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Analysis of substitutable inventory system in supply chain with direct demand and partial backlogging

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Abstract
This paper presents continuous review two-echelon inventory systems with two different substitutable items in stock. The demand for the products follows independent Poisson with rates λ_1 and λ_2 respectively for product and A and B. There is a direct demand at DC follows poison with rate λ_d. The operating policy at the lower echelon for the products are (s_i, S_i) that is whenever the inventory level drops to 's_i' on order for Q_i = (S_i-s_i) items is placed, the ordered items are received after a random time which is distributed as exponential. We assume that the demands accruing during the stock-out period are partially backlogged. The retailer replenishes the stock of products from the supplier which adopts (0,M) policy. The joint probability disruption of the inventory levels of the products, at retailer and the products at supplier are obtained in the steady state case. Various system performance measures are derived and the long run total expected inventory cost rate is calculated. Several instances of numerical examples, which provide insight into the behavior of the system, are presented.

Keywords: Two-echelon inventory, Substitutable Product, Markov process, Partial backlogging

1. Introduction
This paper investigates the substitutable product inventory control systems in supply chain. When different products are sold by a retailer, substitution between these products causes the retailers to manage their order quantities in a competitive environment. In this article, we examine the nature of continuous review inventory control system facing stochastic independent demand in supply chain. Substitutable product inventory problem was first studied by McGillivray and Silver [1] in the Economic Order Quantity (EOQ) context. Later, Parlar and Goyal [19] and Khouja and Mehrez and Rabinowitz [18] gave single-period formulations for an inventory system with two substitutable products independently of each other. In [21], Parlar proposed a Markov Decision Process model to find the optimal ordering policies for perishable and substitutable products from the point of view of one retailer. Parlar's study in [25] is a game theoretic analysis of the inventory control under substitutable demand. He modeled the two-product single period problem as a two-person nonzero-sum game and showed that there exists a unique Nash equilibrium.

Study on multi-echelon systems are much less compared to those on single commodity systems. The determination of optimal policies and the problems related to a multi-echelon systems are, to some extent, dealt by Veinott and Wagner [30] and Veinott [31]. Sivazlian [28] discussed the stationary characteristics of a multi commodity single period inventory system.The terms multi-echelon or multi-level production distribution network and also synonymous with such networks (supply chain) when on items move through more than one steps before reaching the final customer. Inventory exist throughout the supply chain in various form for various reasons. At any manufacturing point they may exist as raw – materials, work-in process or finished goods.

The main objective for a multi-echelon inventory model is to coordinate the inventories at the various echelons so as to minimize the total cost associated with the entire multi-echelon inventory system. This is a natural objective for a fully integrated corporation that operates the entire system. It might also be a suitable objective when certain echelons are managed by either the suppliers or the retailers of the company. Multi-echelon inventory system has been studied by many researchers and its applications in supply chain management has proved worthy in recent literature.

As supply chains integrates many operators in the network and optimize the total cost involved without compromising as customer service efficiency. The first quantitative analysis in inventory studies Started with the work of Harris [13]. Clark and Scarf [8] had put forward the multi-echelon inventory first. They analyzed a N-echelon pipelining system without considering a lot size. One of the oldest papers in the field of continuous review multi-echelon inventory system is written by Sherbrooke in 1968.Hadley, G and Whitin, T. M., [12], Naddor. E [24] analyses various inventory Systems. HP's (Hawlett Packard) Strategic Planning and Modeling (SPaM) group initiated this kind of research in 1977.

Sivazlian and Stanfel [27] analyzed a two commodity single period inventory system. Kalpakam and Arivarignan [15] analyzed a multi-item inventory model with renewal demands under a joint replenishment policy. They assumed instantaneous supply of items and obtain various operational characteristics and also an expression for the long run total expected cost rate. Krishnamoorthy *et al*., [16] analyzed a two commodity continuous review inventory system with zero lead time. A two commodity problem with Markov shift in demand for the type of commodity required, is considered by Krishnamoorthy and Varghese [17]. They obtain a characterization for limiting probability distribution to be uniform. Associated optimization problems were discussed in all these cases. However in all these cases zero lead time is assumed.

A Complete review on multi echelon inventory model was provided by Benito M. Beamon [6]. Sven Axsater [5] proposed an approximate model of inventory structure in SC. He assumed (S-1, S) polices in the Deport-Base systems for repairable items in the American Air Force and could approximate the average inventory and stock out level in bases.

