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E-cordial labeling of bull related graphs and invariance

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Abstract
A bull graph has a copy of C_3 with pendent edge attached at two (adjacent) vertices. Similar structure is obtained on cycles C_4, C_5. We obtain e- cordial labeling of these graphs and one point union of these graphs. Further we show that one point union structures are invariant under e-cordial labeling.

Keywords: E-cordial, labeling, bull, one point union, cycle

1. Introduction

In 1997 Yilmaz and Cahit [4] introduced a weaker version of edge graceful labeling called E-cordial. The word cordial was used first time in this paper. Let G be a graph with vertex set V and edge set E. Let f be a function that maps E into {0, 1}. Define f on V by $f(v) = \sum\{f(uv)/(uv) \in E\}$(mod 2). The function f is called as E cordial labeling if $|e_f(0)-e_f(1)|\leq 1$ and $|v_f(0)-v_f(1)|\leq 1$. Where $e_f(i)$ is the number of edges labeled with i = 0,1 and $v_f(i)$ is the number of vertices labeled with i = 0,1. We also use $v_f(0,1) = (a,b)$ to denote the number of vertices labeled with 0 are a and that with 1 are b in number. Similarly $e_f(0,1)=(x, y)$ to denote number of edges labeled with 0 are x and that labeled with 1 are y in number respectively. A lot of work has been done in this type of labeling and the above mentioned paper gave rise to number of papers on cordial labeling. A graph that admits E-cordial labeling is called as E-cordial graph. Yilmaz and Cahit has shown that Trees T_n with n vertices and Complete graphs K_n on n vertices are E – cordial iff n is not congruent to 2 (modulo 4). Friendship graph $C_3^{(n)}$ for all n and fans F_n for n not congruent to1 (mod 4).One may refer A Dynamic survey of graph labeling for more details on completed work. A bull graph consists of C_3 and two pendent vertices one each at adjacent vertices. It has 5 edges and 5 vertices. This concept we generalize for all C_n. A general bull graph bull(C_n) consists of a cycle and a pendent edge each at two adjacent vertices. It has n+2 vertices and n+2 edges. We obtain e-cordial labeling of G = bull(C_n) for n = 3, 4, 5…n. Also we define one point union of G on k copies and obtain different structures by changing common point of union on G. We show that all these structures are e-cordial. This is referred as invariance under e-cordial labeling.

2. Preliminaries

By one point union of k copies of graph G we fuse k copies of G at fixed vertex of G. It is denoted by $G^{(k)}$. It has kq edges and Pk-k+1 vertices. We choose G as bull(C_n) with n = 3,4,5.
Fusion of vertex. Let G be a (p,q) graph. let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges [5].

3. Theorems Proved

3.1 A bull graph is e-cordial. All structures on one point union of bull graph denoted by $(\text{bull}(C_3))^{(k)}$ are e-cordial.
Proof. Define a function f: E(G)→{0,1}where G = $(\text{bull})^{(k)}$. From the diagram below it follows that the bull graph is e-cordial.

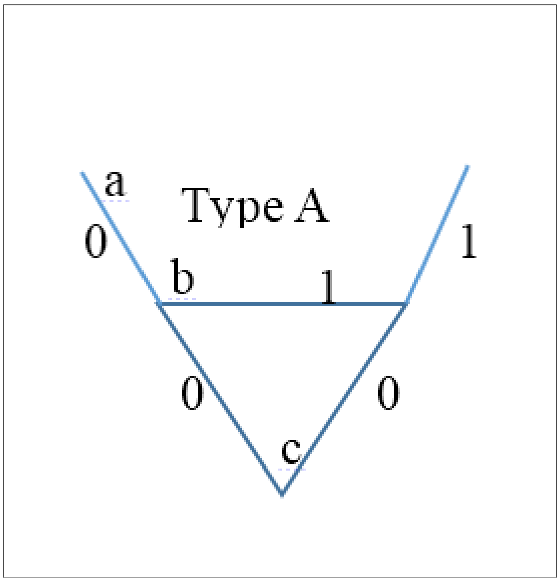

Fig 4.1: $v_f(0,1) = (3,2)$, $e_f(0,1)=(3,2)$.The numbers are edge numbers.

