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Mukund V Bapat
Bapat, Hindale, Tal: Devgad, Sindhudurg, Maharashtra, India

Correspondence
Mukund V Bapat
Bapat, Hindale, Tal: Devgad, Sindhudurg, Maharashtra, India

Invariance under cordiality of C_4 and P_m related graphs under path: unions

Mukund V Bapat

Abstract
We discuss one point union of k copies og G i.e. $G^{(k)}$ where G = crown of C_4, C_4 and Pm for m = 3, 4, 5.We also obtain cordial labeling of C_4 and P_m.

Keywords: cordial, one point union, crown, path, invariance

1. Introduction
The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [6], A dynamic survey of graph labeling by J.Gallian [8] and Douglas West [9]. I. Cahit introduced the concept of cordial labeling [5]. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f(u)-f(v)|. Further number of vertices labeled with 0 i.e $v_f(0)$ and the number of vertices labeled with 1 i.e.$v_f(1)$ differ at most by one. Similarly number of edges labeled with 0 i.e. $e_f(0)$ and number of edges labeled with 1 i.e. $e_f(1)$ differ at most by one. Then the function f is called as cordial labeling. I. Cahit has shown that: every tree is cordial; K_n is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t is not congruent to 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n is not congruent to 3 (mod 4). A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian [8].

Our focus of attention is on path unions on different graphs. For a given graph there are different path unions (up to isomorphism) structures possible. It depends on which point on G is used to fuse with vertex of Pm to obtain path-union. We have shown that for G = bull on C_3, bull on C_4, C_3^+, C_4^+-e then different path union $P_m(G)$ are cordial [4]. It is called as invariance under cordial labeling. We use the convention that $v_f(0,1) = (a,b)$ to indicate the number of vertices labeled with 0 are a in number and that number of vertices labeled with 1 are b in number. Further $e_f(0,1) = (x,y)$ we mean the number of edges labeled with 0 are x and number of edges labeled with 1 are y in number. The graph whose cordial labeling is known is cordial graph.

2. Preliminaries: Path union of G, i.e. Pm(G) is obtained by taking a path p_m and take m copies of graph G Then fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq +m-1 edges where G is a (p,q) graph.

3. Theorems Proved
3.1 Theorem: All structures of $P_m(G)$ are cordial, where G = C^+
Proof: Define f: V(G) →{0,1}.It gives different labeling units of C+.

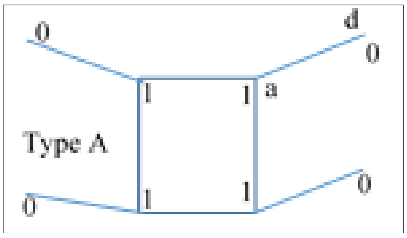

Fig 1: $v_f(0,1) = (4,4)$ $e_f(0,1)=4,4)$

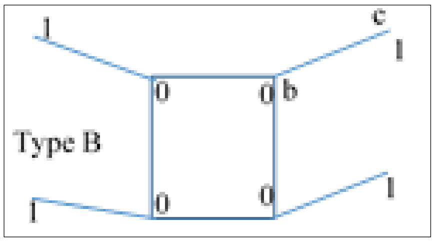

Fig 2: vf(0,1) = (4,4) ef(0,1)=4,4)

To obtain a labeled copy of $P_m(G)$ we start with an unlabeled copy of $P_m=(v_1, v_2,. v_m)$.
To obtain structure 1 we fuse type A label at vertex ‘a’ on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex ‘b’ label at vertex v_i if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of $P_m(C_4^+)$.The label number distribution is $v_f(0,1) = (4m,4m)$, $e_f(0,1)=(4m+x,4m+x)$ for all m =2x+1, x= 0, 1, 2,. When m is even number given by m =2x, x= 1, 2,. $v_f(0,1) = (4m,4m)$, $e_f(0,1)=(4m+x,4m+x-1)$
To obtain structure 2 we fuse type A label at vertex ‘d’ on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex ‘c’ label at vertex v_i if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of $P_m(C_4^+)$.The label number distribution is $v_f(0,1) = (4m,4m)$, $e_f(0,1)=(4m+x,4m+x)$ for all m = 2x+1, x= 0, 1, 2, … When m is even number then m= 2x, x = 1, 2,. $v_f(0,1) = (4m, 4m)$, $e_f(0,1)=(4m+x,4m+x-1)$.
The two structures are non-isomorphic and cordial.

