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On a mathematical model involving aleph-function to study the effects of environmental pollution on biological population

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Abstract
A lot of research work has been carried out pertaining to the adverse effects of environmental pollutant and factors by several researchers. In this paper an attempt has been made to study the effect of environmental pollution on the growth and existence of biological populations by presenting a mathematical model involving Aleph (ℵ)- Function. It is shown here that the habitat still remains asymptotically stable but at much reduced levels implying that if the concentration of pollutant continues to increase in the environment unabatedly, the species may not exist for long. The results established in this paper are general in nature and yield numerous cases of interest on suitable specifications of parameters involved therein.

Keywords: Aleph (ℵ)- function, non-linear partial differential equation

1. Introduction
Environmental pollution is a worldwide problem, having a great potential to influence not only the physiology of human population but also causing grave and irreparable damage to the earth. Environmental Pollution is the contamination of the physical and biological components of the earth/atmosphere to such an extent that normal environmental processes are adversely affected [26, 27, 28].

The biological and ecological consequences of pollution in our environment may be considered in several ways depending upon the toxic level of pollutants (acuteor chronic) and the Eco toxicological situations. One such situation is where the pollutants can adversely affect the natural resources, thereby influencing the growth of other biological populations which may be depending upon these resources. Another such situation is where the pollutants can affect directly the species accompanied by rapid injury to the principal physiological and biochemical systems of the organism. This results in lethal toxication, elimination of individual species and populations or causes profound pathological alterations on the level of individual organisms, individual populations, and occasionally on entire ecosystems which might change the carrying capacity of the environment [1, 3, 6, 7, 8, 25]. Various investigations have also been carried out in this direction, both experimentally and mathematically [6, 7, 8, 12, 13, 14].

The deleterious effect of environmental pollution on interacting biological populations depends upon the toxicity and the level of pollutant, the sort of damage it causes to the physiological and biochemical systems of the populations and their environment.

In light of the above discussion, in this paper, we have studied the effect of environmental pollution on the growth and existence of two interacting biological populations in the situation where the pollutant causes injury to the principal physiological and biochemical systems of the populations and their environment. To study this situation, we presented a mathematical model involving the Aleph (ℵ)- Function [19] by considering thatthe growth rate of species and the carrying capacity of its environment are directlyaffected by pollution and decrease as the concentration of the pollutant increases.

For the present study, we have used the Aleph (ℵ)- function, introduced by Südland *et al*. ([19]; see also [15]), is defined in terms of Mellin Barnes type integrals as:

$$\aleph[z] = \aleph^{m,n}_{p_i,q_i,\tau_i;r}\left\{z\left|\begin{matrix}(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right\} = \frac{1}{2\pi\omega}\int_L \Omega^{m,n}_{p_i,q_i,\tau_i;r}(s)z^{-s}\,ds \tag{1.1}$$

where $\omega = \sqrt{-1}$
and

$$\Omega^{m,n}_{p_i,q_i,\tau_i;r}(s) = \frac{\prod_{j=1}^{m}\Gamma(b_j+B_js)\prod_{j=1}^{n}\Gamma(1-a_j-A_js)}{\sum_{i=1}^{r}\tau_i\prod_{j=m+1}^{q_i}\Gamma(1-b_{ji}-B_{ji}s)\prod_{j=n+1}^{p_i}\Gamma(a_{ji}+A_{ji}s)} \tag{1.2}$$

The integration path $L = L_{\omega\gamma\infty}, \gamma\epsilon\,\Re$ extends from $\gamma - \omega\infty\ to\ \gamma + \omega\infty$, and is such that the poles of $\Gamma(1 - a_j - A_js), j = \overline{1,n}$ do not coincide with the poles of $\Gamma(b_j + B_js), j = \overline{1,m}$. The parameters $p_i\ and\ q_i$ are non-negative integers satisfying the condition $0 \le n \le p_i, 0 \le m \le q_i, \tau_i > 0$ for $i = \overline{1,r}$. The parameters $A_j, B_j, A_{ji}, B_{ji} > 0$ and $a_j, b_j, a_{ji}, b_{ji}\ \epsilon\ C$. An empty product in (1.2) is interpreted as unity. The existence conditions for (1.1) are:

