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Comparison of two weighted goal programming models for financial management of a health care system

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Abstract
In this paper, we presented weighted goal programming model (WGP) for financial management of healthcare system in Hyderabad. The data are collected from the health care system's financial statements from 2010 to 2017. In this study, we considered six financial metrics such as liability, equity, income, asset, profit and proportion of values in the statement. The weights in the goal programming model are obtained by percentage normalization and analytical hierarchy process (AHP). The problem was solved using PM-QM for windows and the solutions are compared.

Keywords: financial management, financial metrics, percentage normalization, analytical hierarchy process (AHP), weighted goal programming model (WGP).

1. Introduction

Now a day's health care systems are facing problems to meet the requirements of the patient's in terms of qualitatively and quantitatively. There is high competition among the health care systems to attract the patients/customers. They are facing problems in human resource planning, other infra-structure resources panning, technical resources planning. To provide high quality services for the patients at low cost for any health care system must overcome these issues. If the health care system is financially good, it can perform well. So, for any health care system financial management is important to run the system effectively. Financial management means not expecting high return/profit on investments. Health care system different from other systems. They always try to provide good service for the patients at reasonable charges. So, for the financial management of selected health care system at Hyderabad, we consider the parameters asset, liability, equity, profit, income and proportion of the values in the data from 2010to 2017.The data has been collected from the annual statements of the health care system certified the financial experts. We used the weighted goal programming in this paper. This method is applied in two ways 1) weights obtained by percentage normalization 2) weights obtained analytical hierarchy process (AHP).

2. Literature Review

Goal programming (GP) is an extension of linear programming introduced by Charnes [7]. Linear programming had been proved to be a good method to solve practical problems. But its drawback is that it can be used only in a single objective optimization. Real life decision making problems are multi-objective rather than single objective. Which led to the development of further programming techniques to deal with multi-objective real-world problems. Goal Programming (GP) is important and widely used in multi-objective optimization approach to formulate and solve many real-life problems with multiple objectives. GP [23] approach is one of the oldest multi-criteria decision-making methods. In GP we consider the constraints as goals and the objective is to minimize the undesired deviational variables. One of the popular variation of GP is weighted goal programming (WGP) [20] in which weights are used in the objective function. The weights can be obtained in different ways. In this study we considered percentage normalization and analytical hierarchy process (AHP) to get the relatives weights for the goals. In our previous studies [1, 2] we have used lexicographic goal programming and weighted goal programming for financial management of health care system. In this paper we compare the two variations of the weighted goal programming models. There are numerous applications of GP. We review few studies. Goal programming is applied for optimization of bank loans [3], financial portfolio management [4], financial structure optimization [26], funding allocation in universities/institutions [9, 12], asset and liability management [24, 11], optimization of liquidity management of banks [18], capital budgeting [13], vehicle routing [6], forest management [8], optimization of pharmacy allocation [10], human resource allocation in hospitals [16], nurse scheduling [19]. AHP is most widely used pair wise comparison method of MCDM category. AHP is useful to prioritize the criteria and select the best alternative. The AHP was introduce by Saaty [21, 22] Vaidya Kumar [27] presented a review paper on AHP. Steuer and Na, [25] applied AHP to the finance sector, Bruno *et al*. [5] evaluated supplier based AHP method, Ivley *et al*. [15] used AHP for medical equipment selection, Wang *et al*. [28] evaluated the relative importance weightings of alternative suppliers supported the delivery liableness, flexibility and responsiveness, cost, and asset. In the next section we presented general GP and AHP methodology.

3. Methodology

In this section we discuss the general goal programming methodology and AHP methodology

3.1 General Goal Programming

The generalized linear goal programming is defined as

$$\textit{Minimize } Z=\sum_{i=1}^{m}\sum_{k=1}^{n_i} P_i(w_{ik}^{-}d_i^{-}+w_{ik}^{+}d_i^{+})$$

$$\textit{subject} \text{ to}$$

$$\text{goal constraints } \sum_{j=1}^{n}\left(a_{ij}x_j+d_i^{-}-d_i^{+}\right)=b_i \quad i=1,2....m,\ j=1,2...n$$

$$\text{system constraints } \sum_{j=1}^{n}\left(a_{ij}x_j\right)\begin{bmatrix}\le\\=\\\ge\end{bmatrix}b_i$$

$$d_i^{-}, d_i^{+}, x_j \ge 0$$

Where w_{ik}^{-} , $w_{ik}^{+} \ge 0$ and represent the relative weights to be assigned to each of the k =1... n_i Different classes within the i^{th} category to which the non-Archimedean transcendental value of P_i is assigned. The P_i are the preemptive priority factors who serve only as a ranking symbol that can be interpreted to mean that no substitutions across categories of goals will be permitted. It is assumed that the ordering of deviation variables in an objective function, will be minimized in order, where $P_i \gg P_{i+1} \gg P_m \gg$

