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Solution of fractional ordinary differential equation by Kamal transform

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Abstract
The fractional calculus for the Kamal transform is introduced and some non- homogenous fractional ordinary differential equation solved by the Kamal transform. We have to get multiple shifting property and periodic function of the Kamal transform.

Keywords: Kamal transform, fractional differential equation, fractional derivatives

1. Introduction

The Kamal transform has fundamental properties which presented in this paper due to its simple formula and consequent and useful properties. It is very useful to solve intricate problem in engineering mathematic and applied science. The Kamal transform can be effecting we solve fractional ordinary differential equation. The purpose of this paper is to show the applicability of this interesting new transform its efficiency in solving the fractional ordinary differential equation.

2. Fundamental properties of Fractional calculus and Kamal transform method

In the field of pure and applied mathematics, the theory of fractional calculus play a significant role different types of differential and integral equation are solved by fractional integrals and derivative, in association with different integral transform.
The description of derivative of fractional order in the same of Abel-Riemann [2013] (A-R) is given by

$$D^{\alpha}\,[\mathrm{f(t)}] = \begin{cases} \frac{1}{\Gamma(m-\alpha)}\frac{d}{dt^m}\int_0^t \frac{f(t)}{(t-\tau)^{\alpha-m+1}}\,d\tau\,, m-1<\alpha\le m \\ \frac{d^m}{dt^m} f(t), \alpha = m \end{cases} \tag{1}$$

Where $m\epsilon\, z^{+}$and $\alpha\epsilon R^{+}$ and the integral operator is defined by implementing an integral of fractional order in Abel-Riemann

$$D^{-\alpha}[f(t)] = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)d\tau\,, t>0, \alpha>0 \tag{2}$$

According to A-R, the integral operator J^{α} is

$$J^{\alpha}[f(t)] = \frac{1}{\Gamma(\alpha)}\int_0^t (t-\tau)^{\alpha-1} f(\tau)d\tau\,, t>0, \alpha>0 \tag{3}$$

We have

$$J^{\alpha}t^{n} = \frac{\Gamma[1+n]}{\Gamma[1+n+\alpha]}t^{n+\alpha} \quad (4)$$

$$D^{\alpha}t^{n} = \frac{\Gamma[1+n]}{\Gamma[1+n+\alpha]}t^{n+\alpha} \quad (5)$$

The fractional calculus derivative is given by

$$J^{\alpha}[D^{\alpha}f(t)] = f(t) - \sum_{k=0}^{\infty} f^{(k)}(0)\frac{t^{k}}{k!} \quad (6)$$

2.1 Kamal Transforam

we can take set A the function is defined in the form

$$\mathrm{A} = \{ f : |f(\mathrm{t})| < pe^{\frac{t}{\emptyset_j}} \text{if t } \varepsilon\ (-1)^{j} \text{ x } [0,\infty)\ , \mathrm{j} = 1,2,;\ \emptyset_j > 0 \} \quad (7)$$

Where $\emptyset_1, \emptyset_2$ may be finite or infinite and the constant p must be finite.

Then Kamal transform is

$$\mathrm{K}(f(t)) = \mathrm{G}(v) = \int_0^{\infty} f(t)e^{\frac{-t}{v}}\,\mathrm{dt}\ , \mathrm{t} \geq 0, \emptyset_1 \leq v \leq \emptyset_2 \quad (8)$$

Derivative of Kamal transform

Let function $f(t)$ then derivative of $f(t)$ with respect to t and the n^{th} order derivative of the same with respect to t are respectively. Then Kamal transform of derivative given by [1]

$$\mathrm{K}[f^{n}(\mathrm{t})] = \frac{1}{v^{n}}\mathrm{G}(v) - \sum_{k=0}^{n-1} v^{k-n+1} f^{k}(0) \quad (9)$$

n = 1, 2,3 ….. in equation (9) give Kamal transform of first and second derivative of f(t) with respect to t

$$\mathrm{K}[f'(\mathrm{t})] = \frac{1}{v}\mathrm{G}(v) - f(0) \quad (10)$$

$$\mathrm{K}[f''(\mathrm{t})] = \frac{1}{v^{2}}\mathrm{G}(v) - \frac{1}{v} f(0) - f'(0) \quad (11)$$