$$\varphi_i > 0, |\arg z| < \frac{\pi}{2}\varphi_i;\ i = \overline{1,r} \tag{1.3}$$

$$\varphi_i \ge 0, |\arg z| < \frac{\pi}{2}\varphi_i\ and\ \Re\{\zeta_i\} + 1 < 0 \tag{1.4}$$

where

$$\varphi_i = \sum_{j=1}^{n}A_j + \sum_{j=1}^{m}B_j - \tau_i\left(\sum_{j=n+1}^{p_i}A_{ji} + \sum_{j=m+1}^{q_i}B_{ji}\right) \tag{1.5}$$

$$\zeta_i = \sum_{j=1}^{m}b_j - \sum_{j=1}^{n}a_j + \tau_i\left(\sum_{j=m+1}^{q_i}b_{ji} - \sum_{j=n+1}^{p_i}a_{ji}\right) + \frac{1}{2}(p_i - q_i);\ i = \overline{1,r} \tag{1.6}$$

Remark 1.1 For $\tau_i = 1, i = \overline{1,r}$ in (1.1) we get the I – Function due to Saxena [16], defined in the following manner

$$I^{m,n}_{p_i,q_i;r}[z] = \aleph^{m,n}_{p_i,q_i,1;r}[z] = \aleph^{m,n}_{p_i,q_i,1;r}\left\{z\left|\begin{matrix}(a_j,A_j)_{1,n},[1(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[1(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right\}$$

$$= \frac{1}{2\pi\omega}\int_L \Omega^{m,n}_{p_i,q_i,1;r}(\xi)z^{-\xi}d\xi \tag{1.7}$$

where the kernel $\Omega^{m,n}_{p_i,q_i,1;r}(\xi)$ is given in (1.2). The existence conditions for the integral in (1.7) are the same as given in (1.3) – (1.6) with $\tau_i = 1, i = \overline{1,r}$.

Remark 1.2 If we set $r = 1$, then (1.7) reduces to the familiar H- Function [5, 10, 11, 18] as

$$H^{m,n}_{p,q}[z] = \aleph^{m,n}_{p_i,q_i,1;1}[z] = \aleph^{m,n}_{p_i,q_i,1;1}\left\{z\left|\begin{matrix}(a_p,A_p)\\(b_q,B_q)\end{matrix}\right.\right\} = \frac{1}{2\pi\omega}\int_L \Omega^{m,n}_{p_i,q_i,1;1}(\xi)z^{-\xi}d\xi \tag{1.8}$$

where the kernel $\Omega^{m,n}_{p_i,q_i,1;1}(\xi)$ can be obtained from (1.2).

2. Mathematical Model

Consider the growth of interacting and dispersing biological species of density N_i (x,t), (i = 1, 2) in a one dimensional linear habitat 0.5 x ≤ L, whose growth rate and the carrying capacity of the environment are decreasing due to the environmental pollution which obey the power-law behaviour (For more details one can see [20, 21, 23, 24]) present in the habitat.

The dynamical equations governing the growth of the species are assumed to be given by the following system of non-linear partial differential equations

$$\frac{\partial N_i}{\partial t} = N_i F_i\big(N_1, N_2, r_i(C), K_i(C)\big) + D_i \frac{\partial^2 N_i}{\partial t^2}; i = 1,2 \tag{2.1}$$

where $F_i\big(N_1, N_2, r_i(C), K_i(C)\big)$determines the interaction function of the species, $r_i(C)\ and\ K_i(C)$are the intrinsic growth rate and the carrying capacity of the environment respectively, which are affected by the concentration $C(x,t)$ of pollutant. The positive constant D_i (i = 1, 2) is the dispersion coefficient of the species.
The dynamics of the concentration $C(x,t)$of the pollutant is considered to be given by the following equation-

$$\frac{\partial C}{\partial t} = Q_0 - \alpha C + D_c \frac{\partial^2 C}{\partial x^2} \tag{2.2}$$

where, $Q_0 > 0$ is the constant determining the exogenous rate of input of pollutantinto the habitat, α > 0 represents the first order decay constant as a result ofbiological (including consumption by the species), chemical or geological processes.$D_c > 0$is the diffusion coefficient of the pollutant.
In the formulation of the mathematical model it has been assumed that the organismal uptake of the pollutant is proportional to the concentration of the pollutant present in the environment of the population. The solution of (2.2)has been obtained in the terms of ℵ- function in the subsequent section of this paper.