Anbazhagan and Arivarignan [2, 3] have analyzed two commodity inventory system undervarious ordering policies. Yadavalli *et al*., [32] have analyzed a model with joint ordering policy and varying order quantities. Yadavalli *et al*., [33] have considered a two commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

In a very recent paper, Anbazhagan *et al*. [4] considered analysis of two commodity inventory system withcompliment for bulk demand in which, one of the items for the major item, with random lead time but instantaneous replenishment for the gift item are considered. The lost sales for major item is also assumed when the items are out of stock. The above model is studied only at single location(Lower echelon). We extend the same in to multi-echelon structure (Supply Chain). The rest of the paper is organized as follows. The model formulation is described in section 2, along with some important notations used in the paper. In section 3, steady state analysis are done: Section 4 deals with the derivation of operating characteristics of the system. In section 5, the cost analysis for the operation. Section 6 provides Numerical examples and sensitivity analysis.

2. Model

2.1 The Problem Description

The inventory control system considered in this paper is defined as follows. Tow Substitutable finished products (A & B) are supplied from manufacturer to supplier which adopts (0,M) replenishment policy then the product is supplied to retailer who adopts (s_i, S_i) policy. The demand at retailer node follows an independent Poisson distribution with rate λ_1 for one product A and λ_2 for product B. When the inventory of one of the product reaches zero the demand for the product is substitutable with the other product with probability ***p*** and similar argument for another product with probability ***q*** so that ***p +q =1.*** The demands that occur when both the products are out of stock are partially backlogged. The replacement of item in terms of product is made from supplier to retailer is administrated with exponential distribution having parameter $\mu > 0$. The maximum inventory level at retailer node for product A and B are $\boldsymbol{S_i}$, and the recorder point is $\boldsymbol{s_i}$ and the ordering quantity is Q(=S_i-s_i) items. The maximum inventory at supplier is M(=nQ).

3. Analysis

Let $1(t)$and $I_2(t)$ denote the on hand Inventory levels of product A and B respectively at retailer and $I_d\ (t)$denote the on hand inventory level of both products at supplier at time t+.

We define I (t) = $\{(I_1(t), I_2(t), I_d(t)): t \geq 0\}$as Markov process with state space E = { (i, j,k) | i = S_1,S_1-1 .....0,-1,-2,…-b j = S_2,S_2-1 .....0,-1,-2,…-b, k=Q,2Q,.....nQ }. Since E is finite and all its states are aperiodic, recurrent, non- null and also irreducible. That is all the states are Ergodic. Hence the limiting distribution exists and is independent of the initial state.

The infinitesimal generator matrix of this process C = (*a* (*i, j, k, :l, m, n*))$^{(i,j,k)(l,m,n)\in E}$ can be obtained from the following arguments.

- The arrival of a demand for product A at retailer make a state transition in the Markov process from (i, j, k) to (i-1, j, k) with the intensity of transition (λ_1>0).i=1,2,….S_1
- The arrival of a demand for product B at retailer make a state transition in the Markov process from (i, j, k) to (i, j-1, k) with the intensity of transition (λ_2 >0).j= 1,2……S_2
- The arrival of common direct demand for product A and B distinct for make a transits in the Markov process from (i, j, k) to (i, j, k-Q) with the indirect of transition λ_d
- When the inventory level of product A is zero, then the the arrival of a demand for product A at retailer make a state transition in the Markov process from (i, j, k) to (i, j-1, k) with the intensity of transition $(p\lambda_1 + \lambda_2)$ >0. i =0
- When the inventory level of product B is zero, then the the arrival of a demand for product B at retailer make a state transition in the Markov process from (i, j, k) to (i, j-1, k) with the intensity of transition $(\lambda_1 + q\lambda_2)$ >0. j =0
- The replacement of inventory at retailer make a state transition in the Markov process from (i, j, k) to (i+Q, j, k-Q) or (i, j, Q) to (i, j+Q, k-Q) with the intensity of transition μ > 0.

