and $v + dv$. We consider the network of points, the position of each point being given by three Cartesian positive integer coordinates $n_{x_1}, n_{x_2}, n_{x_3}$ .Thus the radius vector R from the origin at any point is given by

$$n_{x_1}^2 + n_{x_2}^2 + n_{x_3}^2 = R^2 \quad (9)$$

Comparing equation (8) and (9) gives

$$R = \frac{2av}{\upsilon} \quad (10)$$

$$dR = \frac{2a\,dv}{\upsilon} \quad (11)$$

Since each point occupies on the average a unit volume in the integer space the number of point in the spherical shell between radii R and $R + dR$ is equal to volume of an octant of spherical shell in integer space.

Volume of octant of spherical shell = $\frac{4\pi R^2}{8} dR$ (12)

This give the no of possible modes of vibration in a given range because each point corresponding to a set of $n_{x_1}, n_{x_2}, n_{x_3}$ and these gives the mode of vibration according to equation (8)

$$D(v)dv = \frac{4\pi}{8} R^3 dR \quad (13)$$

Substituting the value of R and dR from (10) and (13) we get,

$$D(v)dv = 4\pi\left(\frac{a^3}{\upsilon^3}\right)v^2 dv \quad (14)$$

Put $a^3 = V$ the volume of solid cube. This implies that the

$$D(v)dv = 4\pi\left(\frac{1}{\upsilon^3}\right)v^3 dv \quad (15)$$

Equation (14) gives the number of modes of vibration between v and v+dv.
For a perfect continuum, frequency vary from 0 to ∞ and here the number of possible vibration increase with square of frequency as shown.
In general there are two type of elastic wave occur in solid i.e.; longitudinal and transverse wave. In such case total no of vibration mode is given by

$$D(v)dv = 4\pi\left(\frac{1}{\upsilon_l^{\ 3}} + \frac{1}{\upsilon_t^{\ 3}}\right)v^3 dv \quad (16)$$

3. Conclusions

This paper is based on the principal of three dimensional Debye's model of specific heat of solids. For perfect continuum frequency vary from 0 to ∞ number of possible vibration is increased with square of frequency and gives the total number of vibrational mode for longitudinal and transverse waves.

4. Acknowledgments

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