Convolution theorem

Let $f(t)$ and g(t) are two function then Kamal transform of convolution theorem of two function is given by

$$\mathrm{K}(f * \mathrm{g}) = \frac{1}{v}\mathrm{K}(f)\,\mathrm{K}(\mathrm{g}) \quad (12)$$

Multiple shift property

Let the function f (t) in set A is multiplied with shift function

$$\mathrm{K}[\mathrm{t} f(\mathrm{t})] = v^{2}\frac{d}{dv}[\mathrm{u}(v)]$$

Proof:

By definition of Kamal transform,

$$\mathrm{K}(f(\mathrm{t})) = \int_0^{\infty} f(t)e^{\frac{-t}{v}}\,\mathrm{dt} = \mathrm{G}(v)\ \mathrm{t} \geq 0 \quad (13)$$

$$\mathrm{G}(v) = \int_0^{\infty} f(t)e^{\frac{-t}{v}}\,\mathrm{dt} \quad (14)$$

Differentiae both side with respect to v, of equation (14), we have

$$\mathrm{G}'(v) = \frac{d}{dv}\int_0^{\infty} f(t)e^{\frac{-t}{v}}\,\mathrm{dt}$$

$$G'(v) = \int_0^\infty \frac{d}{dv}\{f(t)e^{\frac{-t}{v}}\}dt$$

$$G'(v) = \int_0^\infty f(t)e^{\frac{-t}{v}} \left(\frac{t}{v^2}\right) dt$$

$$G'(v) = \frac{1}{v^2} \int_0^\infty (tf(t))e^{\frac{-t}{v}} dt$$

$$G'(v) = K[t\, f(t)]\frac{1}{v^2}$$

$$K[t\, f(t)] = v^2 \frac{d}{dv}[G(v)] \tag{15}$$

Theorem: The Kamal transform of a piece wise periodic function $f(t)$ with period p is

$$K[f(t)] = \frac{1}{1-e^{\frac{-p}{v}}}\int_0^p e^{\frac{-t}{v}} f(t)\, dt\,;\ v > 0 \tag{16}$$

Proof:
Let Function $f(t)$ is said to be a periodic function T>0 if

$$f(t) = f(T + t) = f(2T + t) = \ldots\ldots\ldots = f(nT + t)$$

By definition

$$K[f(t)] = \int_0^\infty f(t)e^{\frac{-t}{v}} f(t)\, dt$$

$$K[f(t)] = \int_0^p e^{\frac{-t}{v}} f(t)\, dt + \int_p^{2p} e^{\frac{-t}{v}} f(t)\, dt + \int_{2p}^{3p} e^{\frac{-t}{v}} f(t)\, dt + \cdots + \int_{np}^\infty e^{\frac{-t}{v}} f(t)\, dt \tag{17}$$

Put
t = u+p in second integral and up to
t= u+(n-1)p in n^{th} integral ,in equation (17),
Now new limit for each integral are 0 to p and equation (17) by periodicity
$f(t + p) = f(t)$, $f(t + 2p) = f(t)$, and so on there for

$$K[f(t)] = \int_0^p e^{\frac{-u}{v}} f(u)\, du + \int_0^p e^{\frac{-(u+p)}{v}} f(u)\, du + \int_0^p e^{\frac{-(u+2p}{v}} f(u)\, du +$$

$$K[f(t)] = \int_0^p e^{\frac{-u}{v}}\left[1 + e^{\frac{-p}{v}} + e^{\frac{-2p}{v}} + e^{\frac{-3p}{v}} + \cdots \ldots\ldots\right] f(u)du$$

$$K[f(t)] = \frac{1}{1-e^{\frac{p}{v}}}\int_0^p e^{\frac{-t}{v}} f(t)\, dt\,;\ v > 0 \tag{18}$$

The Kamal transform of periodic function $f(t)$ of period p is obtained by integrating $e^{\frac{-t}{v}} f(t)$ in the interval (0 , p) with respect to t and multiply the resultant by the factor $(1 - e^{\frac{-p}{v}})^{-1}$.

