International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(2): 260-263

www.mathsjournal.com
Received: 07-01-2018
Accepted: 10-02-2018

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An empirical examination of two-population interaction in urban growth: Tamil Nadu (1901-2011)

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Abstract
The urban-rural nonlinear interaction models help us to understand more about the urban system by chaos theory and meanwhile realize more the nature of chaos by urban growth. The structure of rural – urban interaction of population is empirically studied through Tamil Nadu state rural –urban population for the census year 1901 to 2011 using the logistic model derived from the urban-rural interaction models. This suggests that the logistic equation is a statistical change from the nonlinear interaction models to the simple linear equations. This put forwards the complicated dynamics of logistic growth come from some kind of the nonlinear interaction. The outcome from this study helps to understand urbanization, rural-urban population interaction, chaotic dynamics and spatial complexity of geographical systems.

Keywords: Chaos, Logistic growth, Two-population interaction, urban growth

Introduction
Many studies of chaotic cities are associated to spatial interaction and logistic growth. A two-population interaction model of urbanization gives a new understanding of chaos. Urban systems are complex systems, and the progression of urbanization and urban evolution are nonlinear process coupled with chaos. With numerical calculation, and empirical analysis, we can acquire new knowledge about chaos based on the nonlinear dynamics of urban evolution.

The level of urbanization is the proportion of urban population in its total population (Davis & Hertz, 1951) [7]. The level of urbanization is a helpful sign for projections of some overall development and the study proposes a method for identifying problems in the time series of urban and rural populations. This paper is a worked on basis of on a series of previous studies (Chen, 2009; Chen, 2010, Chen, 2012; Chen, 2014) [6]. The objective of this paper is to analyze the structure of rural – urban interaction of population and empirically studied through Tamil Nadu state rural –urban population for the census year 1901 to 2011 to validate the data reliability and to verify the dynamical mechanism of urbanization, Using the logistic model derived from the urban-rural interaction models. Rural-urban interactions are considered as linkages across space reflected in the flows of people between urban and rural areas.

Two-population *interaction model*
A rural-urban population interaction model can show the way to a new understanding of chaos. The rural-urban population interaction is studied through many mathematical models among that the United Nations non-linear model gives an idea about that the spatial interaction between urban and rural population that causes urbanization. The rural–urban interaction model can be expressed as (Chen, 2009a) [4].

$$\frac{dr(t)}{dt} = ar(t) + \varphi u(t) - b\frac{r(t)u(t)}{r(t)+u(t)} \quad (1)$$

$$\frac{du(t)}{dt} = cu(t) + \psi r(t) + d\frac{r(t)u(t)}{r(t)+u(t)} \quad (2)$$

If parameters $\varphi=\psi=0$, then the UN model becomes logistic model. Population switch over between urban and rural regions depends only on urban-rural population interaction.

$$\frac{dr(t)}{dt} = ar(t) - b\frac{r(t)u(t)}{r(t)+u(t)} \quad 3$$

$$\frac{du(t)}{dt} = cu(t) + d\frac{r(t)u(t)}{r(t)+u(t)} \quad 4$$

The above second equation says that rural population cannot suddenly flow in to urban and vice versa. The size of urban population is influenced by rural population size and in turn reacts on it. So both the growth rate of urban population and that of rural population depend to a great extent on the interaction of urban and rural population.

Logistic Growth Model

The *level of urbanization* is defined as the proportion of urban population to the total population in a region. The level of urbanization ranging from 0 to 1 can often be fitted to the logistic function since it has clear upper and lower limits.

The logistic model can be expressed as

$$L(t) = \frac{1}{1+\left(\frac{1}{L0}-1\right)e^{-kt}} \quad 5$$

Where

*L*0 - the initial value of urbanization level,

L(*t*) - the level of urbanization at time *t*,

k - the intrinsic/ original growth rate.