d_i^{+} = is the positive deviation variable from overachieving the i^{th} goal

d_i^{+} = is the negative deviation variable from underachieving the i^{th} goal

x_j = is the j^{th} decision variable a_{ij} is the decision variable coefficient

b_i = the associated right-hand side value

Where the deviational variables are mathematically defined as follows

$$d_i^{+}=\frac{1}{2}\left[\left|\sum_{j=1}^{n}\left(a_{ij}x_j-b_i\right)\right|+\sum_{j=1}^{n}\left(a_{ij}x_j-b_i\right)\right]$$

$$d_i^{-}=\frac{1}{2}\left[\left|\sum_{j=1}^{n}\left(a_{ij}x_j-b_i\right)\right|-\sum_{j=1}^{n}\left(a_{ij}x_j-b_i\right)\right]$$

The deviational variable cannot be basic variables at once, because by definition they are dependent. This indicates that in any simplex iteration, at most, one of them can assume a positive value. The following table-1 shows the various types of goal achievement.

Table 1: Goal achievement

Minimize	goal	If goal achieved
d_i^{-}	Minimize under achievement	$d_i^{-}=0,\ d_i^{+}\ge 0$
d_i^{+}	Minimize overachievement	$d_i^{+}=0,\ d_i^{-}\ge 0$
$d_i^{-}+d_i^{+}$	Minimize both under and over achievement	$d_i^{-}=0,\ d_i^{+}=0$

3.2 AHP Methodology

In the goal programming, goals are prioritized by analytical hierarchy process. To make a decision in an organized way to generate priorities we need to decompose the decision into the following steps.

Step-1 outline the matter and confirm the sort of information sought-after.

- Step-2 Structure the choice hierarchy from the highest with the goal of the choice, then the objectives from a broad perspective, through the intermediate levels (criteria on that later components depend) to all-time low level (which typically could be a set of the alternatives).
- Step-3 Construct a collection of pairwise comparison matrices. Every part in associate degree higher level is employed to match the weather within the level right away below with regard to it.
- Step-4 Use the priorities obtained from the comparisons to weigh the priorities within the level right away below. Do that for each part. Then for every part within the level below add its weighed values and acquire its overall or international priority. Continue this method of advisement and adding till the ultimate priorities of the alternatives within the bottom most level is obtained. For detailed methodology is refer [22]

4. Data of the problem

The data is collected from the annual reports of the health care system which are uploaded in the website. We considered the asset, liability, equity, income, profit, as parameters. Table-.2 shows the data of the health care system (in Rs. millions).

Table 2: Data of the problem

Item (goal)	year								total
	2010	2011	2012	2013	2014	2015	2016	2017	
asset	6,196.62	5,980.23	9,640.36	14,632.9	13,575.63	16,963.33	17,828.42	21,863.04	1,06,680.53
liability	5,228.46	5,350.15	5,438.24	5,050.31	5,550.09	7,959.93	9,821.75	9,783.53	54,182.46
equity	15,417.78	17,721.65	23,522.66	27,275.97	29,647.25	31,610.71	34,301.31	38,877.81	2,18,375.14
income	18,587.45	23,522.66	28,279.20	33,488.18	38,840.88	46,380.62	54,779.64	72,773.99	3,16,6,52.62
profit	1,519.64	1,817.18	2,309.90	3,091.08	3,307.20	3,465.95	3,694.39	1,311.6	20,516.94
total	46,949.95	54,391.87	69,190.36	83,538.44	90,921.05	1,06,380.54	1,20,425.51	1,44,609.97	6,54,136.69

Table-3 shows the coded values (in Rs. trillions) of the health care system. We coded the values because to enable the analysis with small values.

Table 3: Coded values

Item(goal)	year								total
	2010	2011	2012	2013	2014	2015	2016	2017	
asset	0.0062	0.0060	0.0096	0.0146	0.0136	0.0170	0.0178	0.0219	0.1067
liability	0.0052	0.0054	0.0054	0.0051	0.0056	0.0080	0.0098	0.0098	0.0542
equity	0.0154	0.0177	0.0235	0.0273	0.0296	0.0316	0.0343	0.0389	0.2184
income	0.0186	0.0235	0.0283	0.0335	0.0388	0.0464	0.0548	0.0728	0.3167
profit	0.0015	0.0018	0.0023	0.0031	0.0033	0.0035	0.0037	0.0013	0.0205
total	0.0470	0.0544	0.0692	0.0835	0.0909	0.1064	0.1204	0.1446	0.6541

5. Model Development

In this section we develop the method for the data

5.1 Decision Variables

The decision variables are defined as follows

x_1 = the amount of financial statement in year 2010

x_2 = the amount of financial statement in year 2011

x_3 = the amount of financial statement in year 2012

x_4 = the amount of financial statement in year 2013

x_5 = the amount of financial statement in year 2014

x_6 = the amount of financial statement in year 2015

x_7 = the amount of financial statement in year 2016

x_8 = the amount of financial statement in year 2017

5.2 The goal constraints: The goals to be examined are: (1) asset accumulation, (2) liability reduction, (3) equity wealth, (4) earning and (5) profitability and (6) optimum management item on the financial statement. The goal constraints are defined below.