3.2 Theorem: All structures of $P_m(G)$ are cordial, where G = C_4 AND p_3.Proof: Define f: V(G) →{0,1}. It gives different labeling units of G. Though label numbers on both type A and type B are same the number distribution is different.

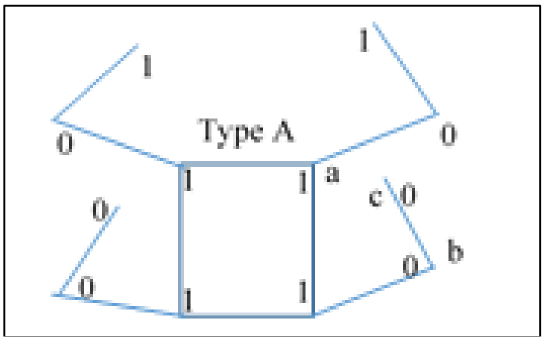

Fig 3: $v_f(0,1) = (6,6)$ $e_f(0,1)=(6,6)$

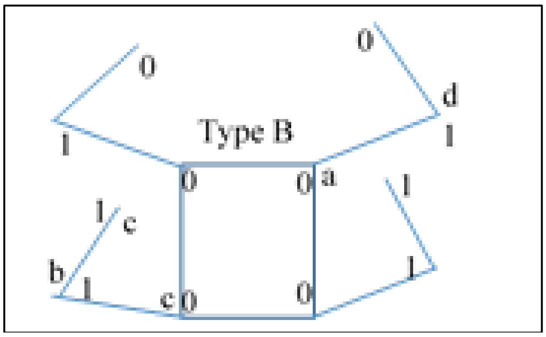

Fig 4: $v_f(0,1) = (6,6)$ $e_f(0,1)=(6,6)$

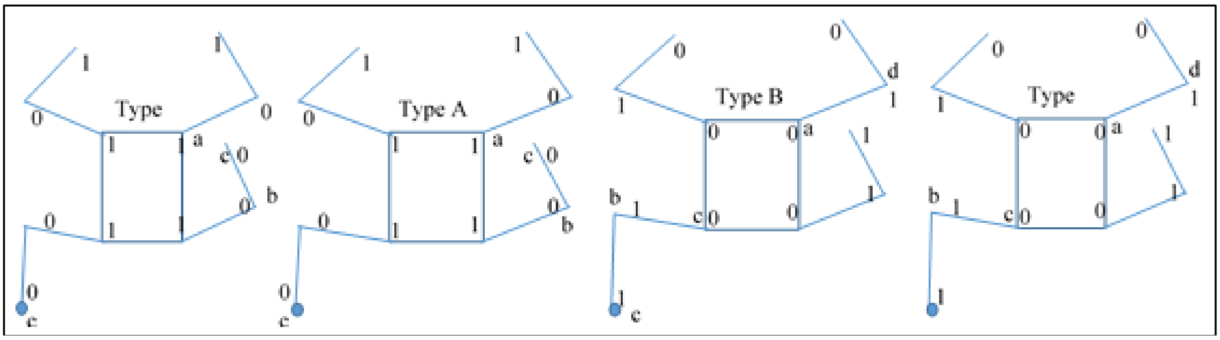

Fig 5: P_4(C_4 and P_2), $v_f(0,1) = (26,26)$ $e_f(0,1)=(26,25)$: structure 3

To obtain a labeled copy of $P_m(G)$ we start with an unlabeled copy of $P_m=(v_1, v_2,. v_m)$.
To obtain structure 1 we fuse type A label at vertex ‘a’ on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex ‘a’ label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of P_m(C_4 and P_3).The label number distribution is $v_f(0,1) = (4m,4m)$, for all m and on edges $e_f(0,1)=(4m+x,4m+x)$ when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then x=1, 2, … then on edges we have $e_f(0,1)=(4m+x,4m+x-1)$.
To obtain structure 2we fuse type A label at vertex ‘b’ on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex ‘b’ label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of P_m(C_4 and P_3).The label number distribution is $v_f(0,1) = (4m,4m)$, for all m and on edges $e_f(0,1)=(4m+x,4m+x)$ when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then x=1, 2, …then on edges we have $e_f(0,1)=(4m+x,4m+x-1)$.
To obtain structure 3 we fuse type A label at vertex ‘c’ on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex ‘c’ label at vertex v_i of P_m if i≡ 2,3 (mod 4). The resultant graph is cordial labeled copy of P_m(C_4 and P_3).The label number distribution is $v_f(0,1) = (4m,4m)$, for all m and on edges $e_f(0,1)=(4m+x,4m+x)$ when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then x=1, 2, …then on edges we have $e_f(0,1)=(4m+x,4m+x-1)$.
Thus the graph is cordial under different structures.

3.3 Theorem: All structures of $P_m(G)$ are cordial, where G = C_4 and p_4.