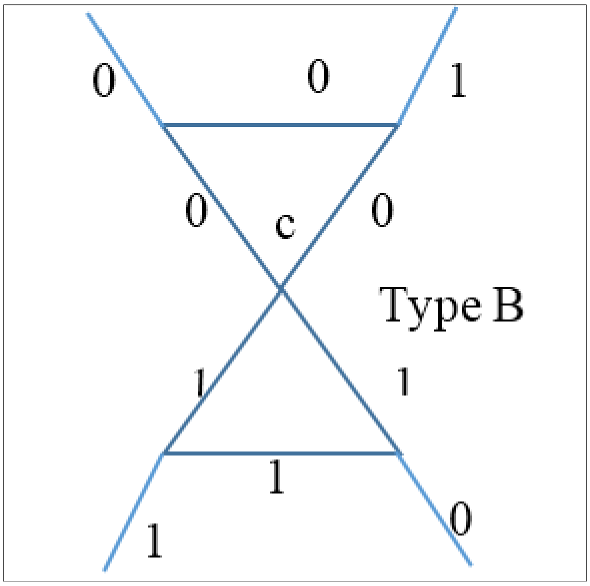

Fig 4.2: $v_f(0,1) = (5,4)$, $e_f(0,1)=(5,5)$.The numbers are edge numbers.

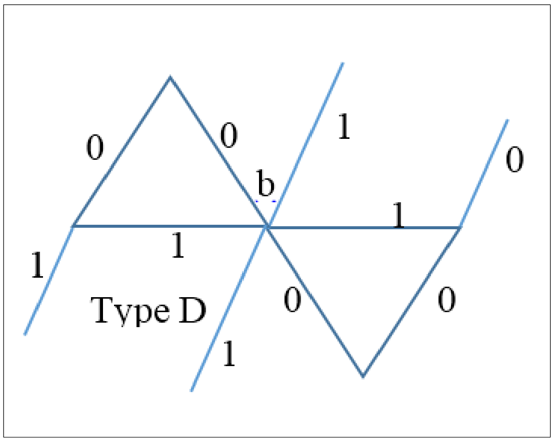

Fig 4.3: $v_f(0,1) = (5,4)$, $e_f(0,1)=(5,5)$.The numbers are edge numbers.

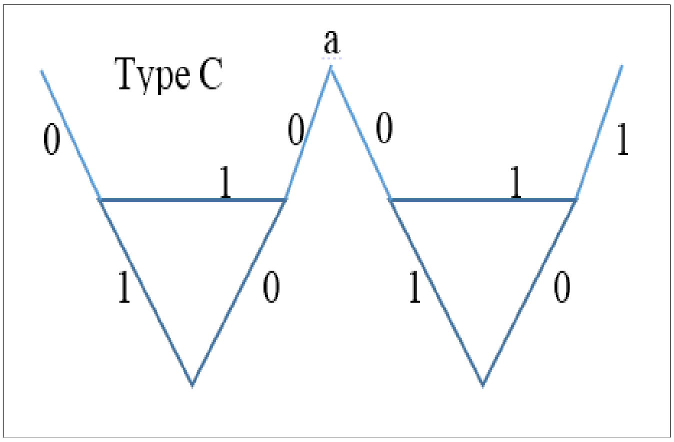

Fig 4.4: $v_f(0,1) = (5,4)$, $e_f(0,1)=(5,5)$.The numbers are edge numbers.

Structure 1: First obtain one point union at point 'c' on bull graph for k = 2x, x=1, 2, 3…This is done by repeatedly fusing type B labeled copy at point c on it. The structure has label number distribution given by $v_f(0,1) = (4x+1,4x)$, $e_f(0,1)=(5x,5x)$.To obtain labeled copy of bull graph on k = 2x+1 copies, x= 0, 1, 2,…We first obtain $(\text{bull})^{(2x)}$ and fuse it at vertex 'c' on it with vertex 'c' on type B label. The resultant structure has label number distribution given by $v_f(0,1) = (4x+3,4x+2)$, $e_f(0,1)=(5x+3,5x+2)$.

Structure 2: First obtain one point union at point 'b' on bull graph for k = 2x, x=1, 2, 3…This is done by repeatedly fusing type C labeled copy at point 'b' on it. The structure has label number distribution given by $v_f(0,1) = (4x+1,4x)$, $e_f(0,1)=(5x,5x)$.To obtain labeled copy of bull graph on k = 2x+1 copies, x= 0, 1, 2,…We first obtain $(\text{bull})^{(2x)}$ and fuse it at vertex 'b' on it with vertex 'b' on type B label. The resultant structure has label number distribution given by $v_f(0,1) = (4x+3,4x+2)$, $e_f(0,1)=(5x+3,5x+2)$.