3. Result in terms of ℵ-function
We choose the concentration $C(x,t)$ in terms of ℵ-function as

$$C(x,t) = \aleph^{m,n}_{p_i,q_i,\tau_i;r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right\} \tag{3.1}$$

$$\sigma > 0, \mu > 0, \varphi_i > 0, |\arg z| < \frac{\pi}{2}\varphi_i;\ i = \overline{1,r} \tag{3.2}$$

$$\varphi_i \geq 0, |\arg z| < \frac{\pi}{2}\varphi_i\ and\ \Re\{\zeta_i\} + 1 < 0 \tag{3.3}$$

where

$$\varphi_i = \sum_{j=1}^{n} A_j + \sum_{j=1}^{m} B_j - \tau_i\Big(\sum_{j=n+1}^{p_i} A_{ji} + \sum_{j=m+1}^{q_i} B_{ji}\Big) \tag{3.4}$$

$$\zeta_i = \sum_{j=1}^{m} b_j - \sum_{j=1}^{n} a_j + \tau_i\Big(\sum_{j=m+1}^{q_i} b_{ji} - \sum_{j=n+1}^{p_i} a_{ji}\Big) + \frac{1}{2}(p_i - q_i);\ i = \overline{1,r} \tag{3.5}$$

Now on differentiating(3.1)with respect to "t" and "x" partially, we get-

$$\frac{\partial C}{\partial t} = \frac{1}{t}\aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i;r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}(0,\mu),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},(1,\mu)\end{matrix}\right.\right\} \tag{3.6}$$

and

$$\frac{\partial^2 C}{\partial x^2} = \frac{1}{x^2}\aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i;r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}(0,\sigma),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},(1,\sigma)\end{matrix}\right.\right\} \tag{3.7}$$

Using (3.1), (3.6) and (3.7) in (2.2) we arrive at

$$\frac{1}{t}\aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i;r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}(0,\mu),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},(1,\mu)\end{matrix}\right.\right\}$$
$$=Q_0-\alpha\aleph^{m,n}_{p_i,q_i,\tau_i;r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r}\end{matrix}\right.\right\}$$
$$+D_c\frac{1}{x^2}\aleph^{m,n+1}_{p_i+1,q_i+1,\tau_i;r}\left\{zx^{\sigma}t^{\mu}\left|\begin{matrix}(0,\sigma),(a_j,A_j)_{1,n},[\tau_i(a_{ji},A_{ji})]_{n+1,p_i;r}\\(b_j,B_j)_{1,m},[\tau_i(b_{ji},B_{ji})]_{m+1,q_i;r},(1,\sigma)\end{matrix}\right.\right\}\tag{3.8}$$

where $\sigma>0,\mu>0$ and $|\arg z|<\frac{\pi}{2}\varphi_i$.

4. Special Case(S)

(i) On taking τ_i=1, $r=1$in (3.8), we readily get the resultin terms of Fox's H-Function due to Singh and Mehta [17].

(ii) On taking τ_i =1 in (3.8), we readily get the result due to Bhargava, Srivastava and Mukherjee [2].

(iii) On suitably specifying the parameters involved in the result obtained here, we can easily get numerous results (known and new) involving several other higher transcendental functions which are of interest in themselves.

5. Conclusion

It is shown here that the habitat still remains asymptotically stable but at much reduced levels implying that if the concentration of pollutant continues to increase in the environment unabatedly, the species may not exist for long. In view of the generality of the ℵ-function, on specializing the various parameters, we can obtain from the result (3.8), several results involving a remarkably wide variety of useful functions, which are expressible in terms of the *H*-function, the *I*-function, the *G*-function of one variable and their various special cases. Thus the result obtained here can prove to be very useful in the literature of Applied Mathematics and other branches as well.

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