Empirical Analysis

The main objective of this study is to have an idea about an empirical analysis for on complex spatial dynamics of urban-rural interaction. So it is necessary to make appropriate statistical analysis of the dynamical equations. Further Empirical analyses for Tamil Nadu census population data from 1910 to 2011 were used to verify data reliability and to determine the dynamical mechanism of urbanization.

The data displayed in Table 1are fitted to the model. The population measures classified as four categories. They are rural population *r*(*t*), urban population *u*(*t*), total population *P*(*t*)=*r*(*t*)+*u*(*t*), and the level of urbanization is the ratio of urban population to the total population L(*t*)=*u*(*t*)/*P*(*t*).

Table 1: Tamil Nadu rural and urban population and their growth rate

Census Year	Rural Population [r(t)]	Urban Population [u(t)]	$\frac{r(t)u(t)}{r(t)+u(t)}$	Rural rate of growth Δr(t)	Urban rate of growth Δu(t)
1901	16528100	2724530	2338968.98	122537.9	42460.7
1911	17753479	3149137	2674695.72	44696.0	27894.2
1921	18200439	3428079	2884734.99	104127.8	80230.3
1931	19241717	4230382	3467939.24	185210.8	94330.0
1941	21093825	5173682	4154666.93	169169.7	215984.3
1951	22785522	7333525	5547924.38	191090.3	165700.3
1961	24696425	8990528	6591094.79	403790.9	347430.6
1971	28734334	12464834	8693590.69	372186.8	348704.1
1981	32456202	15951875	10695266.36	432515.2	312571.7
1991	36781354	19077592	12561992.57	-185967.3	840640.6
2001	34921681	27483998	15379808.80	230790.9	743344.2
2011	37229590	34917440	18018232.70		

Let r(t),u(t), r(t)u(t) and r(t)u(t)/[r(t) + u(t)] be independent variables and Δ(t)/Δt, and Δu(t)/Δt be dependent variables. The parameters of model are estimated by the least squares computation, the parameters and slopes, could tally better with actual state and it describe the growth of the rural population and urban population. But, the estimated parameters should be tested by statistical tests, and logical tests. If the model does not pass the statistical tests, it has in complete or unneeded variables or in accurate parameter values. If the model does not pass the logical tests, it has structure problem so that it cannot explain the trend at present and future. Statistical tests can be made in definite procedure, while the logical tests need to be done with the help of mathematical transform and numerical analyses.

In the statistical analysis, the least square computation analysis for the census data of Tamil Nadu from 1901 to 2011 gives the following model:

$$\frac{\Delta r(t)}{\Delta t} = -0.004r(t) + 0.155\frac{r(t)u(t)}{r(t)+u(t)} - 0.075u(t) \quad 7$$

$$\frac{\Delta u(t)}{\Delta t} = -0.010r(t) + 0.113\frac{r(t)u(t)}{r(t)+u(t)} - 0.023u(t) \quad 8$$

Table 2: ANVOA table of the Tamil Nadu urbanization model

Dependent Variable	Model	Independent Variable	R^2	Std. Erorr of the Estimate	Durbin-Watson
Δr(t)	1	r(t)	0.841	105755.74406	1.306
	2	r(t),r(t)u(t)/P(t)	0.841	111430.97242	1.310
	3	r(t),r(t)u(t)/P(t),u(t)	0.884	101005.30963	2.365
Δu(t)	1	r(t)u(t)/P(t)	0.920	16415.49041	2.324
	2	r(t),r(t)u(t)/P(t)	0.932	112716.60973	3.044
	3	r(t),r(t)u(t)/P(t),u(t)	0.934	118028.59470	2.899

Table 2 indicates the R-squares value with “rural” alone 84 % (0.84) of the variance was accounted for rural growth population and Durbin-Watson test is 1.306 is evidence of positive serial correlation and whereas urban growth it is 92 % (0.92) of variance accounted by rural-urban -ratio alone. Durbin-Watson test is 2.324 which evidence of serial correlation.