The infinitesimal generator **C** is given by

$$\mathbf{C}=\begin{bmatrix} A & B & 0 & .... & 0 & 0 \\ 0 & A & B & .... & 0 & 0 \\ \vdots & \vdots & \vdots & .... & \vdots & \vdots \\ 0 & 0 & 0 & .... & A & B \\ B & 0 & 0 & .... & 0 & A \end{bmatrix}$$

The sub matrices A and B are given by

$$[A]_{mxn} = \begin{cases} A_1 & n = m, \quad m = s_1+1, s_1+2, ...S_1, \\ B_1 & n = m-1, \quad m = S_1, S_1-1, ...1, ...-(b+1) \\ C_1 & n = m, \quad m = s_1, s_1-1, ...1.......-(b+1) \\ D_1 & n = m, \quad m = -b, \\ 0 & \text{otherwise} \end{cases}$$

$$[B]_{mxn=} \begin{cases} M & m = n \quad S_{1,} S_1 -1, ...1, 0, -1.....-b \\ 0 & \text{otherwise} \end{cases}$$

Where

$$[A_1]_{mxn} = \begin{cases} (\lambda_1) & n = m-1, \quad m = S_1, S_1-1, ...1, 0....-b \\ -(\lambda_1+\lambda_2) & n = m, \quad m = s_1+1, s_1+2, ...S_1 \\ -(\lambda_1+\lambda_2+\mu\ ) & n = m, \quad m = 1, 2, ...s_1 \\ -(p\lambda_1+\lambda_2+\mu) & n = m, \quad m = -b \\ 0 & \text{otherwise} \end{cases}$$

$$[B]_{mxn} = \begin{cases} (\lambda_2) & n = m \quad m = S_{1,} S_1-1, ...1, 0....-b \\ (p\lambda_1+\lambda_2) & n = m \quad m = -b \\ & \text{otherwise} \end{cases}$$

$$[C_1]_{mxn} = \begin{cases} (\lambda_1) & n = m-1, \quad m = S_1, S_1-1, ...1, 0...-(b+1) \\ -(\lambda_1+\lambda_2+\mu+\lambda_d) & n = m, \quad m = s_1+1, s_1+2, ...S_1 \\ -(\lambda_1+\lambda_2+2\mu+\lambda_d\ ) & n = m, \quad m = 1, 2, ...s_1 \\ -(p\lambda_1+\lambda_2+2\mu+\lambda_d) & n = m, \quad m = -b \\ 0 & \text{otherwise} \end{cases}$$

$$[D_1] = \begin{cases} (\lambda_1) & n = m-1, \quad m = S_1, S_1-1, ..., 1, 0..., -(b+1) \\ -(\lambda_1+p\lambda_2+\mu+\lambda_d) & n = m, \quad m = s_1+1, s_1+2, ...S_1 \\ -(\lambda_1+p\lambda_2+2\mu+\lambda_d) & n = m, \quad m = 1, 2, ...s_1 \\ (-2\mu+\lambda_d) & n = m, \quad m = -b \\ 0 & \text{otherwise} \end{cases}$$

$$[M]_{mxn=} \begin{cases} \lambda_d & q = p \quad p = S, S_1-1.....0, -1, ...-b \\ \mu & q = p \quad p = S_1, S_1-1.....0, -1, ...-b \\ 0 & \text{otherwise} \end{cases}$$

3.1 Steady State Analysis

The structure of the infinitesimal matrix C, reveals that the state space E of the Markov process { I (t): t ≥ 0 } is finite and irreducible. Let the limiting probability distribution of the inventory level process be

$$\prod_{i,j}^{k} = \lim_{t\to\infty} Pr\{(I_m(t), I_c(t), I_d(t) = (i,j,k))\}$$

where $\prod_{i,j}^{k}$ *is* the steady state probability that the system be in state *(i, j, k)*.

Let $\prod = \{\prod_{i,j}^{nQ}, \prod_{i,j}^{(n-1)Q}, ...\prod_{i,j}^{Q}.\}$ denote the steady state probability distribution. For each (i,j,k), $\prod_{i,j}^{k}$ *can* be obtained by solving the matrix equation $\prod$ C = 0.

By solving the above system of equations, together with normalizing condition $\sum_{(i,j,k)\in E} \prod_{i,j}^{k} = 1$, the steady probability of all the system states are obtained.

4. Operating characteristic

In this section we derive some important system performance measures.