Proof: Define f: V(G) →{0,1}.It gives different labeling units of G. Though label numbers on both type A and type B are same the number distribution is different. There are four different non-isomorphic structures of path-union possible on G

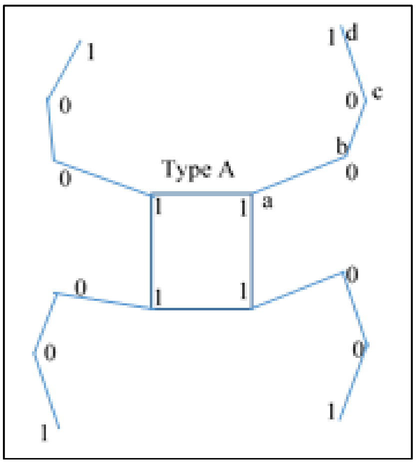

Fig 6: $v_f(0,1)$ = (8,8) $e_f(0,1)$=(8,8)

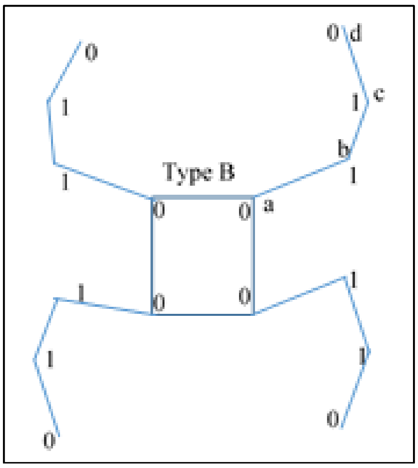

Fig 7: $v_f(0,1)$ = (8,8) $e_f(0,1)$=(8,8)

Depending on which vertex 'a', 'b', 'c', or 'd' we use to fuse on vertex of path P_m.
To obtain structure 1 we fuse type A label at vertex 'a' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'a' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of P_m(C_4 and P_4).The label number distribution is $v_f(0,1)$ = (8m,8m), for all m and on edges $e_f(0,1)$=(8m+x,8m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then x=1, 2, … then on edges we have $e_f(0,1)$=(8m+x,8m+x-1).
To obtain structure 2we fuse type A label at vertex 'b' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'b' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of P_m(C_4 and P_4).The label number distribution is $v_f(0,1)$ = (8m,8m), for all m and on edges $e_f(0,1)$=(8m+x,8m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then x=1, 2, …then on edges we have $e_f(0,1)$=(8m+x,8m+x-1).
To obtain structure 3 we fuse type A label at vertex 'c' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'c' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of P_m(C_4 and P_4).The label number distribution is $v_f(0,1)$ = (8m,8m), for all m and on edges $e_f(0,1)$=(8m+x,8m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then x=1, 2, …then on edges we have $e_f(0,1)$=(8m+x,8m+x-1).
To obtain structure 4 we fuse type A label at vertex 'd' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'd' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of P_m(C_4 and P_4).The label number distribution is $v_f(0,1)$ = (8m,8m), for all m and on edges $e_f(0,1)$=(8m+x,8m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then x=1, 2, …then on edges we have $e_f(0,1)$=(8m+x,8m+x-1).
Thus the graph is cordial under different structures.