Structure 3: First obtain one point union at point 'a' on bull graph for k = 2x, x=1, 2, 3…This is done by repeatedly fusing type C labeled copy at point 'a' on it. The structure has label number distribution given by $v_f(0,1) = (4x+1,4x)$, $e_f(0,1)=(5x,5x)$.To obtain labeled copy of bull graph on k = 2x+1 copies, x= 0, 1, 2,…We first obtain $(\text{bull})^{(2x)}$ and fuse it at vertex 'a' on it with vertex 'a' on type B label. The resultant structure has label number distribution given by $v_f(0,1) = (4x+3,4x+2)$, $e_f(0,1)=(5x+3,5x+2)$.These three structures are pairwise non-isomorphic. All of them are shown to be E- cordial.

3.2 Theorem: A bull graph on C_4 denoted by bull(C_4) is not e-cordial. But all structures on one point union of bull(C_4) graph denoted by $(\text{bull}(C_4))^{(k)}$, for k not congruent to 1,3 (mod 4), are e-cordial.

Proof: Define a function f: $E(G)\rightarrow\{0,1\}$where G = $(\text{bull}(C_4))^{(k)}$.

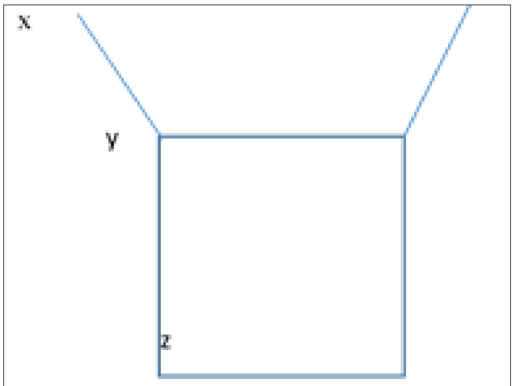

Fig 4.5 bull on C_4: It is not e-cordial.

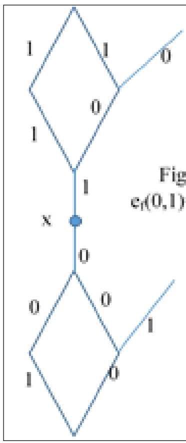

Fig 4.6 : $v_f(0,1) = (5,6)$, $e_f(0,1)=(6,6)$. The numbers are edge numbers.

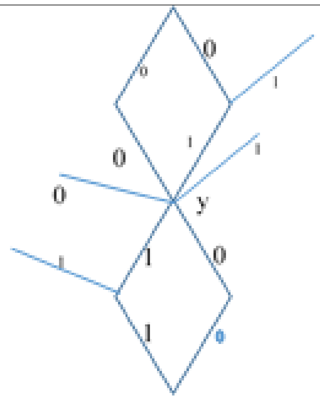

Fig 4.7 : $v_f(0,1) = (5,6)$, $e_f(0,1)=(6,6)$. The numbers are edge numbers.

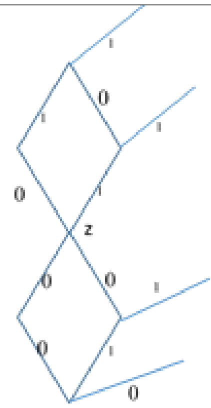

Fig 4.8 : $v_f(0,1) = (5,6)$, $e_f(0,1)=(6,6)$. The numbers are edge numbers.

From fig 4.5 it follows that there are three structures possible on one point union of bull(C_4).Structure1 is due to one point union taken at vertex 'x. Structure2 is due to one point union taken at vertex 'y'. Structure3 is due to one point union taken at vertex 'z'. In all the three cases we have label distribution given by: On vertices $v_f(0,1) = (10x+1,10x)$, when m = 2x, x=0, 1, 2, ..and the label of common vertex as 0. And vf(0,1) = (5m,5m+1), when m =1, 3, 4, .. and the label of common vertex as 1. For all m the distribution on edge labels is $e_f(0,1)=(6m,6m)$. This shows invariance under e-cordial labeling of different structures of G= $(\text{bull}(C_4))^{(k)}$for even k.