Table 3: Coefficients and related statistics of the Tamil Nadu urbanization model

Dependent Variable	Model	Independent Variable	Regression Coefficients	Std. Erorr	t-Statistic	P-Value (sig.)	VIF
Δr(t)	1	r(t)	0.009	0.001	7.275	0.000	1.000
	2	r(t)	0.009	0.004	2.093	0.066	11.783
		r(t),u(t)/P(t)	-0.001	0.014	-0.086	0.934	11.783
	3	r(t)	-0.004	0.009	-0.412	0.691	52.002
		r(t),u(t)/P(t)	0.155	0.092	1.688	0.130	583.575
		u(t)	-0.075	0.044	-1.719	0.124	322.873
Δu(t)	1	r(t)u(t)/P(t)	0.047	0.004	10.714	0.000	1.000
	2	r(t)u(t)/P(t)	0.065	0.015	1.327	4.459	11.783
		r(t)	-0.006	0.005	-0.384	-1.291	11.783
	3	r(t)u(t)/P(t)	0.113	0.107	2.317	1.057	583.575
		r(t)	-0.010	0.010	-0.647	-0.988	52.002
		r(t)u(t)/P(t)	0-.023	0.051	-0.744	-0.456	322.873

This model gives rise to two problems. From Table 3 First, Some values fail to pass the statistic test either. VIF value is far more than 10, which implies that the model cannot avoid multi-collinearity, between the three independent variables. Second, based on the model, the total population will not converge but increase infinitely. The statistics *P*-value (or *t* statistic), variance inflation factor (VIF) value and Durbin-Watson (DW)value can does not pass the test for all the three variables.

A multivariate stepwise regression analysis based on the census data from 1901 to 2011 yielded the following model:

$$\frac{\Delta r(t)}{\Delta t} = 0.009\, r(t) \quad 9$$

$$\frac{\Delta u(t)}{\Delta t} = 0.047 \frac{r(t)u(t)}{r(t)+u(t)} \quad 10$$

The above first equation indicates the growth of rural population depends on rural population and second equation indicates that the growth of urban population is due to urban – rural interaction which can transform the rural population into urban population. The statistical tests, t-statistic, P-value, variance inflation factor (VIF) value can pass the test. The first equation has only rural term which doesn't say about logical test, but the second equation has the on-linear term $\frac{r(t)u(t)}{r(t)+u(t)}$ is positive which indicates that the urban-rural interaction can transform the rural population into the urban population.

Table 4: ANVOA table of the Tamil Nadu urbanization model

Dependent Variable	Model	Independent Variable	R^2	Std. Erorr of the Estimate	Durbin-Watson
Δr(t)	1	r(t)	0.841	105755.74406	1.286
Δu(t)	1	r(t)u(t)/p(t)	0.929	109459.60245	1.908

The R-squares with "rural" alone 84 % (0.84) of the variance was accounted for rural growth population and Durbin-Watson test is 1.286 is evidence of positive serial correlation. Whereas urban growth it is 92 % (0.92) of variance accounted by rural-urban-ratio alone. Durbin-Watson test is close to 2(1.908) which says that it doesn't have autocorrelation

Table 5: Coefficients and related statistics of the Tamil Nadu urbanization model

Dependent Variable	Model	Independent Variable	Regression Coefficients	t-Statistic	P-Value (sig.)	VIF
Δr(t)	1	r(t)	0.009	7.275	0.000	1.000
Δu(t)	1	r(t)u(t)/p(t)	0.047	10.714	0.000	1.000

The statistical test, VIF value is 1, which implies that there doesn't present multi-co linearity. The p-value is 0.000, implies that there is chances of finding the linear relationship between Δr (t) and r (t).and between Δu (t) and $\frac{r(t)u(t)}{r(t)+u(t)}$.

