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Derivation of continuous equations for density of burglars

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Abstract
The discrete model for movement of burglars is given by the equation as:

$$\rho(x,t+h)=\left\{\frac{R(x,t)}{R(x-2,t)+R(x,t)}\rho(x-1,t)+\frac{R(x,t)}{R(x+2,t)+R(x,t)}\rho(x+1,t)\right\}(1-P(x,t))+Gh$$

where R(x, t) is assumed to be strictly positive across the whole spatial domain for all *t.* We can express the relation in terms of discrete spatial laplacian Δ . For this, in continuous limit, we re-express *p*(*x, t*) in terms of *p*(*x, t*) and *R*(*x, t*) ≈ *R*(*X, t*), *p*(*x, t*) ≈ *p*(*X, t*), where *X*=*lx, l*, is a grid spacing.

Keywords: Derivation, continuous equations

Introduction
The discrete model for movement of burglars is given by the equation as:

$$\rho(x,t+h)=\left\{\frac{R(x,t)}{R(x-2,t)+R(x,t)}\rho(x-1,t)+\frac{R(x,t)}{R(x+2,t)+R(x,t)}\rho(x+1,t)\right\}(1-P(x,t))+Gh$$

where $R(x,t)$ is assumed to be strictly positive across the whole spatial domain for all t.We can express the relation in terms of discrete spatial laplacian Δ . For this, in continuous limit, we re-express $\rho(x,t)$ in terms of $\rho(X,t)$ and $R(x,t)\approx R(X,t)$,$P(x,t)\approx P(X,t)$, where $X=lx$, l is a grid spacing.
Since probability is independent of length or time scales, therefore we can write $P(aX,t)=aP(X,t)$. With this notion and assuming l is small, terms of the above relation can be re-written as

$$\rho(x,t+h)=\rho(\frac{X}{l},t+h)=\frac{1}{l}\rho(X,t+h)\approx\rho(X,t+h)$$

$$R(x,t)=R(\frac{X}{l},t)=\frac{1}{l}R(X,t)\approx R(X,t)$$

$$R(x-2,t)=R(\frac{X}{l}-2,t)=\frac{1}{l}R(X-2l,t)\approx R(X-2l,t)$$

$$R(x+2,t) = R(\frac{X}{l}+2,t) = \frac{1}{l}R(X+2l,t) \approx R(X+2l,t)$$

$$\rho(x-1,t) = \rho(\frac{X}{l}-1,t) = \frac{1}{l}(X-l,t) \approx \rho(X-l,t)$$

$$\rho(x+1,t) = \rho(\frac{X}{l}+1,t) = \frac{1}{l}(X+l,t) \approx \rho(X+l,t)$$

Since $P(x,t) = R(x,t)h$ therefore,

$$P(x,t) = R(x,t)h$$

$$\Rightarrow \quad P(\frac{X}{l},t) = R(\frac{X}{l},t)h$$

$$\Rightarrow \quad \frac{1}{l}P(X,t) = \frac{1}{l}R(X,t)h$$

$$\Rightarrow \quad P(X,t) = R(X,t)h,$$

$$\text{thus} \quad \left[1-P(x,t)\right] \approx \left[1-P(X,t)\right]$$

$$= \quad 1-R(X,t)h$$

Thus the above relation is reduced to

$$\rho(X,t+h) = \left\{\frac{R(X,t)}{R(X-2l,t)+R(X,t)}\rho(X-l,t) + \frac{R(X,t)}{R(X+2l,t)+R(X,t)}\rho(X+l,t)\right\}(1-R(X,t)h) + Gh$$

For diffusion of the Risk of victimization and density of burglars to spatial location
The relation can be written as

$$\rho(X,t+h) = \left\{\frac{R(X)}{R(X-2l)+R(X)}\rho(X-l) + \frac{R(X)}{R(X+2l)+R(X)}\rho(X+l)\right\}\{1-R(X)h\} + Gh$$

$$\Rightarrow \quad \rho(X,t+h) = f(l)\{1-R(X)h\} + Gh \qquad (1)$$

$$\text{where} \quad f(l) = \frac{R(X)}{R(X-2l)+R(X)}\rho(X-l) + \frac{R(X)}{R(X+2l)+R(X)}\rho(X+l)$$

Here, $f(0) = \rho(X)$

$$f'(l) = -\frac{R(X)}{R(X-2l)+R(X)}\rho_X(X-l) + \frac{2R(X)R_X(X-2l)}{\{R(X-2l)+R(X)\}^2}\rho(X-l) + \frac{R(X)}{R(X+2l)+R(X)}\rho_X(X+l) - \frac{2R(X)R_X(X+2l)}{\{R(X+2l)+R(X)\}^2}\rho(X+l)$$

$$\therefore \quad f'(0) = -\frac{1}{2}\rho_X(X) + \rho(X)\frac{2R(X)R_X(X)}{4\{R(X)\}^2} + \frac{1}{2}\rho_X(X) - \rho(X)\frac{2R(X)R_X(X)}{4\{R(X)\}^2} = 0$$