Goal 1: Asset Accumulation Goal: The management of healthcare system wants to maximize the asset accumulation. So, we have to minimize the negative deviational variable d_1^-.

$$0.0062x_1 + 0.0060x_2 + 0.0096x_3 + 0.0146x_4 + 0.0136x_5 + 0.0170x_6 + \\ 0.0178x_7 + 0.0219x_8 + d_1^- - d_1^+ = 0.1067$$

Goal 2: Liability Goal: The management wants to minimize the liability. So, we need to minimize the over achievement of the goal, that is positive deviational variable d_2^+

$$0.0052x_1 + 0.0054x_2 + 0.0054x_3 + 0.0051x_4 + 0.0056x_5 + 0.0080x_6 \\ +0.0098x_7 + 0.0098x_8 + d_2^- - d_2^+ = 0.0542$$

Goal 3: Equity Goal: The equity is to be maximized. So, the under-achievement variable d_3^- is to be minimized.

$$0.0154x_1 + 0.0177x_2 + 0.0235x_3 + 0.0273x_4 + 0.0296x_5 + 0.0316x_6 + \\ 0.0343x_7 + 0.0389x_8 + d_3^- - d_3^+ = 0.2184$$

Goal 4: Income Goal: The management wants to maximize the income. We have to minimize the under-achievement variable d_4^-

$$0.0186x_1 + 0.0235x_2 + 0.0283x_3 + 0.0335x_4 + 0.0388x_5 + 0.0464x_6 + \\ 0.0548x_7 + 0.0728x_8 + d_4^- - d_4^+ = 0.3167$$

Goal 5: Profitability Goal: To maximize the profit, the under-achievement variable d_5^- is to be minimized.

$$0.0015x_1 + 0.0018x_2 + 0.0023x_3 + 0.0031x_4 + 0.0033x_5 + 0.0035x_6 + \\ 0.0037x_7 + 0.0013x_8 + d_5^- - d_5^+ = 0.0205$$

Goal 6: Financial Statement Managing Goal: To maximize the proportion of the values in the financial statement the under-achievement variable d_6^- is to be minimized

$$0.0470x_1 + 0.0544x_2 + 0.0692x_3 + 0.0835x_4 + 0.0909x_5 + 0.1064x_6 + \\ 0.1204x_7 + 0.1446x_8 + d_6^- - d_6^+ = 0.6541$$

6. Weights By percentage normalization

In this method each deviation is turned into a percentage value away from its target level. Thus, all deviations are measured in the same units. In our problem, consider that the decision maker esteems the penalization of all unwanted deviations as equally important. The weighted goal programming with percentage normalization is given as follows

6.1 Objective function

$$Minimize Z = (1/0.1067)d_1^- + (1/0.0542)d_2^+ + (1/0.2184)d_3^- + (1/0.3167)d_4^- + \\ (1/0.0205)d_5^- + (1/0.6541)d_6^-$$

We can observe that the weights in the objective function have been divided by the target levels rather than the entire goal. The former gives the same effect as the latter but with less effort in reformulation. Also, the objective function contributions are actually proportions rather than percentages. That is, they are one hundredth of the percentage value. The entire achievement function could be multiplied by 100 to give percentages, but this does not affect the optimal solution found. The achievement function value a now has meaning, the total sum of proportional deviations away from the goals

6.2 Solution

The problem was solved by using POM-QM for windows (formerly DS for windows). The following table-4 shows the results.

Table 4: Solution of WGP

Goal priority	Negative deviation variable (d_i^-)	positive deviation variable (d_i^+)
P_1	0	0
P_2	.00933	0
P_3	0	.00339
P_4	0	0
P_5	0	.00061
P_6	0	.05695

The table-3 shows the values of positive and negative deviational variables related to the goals from P_1 to P_6. The first priority P_1 is to maximize the total assets. The goal is fully achieved because the negative deviational variable $d_1^- = 0$ and $d_1^+ = 0$. This means the asset of the health care system can't be changed. The goal of liability reduction P_2 is also achieved since $d_2^+ = 0$ but $d_2^- = 0.00933$, this means the liability can be reduced to 0.00933 Trillion. The third priority goal P_3 is achieved since the negative deviational variable d_3^- =0 but the positive deviational variable $d_3^- = 0.00339$ this means the equity amount can be increased by 0.00339 trillion. The fourth priority goal P_4 is maximizing income is also achieved, since both the deviational variable d_4^- = 0, $d_4^+ = 0$. The profitability goal is achieved, since $d_5^- = 0$ but $d_5^+ = 0.00061$ this indicates the total profit can be changed increased by0.00061 Trillion. Lastly the goal P_6 of maximizing the proportion of the value given in the financial statement is also achieved, because the negative deviational variable d_6^- =0, but the positive deviational variable d_6^+ =0.05695 indicates that the proportion of the values given in the financial statement can be increased by 0.05695 in the period.