4.1 Average inventory Level

The event I_1, I_2denote the average inventory level for products A and B respectively at retailer node *and* I_d denote the average inventory level at distributor node

(i) $$I_1 = \sum_{k=Q}^{nQ} \sum_{j=0}^{S_2} \sum_{i=0}^{S_1} i.\prod_{i,j}^{k}$$

(ii) $$I_2 = \sum_{k=Q}^{nQ} \sum_{i=0}^{S_1} \sum_{j=0}^{S_2} j.\prod_{i,j}^{k}$$

(iii) $$I_d = \sum_{i=0}^{S_1} \sum_{j=0}^{S_2} \sum_{k=Q}^{nQ} k.\prod_{i,j}^{k}$$

4.2 Mean Reorder Rate

Let R_1, R_2, *denote* the mean reorder rate for products A and B, at retailer and R_d denote the mean reorder rate for products at distributor respectively,

(i) $$R_1 = (\lambda_1 + (s_1+1)\gamma)\sum_{k=Q}^{nQ} \sum_{j=o}^{S_2} \prod_{s_1+1,j}^{k}$$

(ii) $$R_2 = (\lambda_2 + (s_2+1)\gamma)\sum_{k=Q}^{nQ} \sum_{i=o}^{S_1} \prod_{i,s_2+1}^{k}$$

(iii) $$R_d = (\mu + \lambda_d)\sum_{i=0}^{S_1} \sum_{j=0}^{S_2} \prod_{i,j}^{Q}$$

4.3 Shortage rate

Shortage occur at retailer only for main product. LetS_r be the shortage rate at retailer for main product, then

(i) $$S_r = (\lambda_1 + \lambda_2)\sum_{k=Q}^{nQ} \prod_{-b,-b}^{k}$$

5. Cost analysis

In this section we impose a cost structure for the proposed model and analyze it by the criteria of minimization of long run total expected cost per unit time. The long run expected cost rate *TC(s₁, s₂, Q*is given by

$$TC(s_1, s_2, Q) = I_1.H_1 + I_2.H_2 + I_d.H_d + R_1.O_1 + R_2.O_2 + R_d.O_d + S_r.T_r$$

Although we have not proved analytically the convexity of the cost function $TC(s_1, s_2, Q$our experience with considerable number of numerical examples indicate that $TC(s_1, s_2, Q)$ for fixed 'S_1 *and* S_2' appears to be convex in*s*. In some cases it turned out to be increasing function of *s*. For large number case of $TC(s_1, s_2, Q)$ revealed a locally convex structure. Hence we adopted the numerical search procedure to determine the optimal value of '*s*'

6. Numerical Example and Sensitivity Analysis

6.1 Numerical Example

In this section we discuss the problem of minimizing the structure. We assume $H_2 \leq H_1 \leq H_d$, i.e, the holding cost for product B at retailer node is less than that of product A and the holding cost of both products is less than that of products at distributor node. Also $O_2 \leq O_1 \leq O_d$ the ordering cost at retailer node for product B is less than that of product A. Also the ordering cost at the distributor is greater than that of both products at retailer node.

The results we obtained in the steady state case may be illustrated through the following numerical example,

$S_1 = 16, S_2 = 16$ M = 80,

$\lambda_1 = 3$, $\lambda_2 = 2$, $\lambda_d = 4$, $\mu = 3$

$H_1 = 1.1,\ H_2 = 1.2,\ H_d = 1.3$

$O_1 = 2.1,\ O_2 = 2.2,\ O_d = 2.3$

$T_r = 3.1, b = 3$

The cost for different reorder level are given by

Table 1: Total expected cost rate as a function s_1, s_2 and Q

s1	**1**	**2**	**3**	**4***	**5**	**6**	**7**
s2	1	2	3	4*	5	6	7
Q	15	14	13	12*	11	10	9
TC(s1, s2, Q)	155.835611	90.63277	77.55012	75.28077	80.75253	94.54283	105.9115

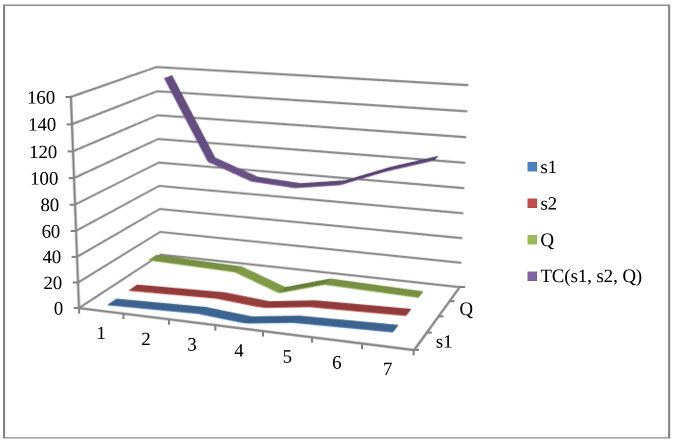

Fig 1

For the inventory capacity S_1 and S_2, the optimal reorder level s_1, and s_2 and optimal cost $TC(s_1, s_2, Q)$ are indicated by the symbol *. The Convexity of the cost function is given in the graph with common reorder point *s* (both s_1, and s_2).