Logistic growth model

The level of urbanization should follow the logistic curve. It is easy to calculate the urbanization ratio using the data in Table 1. For convenience, we set time dummy *t*=year-1901. A least squares computation involving the percentage urban data gives the following results.

$$L(t) = \frac{1}{1 + 6.0664 e^{-0.016t}} \quad 11$$

The estimated value of the intrinsic growth rate value is K=0.016

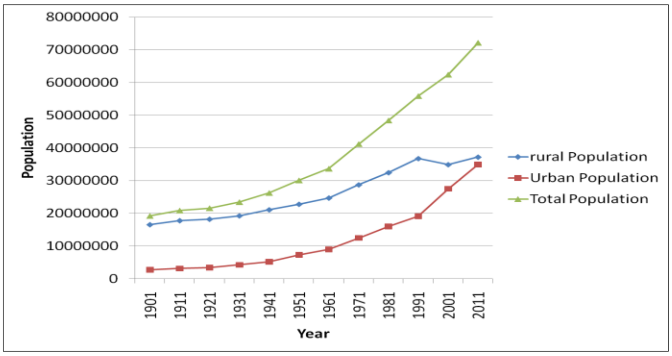

Fig 1: The changing trend of the Tamil Nadu urban, rural and total population (1901-2011)

The above figure 1 shows the census rural, urban and total popuation. It shows that the urban population is increasing fastly and rural population is gradualluy started decreasing in the last towo to three decades.

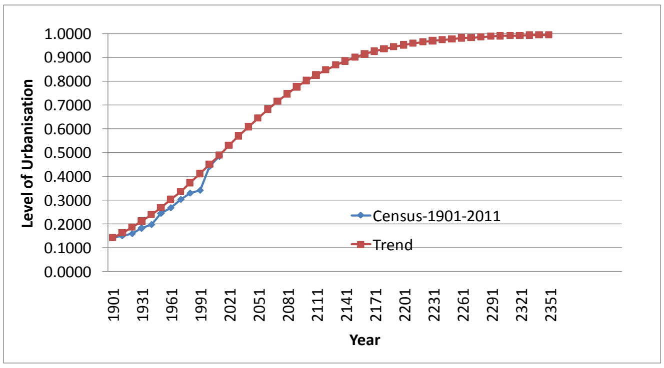

Fig 2: Logistic process of the Tamil Nadu level of urbanization

Figure 2 shows the level of urbanization of Tamil Nadu for both census and trend line. The level of urbanization is growing fast in the last two censuses. By the year 2351 Tamil state will become completely urban population state.

Conclusion

This study analyzed the trends in urbanization levels in Tamil Nadu census population data. Based on the census, urbanization dynamics were examined by using the rural-urban interaction model. Insight into the patterns and processes of the urbanization trends in the Tamil Nadu were obtained by systematically examining of the census data. The theoretical background and empirical base of this paper is to understand more about the urban system by chaos theory. By means of numerical computation, we can reveal that the rate of spatial replacement dominate the nonlinear dynamics of fractal dimension evolution. Chaotic behavior can be linked by spatial replacement dynamics. It is noted that least square computation of the Tamil Nadu census results in multi-collinearity among the independent variables in the model.

A multivariate stepwise regression analysis based on the census data yielded the model as rural population growth increases because of rural population and urban population increase due to urban-rural interaction. The intrinsic growth rate is estimated by using logistic growth model and the level of urbanization if projected for the year 1901 to 2351. In the year 2351 Tamil state will become completely urban population state. The urban-rural nonlinear interaction models could help us better understand the urban system by chaos theory, and meanwhile comprehend more the nature of chaos by urban evolution. Finally, the study has provided an assessment of the validity of the prey-predator model for the interpretation of urban dynamics at the empirical level. The prey-predator model chosen for the empirical analysis gives an idea for demographers, geographers and policymakers to get an idea into the urbanization process is the base for social, economic, cultural, and environmental planning and policy making.

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