Preposition 1: if $f(t)$ is a function and $G(v)$ is a Kamal transform then fractional integral for Kamal transform of order α is

$$K[D^{-\alpha} f(t)] = \frac{1}{\Gamma(\alpha)}(\alpha - 1)!\, v^\alpha\, G(v)$$

Proof:
The fractional integral foe the function (t) , is

$$D^{-\alpha}[f(t)] = \frac{1}{\Gamma(\alpha)}(t^{(\alpha-1)} * f(\tau) \tag{19}$$

Taking the Kamal transform in the equation

$$K[D^{-\alpha} f\ (t)]\ = K\left[\frac{1}{\Gamma(\alpha)}(t^{(\alpha-1)}\ * f\ (\tau)\right]$$

$$K[D^{-\alpha} f\ (t)] = \frac{1}{\Gamma(\alpha)}(\alpha-1)!\, v^{\alpha}\, G(v) \tag{20}$$

Proposition 2: If function $f(t)$ and $G(v)$ is Kamal transform then fractional derivative for Kamal transform of order n is

$$K[f^{n}(t)]\ =\ \frac{1}{v^{n}} G(v) - \sum_{k=0}^{n-1} v^{k-n+1} f^{k}(0\ ) \tag{22}$$

If $f(t)$ is a function and $G(v)$ is Kamal transform then Riemann-Liouville fractional derivative is

$$K\left[\ \frac{\partial^{n} y(t)}{\partial t^{n}}\ \right] = K[D^{n} f(t)] = \frac{1}{v^{n}}\left[\ G(v) - \sum_{k=1}^{n} v^{n-(k+1)}\, [D^{n-k}\big(f(t)\big)\,]_{t=0}\ \right] \tag{23}$$

3. Application of Kamal transform
In this section, we discuss the solution of fractional ordinary differential equation using general properties with initial condition.

3.1 Solve the non - homogeneous fractional ordinary differential equation as,

$$\frac{\partial^{n} y(t)}{\partial t^{n}} = \ \frac{\partial^{2} y(t)}{\partial t^{2}} + \ \frac{\partial\ y(t)}{\partial t} + y(t) + a \tag{24}$$

And initial condition, $(0) = f(0)$, (25)

We appling the Kamal transform of equation (24),

$$K\left[\ \frac{\partial^{n} y(t)}{\partial t^{n}}\ \right] = \ K\left[\ \frac{\partial^{2} y(t)}{\partial t^{2}} + \frac{\partial\ y(t)}{\partial t} + y(t) + a\ \right]$$

$$\frac{1}{v^{n}}\left[\ G(v) - \sum_{k=1}^{n} v^{n-(k+1)}\, [D^{n-k}\big(f(t)\big)\,]_{t=0}\ \right] = \ \frac{1}{v^{2}} G(v) - \frac{1}{v} f(0) - f'(0) + \frac{1}{v} G(v) - f(0) + av$$

$$\left[\ G(v) - \sum_{k=1}^{n} v^{n-(k+1)}\, [D^{n-k}\big(f(t)\big)\,]_{t=0}\ \right] = \frac{v^{n}}{v^{2}} G(v) - \frac{v^{n}}{v} f(0) - v^{n} f'(0) + \frac{v^{n}}{v} G(v) - v^{n} f(0) + v^{n} av$$

$$G(v) - \sum_{k=1}^{n} v^{n-(k+1)}\, [D^{n-k}\big(f(t)\big)\,]_{t=0} = v^{n-2} G(v) - v^{n-1} f(0) - v^{n}\ f'(0) + v^{n-1} G(v) - \ v^{n} f(0) + v^{n} av$$

$$G(v)\left[1 - \ v^{n-2} - v^{n-1}\ \right] = \ \sum_{k=1}^{n} v^{n-(k+1)}\, [D^{n-k}\big(f(t)\big)\,]_{t=0} - v^{n-1} f(0) - v^{n}\ f'(0)\ - v^{n} f(0) + \ v^{n} av \tag{26}$$

Solve the equation (26) and find out the value of $G(v)$, with initial condition.

3.2 Solve the non - homogeneous fractional ordinary differential equation as,

$$D^{\frac{3}{2}} y(t) + D\ y(t)\ = 1 + t$$

With initial condition, y(0) = y(0) = 0, and

$$\left[D^{\frac{-1}{2}}\big(f(t)\big)\,]_{t=0}\ \right] = 0$$

Solution;
Given equation;

$D^{\frac{3}{2}}y(t) + D\,y(t) = 1 + t$ (27)

And, y(0) = y'(0) = 0, and $\left[D^{\frac{-1}{2}}\left(f(t)\right)]_{t=0}\right] = 0$ (28)