6.2 Sensitivity Analysis

Below tables are represented a numerical study to exhibit the sensitivity of the system on the effect of varying different parameters.

$$\lambda_1 \& \mu,\ \lambda_2 \& \mu,\ H_2 \& H_1,\ H_1 \& H_d,\ O_1 \& O_2,\ O_1 \& O_d;$$

For the following cost structure $S_1 = 16$, $S_2 = 16$ M = 80,

$\lambda_1 = 3$, $\lambda_2 = 2$, $\lambda_d = 4$, $\mu = 3$

$H_1 = 1.1,\ H_2 = 1.2,\ H_d = 1.3$

$O_1 = 2.1,\ O_2 = 2.2,\ O_d = 2.3$ $T_r = 3.1, b = 3$

Table 2: Effect on Replenishment rate& Demand rates $\mu \backslash \lambda_1$

$\mu \backslash \lambda_1$	**1**	**2**	**3**	**4**	**5**
1	44.9562	105.736	183.073	262.016	341.248
2	64.192	80.6682	132.526	204.329	281.382
3	83.6582	100.861	120.95	167.608	234.24
4	99.8989	127.775	141.663	165.368	210.307
5	114.624	151.974	171.738	187.085	213.553

Table 3: Effect on Replenishment rate& Demand rates $\mu \backslash \lambda_2$.

$\mu \backslash \lambda_2$	**1**	**3**	**5**	**7**	**9**
1	264.4122	186.988	225.608	308.413	388.178
3	261.1814	161.193	171.85	229.599	285.558
5	260.5082	155.149	158.612	209.956	259.907
7	260.2163	152.461	152.627	201.05	248.266
9	260.0576	150.94	149.217	195.96	241.605

Table 4: Effect on Holding cost ($H_1 \backslash H_d$)

$H_1 \backslash H_d$	**1.1**	**2.1**	**3.1**	**4.1**	**5.1**
1.1	167.7286	169.917	171.935	173.955	175.974
2.1	171.927	173.947	175.959	177.987	180.004
3.1	175.959	177.979	179.996	182.016	184.036
4.1	179.9885	182.001	184.028	186.045	188.065
5.1	184.0205	186.038	188.058	190.077	192.097

Table 5: Effect on Ordering Cost ($H_2 \backslash H_1$)

$H_2 \backslash H_1$	**2.1**	**2.2**	**2.3**	**2.4**	**2.5**
2.1	167.7286	168.2995	168.778	169.306	169.83
2.2	168.2995	168.8269	169.352	159.887	170.404
2.3	168.873	169.4003	169.925	170.45	170.977
2.4	169.4464	169.4003	170.499	171.023	171.551
2.5	170.0198	170.5446	171.072	171.597	172.122

Table 6: Effect on Penalty Cost ($O_1 \& O_d$)

$O_1 \backslash O_d$	**3.1**	**3.2**	**3.3**	**3.4**	**3.5**
3.1	167.7286	168.3	168.778	169.306	169.83
3.2	168.2995	168.827	169.352	159.887	170.404
3.3	168.873	169.4	169.925	170.45	170.977
3.4	169.4464	169.4	170.499	171.023	171.551
3.5	170.0198	170.545	171.072	171.597	172.122

Table 7: effect on Holding cost & ordering cost ($H_1 \& O_d$;)

$H_1 \backslash O_d$;	**1.1**	**1.2**	**1.3**	**1.4**	**1.5**
1.1	166.1952	166.623	167.025	167.429	167.831
1.2	166.8557	167.265	167.662	168.067	168.468
1.3	167.4957	167.898	168.3	168.704	169.106
1.4	168.1331	168.535	168.94	169.341	169.743
1.5	168.7706	169.172	169.577	169.979	170.381

It is observed that from the table, the total expect cost $TC(s_1, s_2, Q)$is increases with the cost increases.

7. Conclusion

This paper deals with a two echelon Inventory system with two substitutable products A and B.. The structure of the chain allows vertical Movements of goods. It would be calculating to analyze the problem with general distribution. The author is working in their directions and further results will be reported soon.

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