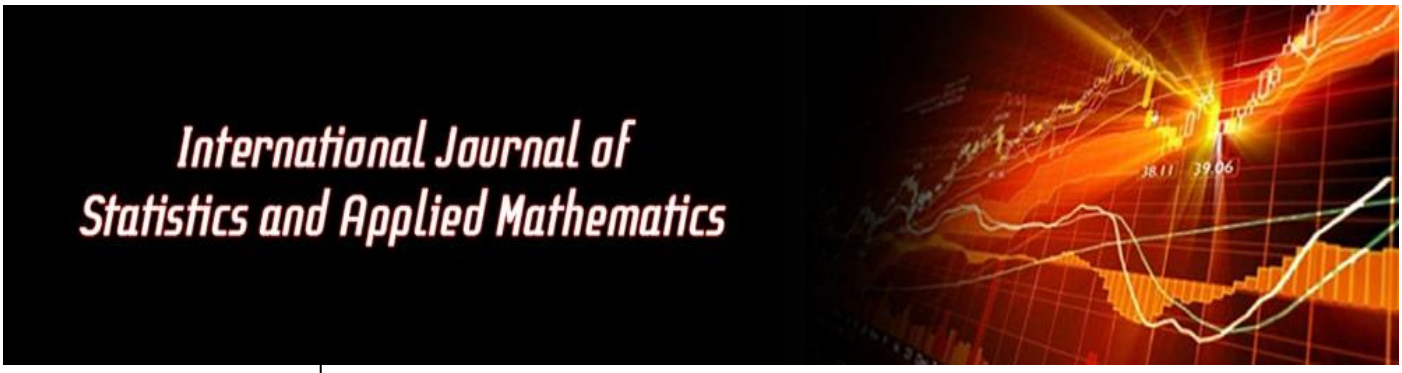
International Journal of Statistics and Applied Mathematics 2018; 3(2): 402-407



ISSN: 2456-1452
Maths 2018; 3(2): 402-407

www.mathsjournal.com
Received: 04-01-2018
Accepted: 07-02-2018

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Exploring the newman modified error hierachical model as a diagnostic approach in the teaching and learning of quadratic equation at the senior secondary school level in Zaria, Nigeria

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Abstract
The study explored the Newman's modified error hierachical model as a diagnostic approach in the teaching and learning of quadratic equation at the senior secondary school level in Zaria, Nigeria. A total of 120 Senior Secondary 2 mathematics students randomly selected from two private schools in Zaria with a mean age of 17 constituted the sample size for the study. The Mathematics achievement tests (MAT) and Quadratic diagnostic test (QDT) were used as the instruments of this study that included three components: the use of factorization, completing the square and formulae. Diagnostic interview in the form of semi-structured questionnaire were also used to identify at which level students' errors occur in solving problems and the perception of students towards quadratic equation was that success can be achieved in problem solving with a good grasp of the subject. The type of error is based on Newman Error Hierarchy Model that includes reading type error, comprehension, transformation, process skill, and encoding error. Data was analyzed using mean, percentage, frequency, Duncan's (Multiple-Range) and Wilcoxon test at $P<0.05$. The study found out among others that most students make error in transformation and process skill in solving problems in quadratic equations. There was no error found in reading. The number of students who made encoding error and carelessness was small. The students' error in solving quadratic equation problems was due to their weaknesses in basic arithmetical and algebraic operations. The implication of the study was highlighted and recommendations was made based on findings.

Keywords: Quadratic equation, comprehension error, transformation error, process skill error

Introduction
Mathematics concepts are vast, interrelated and possess interconnected elements. The interrelationship of mathematical concepts can be identified in the use of elementary operations of division, ratios, percentage, addition, subtraction as well as translation of word problems (Charles-Ogan and George, 2015) [4]. Students of low logical reasoning and analytical prowess would, therefore, find certain concepts difficult especially when it has to do with problem solving.

Problem-solving is an activity that can generate Higher Order Thinking Skills (HOTS) among students. However, only some of the students are capable of solving problems and some are having difficulties. The difficulties have caused students to make various kinds of errors. Hence, this study conducted is to identify and analyze students' errors in solving problems that involve HOTS in the topic of Fraction (Abdullah *et al*. 2015) [1]. Problem-solving is an activity that involves various actions in the mind of thought including accessing and using knowledge and experience (Lester & Kehle, 2003) [10]. Thus, teaching strategies that involve the use of non-routine problems in the classroom give students the opportunity to develop higher order thinking skills in the process of understanding, exploration and application of mathematical concepts (Polya, 1973) [16]. Diagnostic interview was also used to identify at which level students' errors occur in solving problems (Huang and Cheng, 2010) [8]. The type of error is based on Newman Error Hierarchy Model that includes reading type error, comprehension, transformation, process skill, and encoding error.