4.3 Theorem: A bull graph on C_5 denoted by G=bull(C_5) is e-cordial. Further all structures of $G^{(k)}$ obtained by changing the common point on one point union of k copies of G are e- cordial.
Proof: Define a function:$E(G)^{(k)} \rightarrow \{0,1\}$as follows. f gives labeled units as follows:

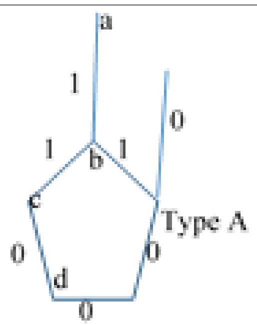

Fig 4.9 : : $v_f(0,1) = (3,4)$, $e_f(0,1)=(4,3)$. The numbers are edge numbers.

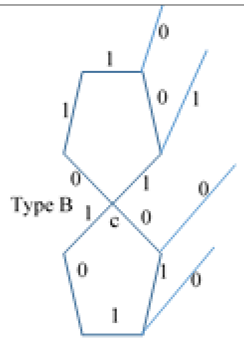

Fig 4.10 : : $v_f(0,1) = (7,6)$, $e_f(0,1)=(7,7)$. The numbers are edge numbers.

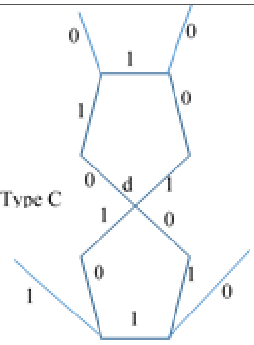

Fig 4.11 : : $v_f(0,1) = (7,6)$, $e_f(0,1)=(7,7)$. The numbers are edge numbers.

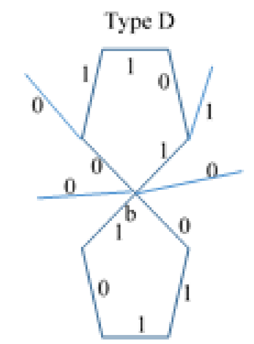

Fig 4.12 :: $v_f(0,1) = (7,6)$, $e_f(0,1)=(7,7)$.The numbers are edge numbers.

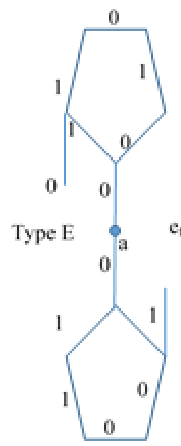

Fig 4.13 :: $v_f(0,1) = (7,6)$, $e_f(0,1)=(7,7)$.The numbers are edge numbers.

It follows from copy of bull(C_5) in fig 4.9 that we can take one point union at vertices ‘a’, ‘b’, ‘c’ and ‘d’ to get four different (pair wise non-isomorphic) structures. In all structures Type A label will serve as $G^{(k)}$ at k =1.For rest of values of k we first obtain $G^{(2x)}$ first and append it with copy type A by fusing with appropriate vertex on it.

To obtain structure 1 we fuse Type B label at vertex ‘c’ on it for x times and we get labeled copy of $G^{(2x)}$, x = 1, 2, 3, ...To get labeled copy of $G^{(2x+1)}$ we first obtain $G^{(2x)}$ and append it with copy of type A label by fusing both at vertex ‘c’ on it.

To obtain structure 2 we fuse Type C label at vertex ‘d’ on it for x times and we get labeled copy of $G^{(2x)}$, x = 1, 2, 3, ...To get labeled copy of $G^{(2x+1)}$ we first obtain $G^{(2x)}$ and append it with copy of type A label by fusing both at vertex ‘d’ on it.

To obtain structure 3 we fuse Type D label at vertex ‘b’ on it for x times and we get labeled copy of $G^{(2x)}$, x = 1, 2, 3, ...To get labeled copy of $G^{(2x+1)}$ we first obtain $G^{(2x)}$ and append it with copy of type A label by fusing both at vertex ‘b’ on it.

To obtain structure 4 we fuse Type E label at vertex ‘a’ on it for x times and we get labeled copy of $G^{(2x)}$, x = 1, 2, 3, ...To get labeled copy of $G^{(2x+1)}$ we first obtain $G^{(2x)}$ and append it with copy of type A label by fusing both at vertex ‘a’ on it. For all structures resultant label number distribution is for vertices $v_f(0,1) = (6x+1,6x)$, and for edges $e_f(0,1)=(7x,7x)$ when k = 2x, x=1, 2, 3, ..And when k = 2x+1 we have on vertices $v_f(0,1) = (6x+3,6x+4)$ and on edges $e_f(0,1)=(7x+4,7x+3)$. Thus in all structures we get e-cordial labeling of $G^{(K)}$. We conclude our paper showing that G= bull(Cn) is e-cordial.(for all n is not divisible by 4).