3.4 Theorem: All structures of $P_m(G)$ are cordial, where G = C_4 and p_5.
Proof: Define f: V(G) →{0,1}.It gives different labeling units of G. Though label numbers on both type A and type B are same the number distribution is different. There are five different non-isomorphic structures of path-union possible on G

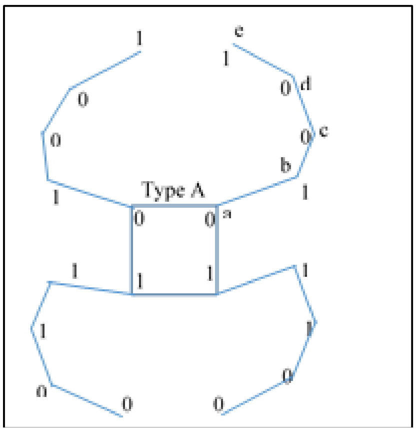

Fig 8: $v_f(0,1)$ = (10,10) $e_f(0,1)$=(10,10)

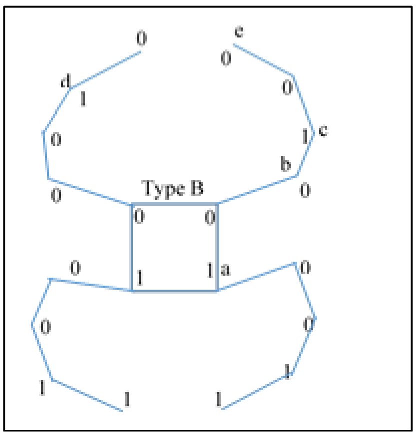

Fig 9: $v_f(0,1)$ = (10,10) $e_f(0,1)$=(10,10)

Depending on which vertex 'a', 'b', 'c', 'd' or 'e' we use to fuse on vertex of path P_m.
To obtain structure 1 we fuse type A label at vertex 'a' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'a' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of P_m(C_4 and P_4).The label number distribution is $v_f(0,1)$ = (10m,10m), for all m and on edges $e_f(0,1)$=(10m+x,10m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then m = 2x, x=1, 2, …then on edges we have $e_f(0,1)$=(10m+x,10m+x-1).

To obtain structure 2 we fuse type A label at vertex 'b' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'b' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of $P_m(C_4$ and $P_4)$.The label number distribution is $v_f(0,1)$ = (10m,10m), for all m and on edges $e_f(0,1)$=(10m+x,10m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then m = 2x, x=1, 2, …then on edges we have $e_f(0,1)$=(10m+x,10m+x-1).
To obtain structure 3 we fuse type A label at vertex 'c' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'c' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of $P_m(C_4$ and $P_4)$.The label number distribution is $v_f(0,1)$ = (10m,10m), for all m and on edges $e_f(0,1)$=(10m+x,10m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then m = 2x, x=1, 2, …then on edges we have $e_f(0,1)$=(10m+x,10m+x-1).
To obtain structure 4 we fuse type A label at vertex 'd' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'd' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of $P_m(C_4$ and $P_4)$.The label number distribution is $v_f(0,1)$ = (10m,10m), for all m and on edges $e_f(0,1)$=(10m+x,10m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then m = 2x, x=1, 2, …then on edges we have $e_f(0,1)$=(10m+x,10m+x-1).
To obtain structure 5 we fuse type A label at vertex 'e' on it at v_i on P_m if i≡1,2 (mod 4) and type B with vertex 'e' label at vertex v_i of P_m if i≡ 2,3 (mod 4).The resultant graph is cordial labeled copy of $P_m(C_4$ and $P_4)$.The label number distribution is $v_f(0,1)$ = (10m,10m), for all m and on edges $e_f(0,1)$=(10m+x,10m+x) when m = 2x+1, where m is odd number and x =0, 1, 2,. and if m is even number then m = 2x, x=1, 2, …then on edges we have $e_f(0,1)$=(10m+x,10m+x-1).
Thus the graph is cordial under different structures.
We conclude our paper with showing that C_4 and P_m is cordial. We do this in two cases in the theorem below by taking m = 2x, x=1, 2,. and m = 2x+1, x= 0, 1, 2, …