Guided by the phases described by Polya and Newman, students can solve problems more easily and systematically even if they are given problems with various levels of difficulties. However, not all students are able to solve the problems as they are having difficulties in the specific phases. Based on the studies conducted by Effandi and Siti Mistima (2010) [7] and Wijaya *et al*. (2014) [22] students have difficulties in the transformation and process skills phases while solving problems of Quadratic Equations. Students' difficulties in problem-solving process have caused them to make various errors. Widiharto (2008) [23] observed that mathematical concepts and skills that are not fully mastered by students have led to difficulties and errors in solving mathematical problems. A study conducted by Susanti *et al*. (2014) [20] found out that students found it difficult to solve problems that involve the use of HOTS and among the difficulties faced by them are a) reading and interpreting data, b) determining and delegating data and c) making conclusions and arguments. Therefore, teachers need to realize the difficulties and errors encountered by students and take appropriate approaches to improve their teaching practices (Ashlock, 2005) [3]. In conclusion, teachers are responsible to identify in advance the difficulties in learning mathematics, which led to errors before teaching other mathematical topics. According to Effandi and Siti Mistima (2010) [7], the Newman's Error Analysis Model [13] has the hierarchy that categorizes types of error based on the levels of problem-solving by the students. The above statement is in line with Ellerton and Clements (1996) [5], who stated that Newman used the "hierarchy" as he gave the reason that students who fail at any level of problem-solving prevented them from getting the required solution. In the process of problem-solving, there are two factors that make students unable to produce the correct answer, namely a) problems in language fluency and conceptual understanding (reading and comprehension) and b) problems in mathematical processing (transformation, process skills and encoding) (Prakitipong & Nakamura, 2006) [17]. The body of academic literature in mathematics education research has shown that students have misconceptions and make errors (Effandi and Siti Mistima, 2010) [7]. This clearly shows that students need to understand the meaning of the questions before going through the mathematical processing in order to produce the correct answers. In addition, the use of the Newman's model in error analysis has overtime attracted considerable attention (Huang and Cheng, 2010) [8]. The first type of error in the Newman's Error Analysis Model is reading, which is the ability of students to read the mathematical problems given and to identify the sentences and mathematical symbols used. The second type of error is comprehension, which is the ability of students to understand the mathematical problems given. The next error is transformation, which sees the ability of students to choose the appropriate mathematical solution methods. The following error is process skills, where students can perform mathematics process correctly or not, and lastly the error of encoding, which is the ability of students to express the final answer.

In this exploratory study, the focus is on the utilization of the Newman's model as a diagnostic approach to analyze the error students face in solving problem involving quadratic equations at the senior secondary school level in Zaria, Nigeria.

Purpose of the study

The main purpose of the study was to analyze students' error in learning of quadratic equation which focused on the manipulation of quadratic equation using factorization, completing the square and formulae. Specifically, the study sought to:

1. Determine the extent to which individual students commit error in the learning of quadratic equation
2. Find out the possible categorization of these errors in the learning of quadratic equation.

Research question

Three research questions were posed for this study:

i. To what extent will individual students' error influence the learning of quadratic equation?
ii. What are the possible categorizations of these errors in the learning of quadratic equation?
iii. What are the student's answers that help teachers explore the students' thinking and reflection about learning of quadratic equation?

Hypotheses

The following null hypotheses were tested at $P<0.05$

H_o There is no significant difference in the type of error committed by students in solving problems involving quadratic equation.

H_A there is no significant difference in the perception of students in solving problems involving quadratic equation.

Methodology

The population for the study consists of all senior secondary schools in Zaria, Nigeria. Zaria and it's environ is made up of over twenty senior secondary schools with a population of over seventeen thousand (17,000) students. The Government funded schools unlike the privately own ones are non-coeducational. Apart from having qualified teachers, the students offer mathematics as a major subject. The private schools were therefore considered appropriate for the study and it is from this population that the study sample was drawn.

Sample

The study was conducted using 120 senior secondary (SS2) students randomly drawn from two non-governments funded (private) secondary schools in Zaria metropolis in Nigeria. The mean age of the students was 17years.