7. Weights by Analytical Hierarchy Process

Analytic hierarchy process (AHP) is a more generally accepted remedy by which the priorities of preemptive goals can be established. AHP utilizes hierarchical structures to represent a decision-making problem and then develops priorities for the alternatives on the basis of decision-makers' judgments throughout the decision-making process. The procedure requires the decision maker to judge the relative importance of each criterion and specify a preference on each criterion for decision alternatives based on pairwise comparisons for elements in hierarchy using the pairwise comparison matrix. For estimation of relative importance for the decision problems, the decision-makers perform synthetization and compute eigenvalues and eigenvectors that are used for measuring consistency. The value of consistency in judgment is determined by the smallest eigenvector. The result is that the smaller the value of consistency, the smaller the value of eigenvector. The value of the consistency ratio of 0.10 or less is considered to be acceptable. To prioritize the financial metrics, we have prepared set of questionnaires and was distributed among a great number of financial analyst/accountant and the judgements are considered. The AHP weights are in the following table-5

Table 5: AHP weights

Criteria	Weightage	Ranking	
Asset	0.211248	1	$\lambda_{\max} = 11.1506$
Equity	0.172429	3	
Liability	0.174445	2	$C.I. = 1.03012$
Income	0.141067	5	
Profit	0.16732	4	
Financial management	0.13349	6	$CR = 0.824096$

7.1 Objective function of AHP-GP: The objective function of combined AHP-GP is defined as follows

$$\text{Minimize Z} = (0.211248)d_1^- + (0.174445)d_2^+ + (0.172429)d_3^- + (0.141067)d_4^- + (0.16732)d_5^- + (0.13349)d_6^-$$

7.2 Solution: the solution of AHP-GP presented in the following table

Table 6: Solution by AHP-GP

Goal priority	Negative deviation variable (d_i^-)	positive deviation variable (d_i^+)
P_1	0	0
P_2	0	0
P_3	0	.013094
P_4	0	0
P_5	0	0.001878
P_6	0	0.0564427

The table-6 shows the values of positive and negative deviational variables related to the goals from P_1 to P_6. The first priority P_1 is to maximize the total assets. The goal is fully achieved because the negative deviational variable $d_1^- = 0$ and $d_1^+ = 0$.This means the asset of the health care system can't be changed. The goal of liability reduction P_2 is also achieved since $d_2^+ = 0$ but $d_2^- = 0$, this means the liability cannot be changed. The third priority goal P_3 is achieved since the negative deviational variable d_3^- =0 but the positive deviational variable $d_3^+ = 0.013094$ this means the equity amount can be increased by 0.013094 trillion. The fourth priority goal P_4 is maximizing income is also achieved, since both the deviational variable d_4^- = 0, $d_4^+ = 0$. The profitability goal is achieved, since $d_5^- = 0$ but $d_5^+ = 0.001878$ this indicates the total profit can be increased by0.001878 Trillion. Lastly the goal P_6 of maximizing the proportion of the value given in the financial statement is also achieved, because the negative deviational variable d_6^- =0, but the positive deviational variable d_6^+ =0.0564427 indicates that the proportion of the values given in the financial statement can be increased by 0.0564427trillion per period.

8. Conclusion

In this chapter first, we have applied weighted goal programming method in two ways: weights by percentage normalization and weights by AHP. The solutions are compared in the following table-7

Table 7: Comparison of solutions

Goal	**Method-1**		**Method -2**	
	(d_i^-)	(d_i^+)	(d_i^-)	(d_i^+)
asset	0	0	0	0
liability	.00933	0	0	0
equity	0	.00339	0	.013094
Income	0	0	0	0
Profit	0	.00061	0	0.001878
Financial management	0	.05695	0	0.0564427

In the above table,

- Method-1 weighted GP(weights by percentage normalization)
- Method-2 Combined AHP-GP

It is clear that both the solutions state that asset, income cannot be changed. The positive deviational variable corresponding to financial management goal, profit and equity is not zero in both the methods. That is all the methods suggests that the values in the date should be increased. All the goals are achieved by both the methods. The methods are very useful and can be applied in other areas also

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