3.5 Theorem: C_4 and P_m is cordial.
Proof: case m = 2p.The ordinary label is shown in figure below.
Define f: V(G)→{0,1} as follows:
$f(x_i)$=0 for i is an odd number.
$f(x_i)$ = 1 for i is an even number.
$f(y_i)$ = 0 for i is an odd number.
$f(y_2)$ = 0; $f(y_i)$=1 for i is an even number ≥ 4.
$f(u_i)$ = 1 for i =1, 2 and $f(u_i)$=0 for all other i.
$f(v_i)$ = 0 for i =2x and $f(v_i)$=1 for all other i. The label number distribution is is $v_f(0,1)$ = (2m+2,2m+2), for all m and on edges $e_f(0,1)$ = (2m+2,2m+2) Case m = 2p+1 Follow the labeling of C_4 and P_m for m= 2p as above. The four pendent verticesat $(2p+1)^{th}$ place are labeled as follows:
$f(x_{2p+1})$ = 1;
$f(y_{2p+1})$ = 1;
$f(u_{2p+1})$ = 0;
$f(v_{2p+1})$ = 0.
The label number distribution is $v_f(0,1)$ = (2m+3,2m+3), for all m and on edges $e_f(0,1)$=(2m+3,2m+3).
Thus the graph is cordial.

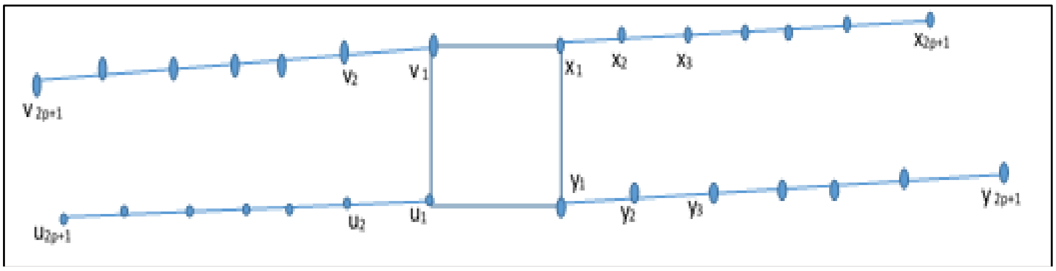

Fig 10: Ordinary labeling of C_4 and P_{2x} C_4 and P_m is cordial. Proof: case m = 2p.

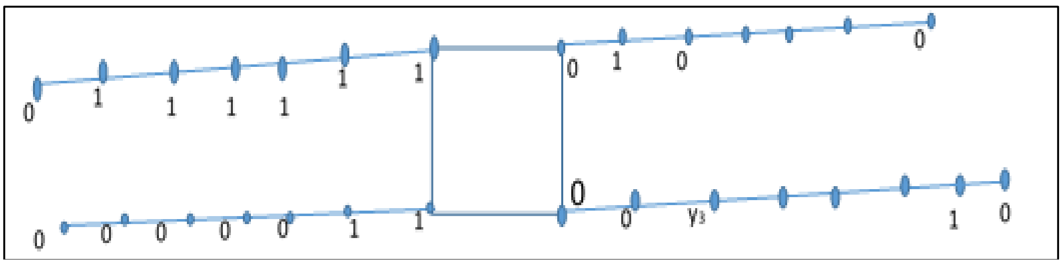

Fig 11: labeled copy of C_4 and P_{2x} cordial, Proof: case m = 2p.

4. Conclusions

We have shown invariance under cordial labeling of different structures obtained on path union of $P_k(G)$ where G= C_4 and P_m (m = 2,3,4,5). These different non isomorphic structures are due to we use different vertex on C_4 and P_m to construct path-union. It remains to do the general case of P_k (C_4 and P_m), for given m. We also obtain particular cordial labeling of C_4 and P_m. We predict that for this graph the path union is invariant under cordial labeling.

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