Instrumentation

The study used a set of quadratic equation diagnostic test (QEDT) questions designed by the investigator to identify type of error committed by students. Prior to the commencement of the study, the questions were subjected to face and Pearson's product moment content validated test respectively. The reliability coefficient as computed from Pearson's product moment correlation for the Diagnostic test (QEDT) was r= 0.89. The value suggest that the test question were reliable and as such would test what it was designed for. In addition, content and face validity of the instrument was carried by the investigator who is a specialist in mathematics and other specialists in cognate field especially senior colleagues in the department of mathematics, Federal College of Education, Zaria as compared to the overall objective of the curriculum.

Prior to the investigation, the respondents were subject to general mathematics achievement test. The primary objective of the MAT items was to categorize the students into high achievers (A) for those that scored 65% and above and achievers (B) for those that scored 50-64%. This is shown in Table 1.

The investigator also interviewed the respondent based on the modified procedure of Newman Error Hierarchical Model from the outcome of the written test. The questions for the respondent included:

i) Can you read the problem? (Reading level)
ii) What does the question ask you to do? (Comprehension level)
iii) What do you use to solve the question? (Transformation level)
iv) Can you show me the working steps that you have used in order to find the answer? (Process Skills)
v) Tell me what is your answer? (Encoding)

Table 1: Distribution of Students According to Different Achievement Levels

Profile	High Achievers	Achievers Total Percentage (%)		
	(A)	(B)		
Male	32	30	62	51.7
Female	30	28	58	48.3
Total	62	58	120	100

Table 2: Number of Items (QEDT) according to Skills

Solving method	Skill	Item number	Number of Item
Quadratic Equation by factorization	Solving quadratic equation by factorization	1-6	6
Quadratic Equation by completing the square	Solving quadratic equation by completing the square	7-12	6
Quadratic Equation by formula	Solving quadratic equation by formula	13-18	6

Table 3: Error in solving quadratic equation by factorization Frequency of Error

Item	Reading	Comprehension	Transformation	Process Skill	Encoding	Carelessness
1	0	18	40	46	10	6
2	0	14	46	51	9	-
3	0	16	45	47	8	4
4	0	9	46	60	5	-
5	0	10	50	56	4	-
6	0	8	48	62	2	-
Total	0	67*	275**	321**	38*	6

** Student error significant at $P<0.05$ with Duncan's (Multiple-Range) test.
* Student error not significant at $P<0.05$

Table 4: Error in solving quadratic equation by use of completing the square Frequency of Error

Item	Reading	Comprehension	Transformation	Process Skill	Encoding	Carelessness
7	0	10	40	60	6	4
8	0	6	48	61	3	2
9	0	6	45	65	4	-
10	0	7	40	68	5	-
11	0	8	44	62	6	-
12	0	7	44	64	5	-
Total	0	44^{*}	261^{**}	380^{**}	29^{*}	6

** Student error significant at $P<0.05$ with Duncan's (Multiple-Range) test.
* Student error not significant at $P<0.05$

Table 5: Error in solving quadratic equation by use of formula Frequency of Error

Item	Reading	Comprehension	Transformation	Process Skill	Encoding	Carelessness
13	0	5	45	60	6	4
14	0	6	48	61	3	2
15	0	5	45	65	5	-
16	0	6	40	68	5	-
17	0	7	43	62	6	2
18	0	7	44	64	5	-
Total	0	36	265^{**}	380^{**}	30^{*}	8

** Student error significant at $P < 0.05$ with Duncan's (Multiple-Range) test.
* Student error not significant at $P < 0.05$

Table 6: Students' perception of quadratic equation

Item	Agree %	Neutral %	Disagree %	Total
Interesting but extremely challenging Very important topic in mathematics	75	15	10	100
Could very difficult due to it	70	24	6	100
Abstractness Has a lot of misconception Teacher makes it more simpler	65	21	14	100
Success in solving problem is due to:	70	15	15	100
	69	14	15	100
-translating worded problem to equation	80	12	8	100
- writing appropriate equation	80	10	10	100
- using appropriate formula/method	72	18	10	100
Conceptual understanding help me to Solve problems easily Mathematical	85	8	7	100
ability help in problem solving	70	20	10	100

N= 120. Ratings were scored on a 5-point likert scale, where 5= strongly agree, 4= agree, 3= neutral, 2= disagree and 1= strongly disagree.
*Significant deviations of student responses to questions from the neutral score of *3* were analyzed using a one-sample Wilcoxon sign-rank test P value of <0.05 were considered to be of statistical significance.