We applying the Kamal transform both side equation (27)

$$K[D^{\frac{3}{2}}y(t)] + K[D\,y(t)] = K\,[1] + K[\,t\,]$$

$$\frac{1}{v^{\frac{3}{2}}}\left[\,G(v) - \sum_{1}^{\frac{3}{2}} v^{\frac{3}{2}-(k+1)}\left[D^{\frac{3}{2}-K}\left(f(t)\right)]_{t=0}\right]\right] + \frac{1}{v}G(v) - f(0) = v + v^2$$

Now taking the k=1, then

$$\frac{1}{v^{\frac{3}{2}}}\left[\,G(v) - \; v^{-1}\left[D^{\frac{1}{2}}\left(f(t)\right)_{t=0}\right]\right] + \frac{1}{v}G(v) - f(0) = v + v^2$$

$$\frac{1}{v^{\frac{3}{2}}}\left[\,G(v) - \; v^{-1}\left[\frac{G(v)}{v^{\frac{1}{2}}} - v^{\frac{-1}{2}}\left[D^{\frac{-1}{2}}\left(f(t)\right)_{t=0}\right]\right]\right] + \frac{1}{v}G(v) - f(0) = v + v^2$$

We applying the initial condition,

$$\frac{1}{v}G(v) = v + v^2$$

We applying the invers Kamal transform both side for the value of y(t)

$$K^{-1}[G(v) = K^{-1}\left[v[v + v^2]\right]$$

We get exact solution by the Kamal transform method as follows:

$$y(t) = t + \frac{1}{2!}t^2$$

3.3 Solve the non - homogeneous fractional ordinary differential equation as,

$D^{\frac{1}{2}}y(t) + y(t) = \frac{1}{2}t + t^{\frac{1}{2}}$, and $\left[D^{\frac{-1}{2}}\left(f(t)\right)]_{t=0} = 0\right]$

Solution:
Given equation;

$D^{\frac{1}{2}}y(t) + y(t) = \frac{1}{2}t + t^{\frac{1}{2}}$ (29)

With initial condition;

$\left[D^{\frac{-1}{2}}\left(f(t)\right)]_{t=0} = 0\right]$ (30)

We applying the Kamal transform both side equation (29),

$$K\,[\;D^{\frac{1}{2}}y(t)\,] + K\,[y(t)] = \;K\;\left[\frac{1}{2}t + t^{\frac{1}{2}}\right]$$

$$\frac{1}{v^{\frac{1}{2}}}\left[\,G(v) - \sum_{k=1}^{\frac{1}{2}} v^{\frac{1}{2}-(k+1)}\left[D^{\frac{1}{2}-k}\left(f(t)\right)]_{t=0}\right]\right] + G(v) \;=\; \frac{1}{2}v^2 + \frac{1}{2!}\;v^{\frac{3}{2}}$$

Now taking the k=1,

$$\frac{1}{v^{\frac{1}{2}}}\left[\ G(v) - \ v^{-\frac{1}{2}}\left[D^{\frac{-1}{2}}\ [f(t)\ ]_{t=0}\ \right] + G(v)\ =\ \frac{1}{2}v^2 + \frac{1}{2!}\ v^{\frac{3}{2}}$$

Apply the initial condition,

$$\frac{1}{v^{\frac{1}{2}}}\ G(v) + G(v) = \ \frac{1}{2}v^2 + \frac{1}{2}\ v^{\frac{3}{2}}$$

$$G(v)\left[1 + \frac{1}{v^{\frac{1}{2}}}\right] = \frac{1}{2}v^2\left[1 + \ v^{\frac{-1}{2}}\ \right]$$

$$G(v) = \frac{1}{2}v^2$$

Apply the inverse Kamal transform for the value of y(t)

$$k^{-1}[G(v)] = k^{-1}\left[\frac{1}{2}v^2\right]$$

We get exact solution by the Kamal transform method as follows:

$$y(t) = \ \frac{1}{2}t$$

4. Discussion and Conclusion

We observed that Kamal transform solves fractional ordinary differential equation with a few computations as well as time unlike the Laplace transform and other we observed that the Kamal transform is defind on the interval $[0, \infty]$. We have applied Kamal transform for fractional ordinary differential equation as well as periodic function. It is found that the Kamal transform has an extensive affinity with the solutions of differential and integral equations, and more specifically with the Fractional differential equations which has been the centre forum of this paper. We found that the solution of fraction ordinary differential equation can be obtained in the form of distribution fractional ordinary differential equations when distributed Kamal transform are invoked.

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