4.4 Theorem: A bull graph on C_5 denoted by G=bull(C_5) is e-cordial. We take three cases on n. At the case n = 4x the e-cordial labeling is not available.

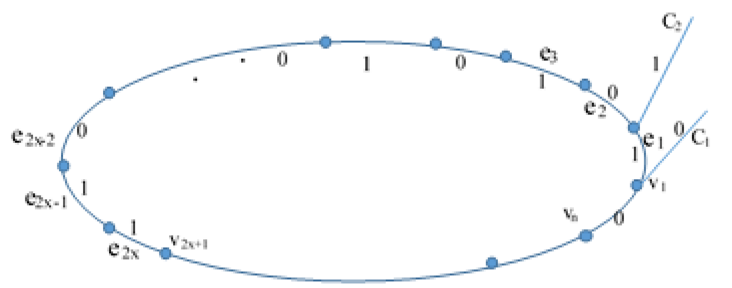

Fig 4.14 : Bull(C_{4x+1}): labeled copy

Define a function f:E(G)→{0,1} as follows.

Case n = 4x+3.G=bull(C_{4x+3}) has 4x+5 edges and 4x+5 vertices.We start with a cycle $C_{4x}=(v_1, e_1, v_2, e_2, v_3, e_3..v_{4x}, e_{4x})$, with pendent edges c_1 and c_2 attached respectively at v_1 and v_2.Define f:V(G)→{0,1} as: $f(e_i)=1$, for i = 1,3,5,7…2p-1 (p=1,2,..x).f(ei)=0 for i=2,4,..2x; $f(e_{2x+J})=1$ for j=1, 2, ..x,x+1, x+2.And for all rest i. $f(e_i) = 0$.Further $f(c_1)=0, f(c_2)=1$.The label number distribution is $v_f(0,1)=(2x+2,2x+3)$, $e_f(0,1) = (2x+3,2x+2)$.

Case n = 4x+2.G=bull(C_{4x+2}) has 4x+4 edges and 4x+4 vertices.We start with a cycle $C_{4x}=(v_1, e_1, v_2, e_2, v_3, e_3..v_{4x}, e_{4x})$, with pendent edges c_1 and c_2 attached respectively at v_1 and v_2Define f:V(G)→{0,1} as: $f(e_i)=1$, for i = 1, 3, ..2x-1, f(ei)=0 for i= 2,4,6,..2x.f(ei)=1 for i = 2x+1 to 2x+7.f(ei)=0 for I 2x+8 to 2x+n.$f(c_1)=0, f(c_2)=1$. The label number distribution is $v_f(0,1)=(2x+2,2x+2)$, $e_f(0,1) = (2x+2,2x+2)$

Case n = 4x+1.G=bull(C_{4x+1}) has 4x+3 edges and 4x+3 vertices. We start with a cycle $C_{4x}=(v_1, e_1, v_2, e_2, v_3, e_3..v_{4x}, e_{4x})$,with pendent edges c_1 and c_2 attached respectively at v_1 and v_2.Define f:V(G)→{0,1} as: $f(e_i)=1$, for i = 1, 3, ..2x-1, f(ei)=0 for i= 2,4,6,..2x.f(ei)=1 for i = 2x+1 to 2x+7.f(ei)=0 for I 2x+8 to 2x+n.$f(c_1)=0, f(c_2)=1$. The label number distribution is $v_f(0,1)=(2x+1,2x+2)$, $e_f(0,1) = (2x+1,2x+2)$. Conclusions: We have defined bull graph bull(G) and shown that G=Bull(C_n), n= 3,4,5. is E-cordial. Also we have shown for n = 3, 4, 5 that $G^{(k)}$ with all non-isomorphic structures are E- cordial. It is necessary to investigate e-cordiality for all $G^{(K)}$ for all k where G = bull(C_n).We predict that all structures on$G^{(k)}$ are e-cordial. When G has vertices say q and q-2 is divisible by 4 then $G^{(k)}$ is e-cordial for k is even number.

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