Discussion

The most frequent errors made by student in solving problems in quadratic equation include comprehension error, transformation error and process skill error. Error in carelessness was minimal. Most comprehension errors occur when students do not understand how to approach a given problem from the concept. For instance, in solving quadratic equation by the use of factorization the predominant error were those involving comprehension, transformation and process skill. Students' often misunderstand the demand of the question. This may be due to the lack of emphasis by teachers in teaching the simplification of concepts as they appear. It may also be due to rote learning on the part of the learner. In fact, students usually don't immediately know what to do when a novel problem is encountered for the first time. Therefore, they might attempt to recall a similar problem encountered in the past and try to use that to solve the current one (Huang and Cheng, 2010) [8]. This means that students' problem solving is based on their past experience in a problem solving episode (Robertson, 2001) [18]. Anderson (1993) [2] argued that all skill learning occurs through analogical problem solving, in which examples have an important rule. Therefore, teachers must ensure that the teaching of mathematical concepts must be balanced with the arithmetic skills. The findings from present study is consistent with previous reports (Delice, 2002; Intanku 2003; Weber, 2005; Effandi and Siti Mistima, 2010) [6, 9, 21, 7].

The error type in transformation occurred during computation process especially during multiplication. This takes place due to computation problem especially among achievers. Most students make error at the process skill level especially in the manipulation of quadratic equation using formula. For example in item 3 and 12 that were virtually the same the students were required to find the roots of the equation $x^2 + 7x + 10=0$, it was found that most students who got the item by the use of factorization method could not replicate same answer using the formula as a result of appropriate placing and manipulation. The findings of the study support the research of Norasiah (2002) who noted that most average students face difficulty in performing algebraic operations especially when square roots are involved. In a related study involving the use of completing the square method, learners were unable to divide by the coefficient of x^2 if the equation was greater than 1 and less than zero,multiplying by half the coefficient of x and also additive inverses of any constant given (Makgakga,..) [12]. some of the reasons which contributed to these difficulties were learners' laziness, lack of participation, teachers commitment and conduct towards learners' among others. The author is of the view that with periodical interaction with learners' and encouragement, some of these diagnosed errors could be used as a basis to understand learners' challenges in the aspect of mathematics. This will no doubt go a long way in making up for appropriate problem solving strategies that will minimize errors in subsequent interactions.

The error in process skill seems more pronounced in the use of formula than the factorization method. This error may be due to the fact that students failed to understand and describe what is required by the questions. This results in failure to solve the problems. Results of this study concurred with the findings of Norasiah (2002) [14] in which problematic students failed to translate mathematical problems into mathematical form and also having problem in understanding the special terms in mathematics.

Results of this study are in line with the research findings of Liew and Wan Muhammad (1991) [11] who observed that emphasis on algorithmics skills without explanation on the concept or principle are the factors that cause difficulties in mathematics and by extension poor achievement in quadratic equation.

Conclusion and Recommendations

The findings suggest that in the exploration of the Newman's model, most errors that are committed by students irrespective of the method used are transformation errors and process skills errors. It further shows that generally, students irrespective of different cognitive ability are susceptible to error in solving problems in quadratic equation. This implies that academic performance is gender bias. In summarizing our findings, we conclude that these results apply to only Secondary School in Zaria, Nigeria. One cannot say whether these relationships apply to other institutions of secondary education. The results certainly suggest avenues for further research and it would be desirable to examine these relationships using data from more than one institution. Nonetheless, the novelty of the findings from this study will undoubtedly enable mathematics teachers to diligently plan their teaching in an effective and meaningful way. Apart from also identifying the roots of the problems in students' learning, the findings will further add to the library of data in this line of research for diversified group of respondents.

In view of the findings above, it is therefore recommended that:

1. Teachers as facilitator of the teaching–learning process should encourage the learners to concentrate on one point at a time and proceed stepwise in a logical manner to reduce attendant difficulty faced in quadratic equation.
2. Teacher should make quadratic equation lessons exciting by encouraging group work with frequent activity-based demonstrations so as to demystify the difficulty encountered in problems involving quadratic equation.
3. Learners should be given enough opportunities to do regular problem exercises as this will go a long way in assisting them and increasing their reasoning skills.
4. This study should be repeated with other similar mathematically based concepts so as to compare findings.

Appendix A: Quadratic Equation Diagnostic Test (QEDT)

1. $X^2-4x+3=0$
2. $X^2-5x-6=0$
3. $X^2+7x+10=0$
4. $6x^2-5x+1=0$
5. $6x^2-7x+2=0$
6. $3x^2+14x+8=0$
7. $x^2-2x-7=0$
8. $u^2-u-3=0$
9. $3z^2-z-1=0$
10. $5x^2-8x+2=0$
11. $3x^2+5x+1=0$
12. $x^2-5x+6=0$
13. $X^2+7x+10=0$
14. $6x^2-5x+1=0$
15. $3x^2-5x+1=0$
16. $x(x+2)+4=0$

17. $2x(x+1)=6$
18. $v(v-1)=12$

References

1. Abdullah AH, Abidin ZN, Ali M. Analysis of Students' Errors in Solving Higher Order Thinking Skills (HOTS) Problems for the Topic of Fraction. Asian Social Science. 2015; 11(21):133-142
2. Anderson JR. Rules of the Mind. Hillsdale, NJ: Erlbaum, 1993.
3. Ashlock RB. Error patterns in computation (8th Ed.). New York: Merrill, 2005.
4. Charles-Ogan G, George NR. Investigating difficult concepts in senior secondary school Mathematics curriculum as perceived by students. International Journal of Academic Research and Reflection. 2015; 3(6):67.
5. Ellerton NF, Clements MA. Newman error analysis. A comparative study involving Year 7 students in Malaysia and Australia. Technology and mathematics education, 1996, 186-193.
6. Delice A. Recognizing, recalling and doing in the 'simplification' of trigonometric expressions. The 26th Annual Conference of the International Group for the Psychology of Mathematics Education (PME26), the School of Education and Professional Development at the University of East Anglia, Norwich: England. 2002; 1:247.
7. Effandi Z, Siti Mistima. Analysis of Students' Error in Learning Quadratic Equations. International Education studies. 2010; 3(3):105-110.
8. Huang X, Cheng L. Analyzing Errors Made by Eighth-Grade Students in Solving Geometrical Problems in China The Mathematics Educator. 2010; 12(2):63-80
9. Intanku SS. Diagnosis for the type of error in differentiation. Unpublished Master of Education Research Project. Universiti Kebangsaan Malaysia, 2003.
10. Lester FK, Kehle PE. From Problem Solving to Modeling: The Evolution of Thinking About Research on Complex Mathematical Activity. In R. Lesh, & H. M. Doerr (Eds.), Beyond Constructivism-Models and Modeling Perspectives on Mathematical Problem Solving, Learning, and Teaching. Mahwah, NJ: Lawrence Erlbaum Associates, 2003, 501-517.
11. Liew ST, Wan Muhamad Saridan, Wan Hasan. Understanding and minimizing difficulty in learning mathematics Berita Matematik. 1991; 38:22-29
12. Makgakga S. Errors and Misconception in Solving Quadratic equation by completing the square Mathematics, 2014. Education.www.amesa,org.za/proceedings/paper.
13. Newman MA. An analysis of 6th grade pupils' errors on written mathematical task. Dlm. Clements, MA. & Foyster, J (Eds). Research in Mathematical Education in Australia, 1977, 239-258.
14. Norasiah A. Error type diagnosis in learning simultaneous equation. Unpublished Master of Education Research Project, Universiti Kebangsaan Malaysia, 2002.
15. Orhun N. The effects of learning styles on high school students' achievement on a mathematics course. Educational Research and Reviews. 2013; 8(14):1158-1165.
16. Polya G. How to solve it: A new aspect of mathematical method. Princeton, NJ. Princeton University Press, 1973.
17. Prakitipong N, Nakamura S. Analysis of mathematics performance of Grade 5 students in Thailand using Newman procedure. Journal of International Cooperation in Education. 2006; 9(1):111-122.
18. Robertson SI. Problem Solving. Philadelphia, PA: Psychology Press, 2001.
19. Ryan JT, Williams JS. Mathematical discussions with children: exploring methods and misconceptions as a teaching strategy. Manchester: University of Manchester, 2000.
20. Susanti E, Kusumah YS, Sabandar J. Computer-Assisted Realistic Mathematics Education for Enhancing Students' Higher-Order Thinking Skills (Experimental Study in Junior High School in Palembang, Indonesia). Journal of Education and Practice. 2014; 5(18):51-58
21. Weber K. Students' understanding of trigonometric functions. Mathematics Education Research Journal, 2005; 17(3):91-112.
22. Wijaya A, Heuven P, Doorman M, Robitzsch. Difficulties in solving context-based PISA mathematics tasks: An analysis of students' errors. The Mathematics Enthusiast. 2014; 11(3):8.
23. Widdiharto R. Diagnosis Kesulitan Belajar Matematika SMP dan Alternatif Proses Remidinya. Yogyakarta: Pusat Pengembangan dan Pemberdayaan Pendidik dan Tenaga Kependidikan Matematika, 2008.