$$f''(l) = -\left[-\frac{R(X)}{R(X-2l)+R(X)}\rho_{XX}(X-l) + \frac{2R(X)R_X(X-2l)}{\{R(X-2l)+R(X)\}^2}\rho_X(X-l)\right] + 2R(X)\left[-\rho_X(X-l)\frac{R_X(X-2l)}{\{R(X-2l)+R(X)\}^2} + \rho(X-l)\frac{\{R(X-2l)+R(X)\}^2\{-2R_{XX}(X-2l)\}}{\{R(X-2l)+R(X)\}^4} + \rho(X-l)\frac{4\{R_X(X-2l)\}^2\{R(X-2l)+R(X)\}}{\{R(X-2l)+R(X)\}^4}\right]$$

$$+\rho_{XX}(X+l)\frac{R(X)}{R(X+2l)+R(X)} + \rho_X(X+l)\frac{\{-2R(X)R_X(X+2l)\}}{\{R(X+2l)+R(X)\}^2} - 2R(X)\left[\rho_X(X+l)\frac{R_X(X+2l)}{\{R(X+2l)+R(X)\}^2} + \rho(X+l)\frac{\{R(X+2l)+R(X)\}^2 2R_{XX}(X+2l)}{\{R(X+2l)+R(X)\}^4} - \rho(X+l)\frac{4\{R_X(X+2l)\}^2\{R(X+2l)+R(X)\}}{\{R(X+2l)+R(X)\}^4}\right]$$

$$\Rightarrow \quad f''(l) = \frac{R(X)}{R(X-2l)+R(X)}\rho_{XX}(X-l) - \frac{4R(X)R_X(X-2l)}{\{R(X-2l)+R(X)\}^2}\rho_X(X-l) + \frac{\{R(X-2l)+R(X)\}^2\{-4R(X)R_{XX}(X-2l)\}}{\{R(X-2l)+R(X)\}^4}\rho(X-l) + \frac{8R(X)R_X{}^2(X-2l)\{R(X-2l)+R(X)\}}{\{R(X-2l)+R(X)\}^4}\rho(X-l)$$

$$+\frac{R(X)}{R(X+2l)+R(X)}\rho_{XX}(X+l) - \frac{2R(X)R_X(X+2l)}{\{R(X+2l)+R(X)\}^2}\rho_X(X+l) - \frac{2R(X)R_X(X+2l)}{\{R(X+2l)+R(X)\}^2}\rho_X(X+l) - \frac{\{R(X+2l)+R(X)\}^2 4R(X)R_{XX}(X+2l)}{\{R(X+2l)+R(X)\}^4}\rho(X+l) - \frac{8R(X)R_X{}^2(X+2l)\{R(X+2l)+R(X)\}}{\{R(X+2l)+R(X)\}^4}\rho(X+l)$$

$$\therefore \quad f''(0)=\frac{1}{2}\rho_{XX}(X)-\frac{R(X)R_X(X)}{\{R(X)\}^2}\rho_X(X)$$

$$+\left[\frac{-16\{R(X)\}^3\{R_{XX}(X)\}+16R_X^2(X)\{R(X)\}^2}{16\{R(X)\}^4}\right]\rho(X)$$

$$+\frac{1}{2}\rho_{XX}(X)-\frac{2R(X)R_X(X)}{4\{R(X)\}^2}\rho_X(X)$$

$$-\frac{2R(X)R_X(X)}{4\{R(X)\}^2}\rho_X(X)$$

$$+\frac{16R(X)\{R(X)\}^2R_{XX}(X)-16R_X^2(X)\{R(X)\}^2}{16\{R(X)\}^4}\rho(X)$$

$$\Rightarrow \quad f''(0)=\rho_{XX}(X)-\frac{R_X(X)}{R(X)}\rho_X(X)-\frac{R_X(X)}{R(X)}\rho_X(X)-\frac{2R_{XX}(X)}{R(X)}\rho(X)+\frac{2R_X^2(X)}{R^2(X)}\rho(X)$$

$$\Rightarrow \quad f''(0)=\rho_{XX}(X)-\frac{2R_X(X)}{R(X)}\rho_X(X)-\frac{2R_{XX}(X)}{R(X)}\rho(X)+\frac{2R_X^2(X)}{R^2(X)}\rho(X)$$

For simplicity, it can be written as

$$\Rightarrow \quad f''(0)=\rho_{XX}-\frac{2R_X}{R}\rho_X-\frac{2R_{XX}}{R}\rho+\frac{2R_X^2}{R^2}\rho$$

$$\Rightarrow \quad f''(0)=\rho_{XX}-[\frac{2R_X}{R}\rho]_X$$

By maclaurians' series expansion, we have

$$f(l)=f(0)+lf'(0)+\frac{l^2}{2!}f''(0)+O(l^3)$$

where $O(l^3)$ represents the terms containing higher power of l. As l is small, neglecting the term $O(l^3)$ and substituting the value of $f(0)$, $f'(0)$ and $f''(0)$ in the above relation, we have

$$f(l)=\rho(X,t)+\frac{l^2}{2}\left[\rho_{XX}(X,t)-\left\{\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right\}_X\right] \quad (2)$$

With the help of relation (2), the equation (1) can be re-written as

$$\rho(X,t+h)=\left[\rho(X,t)+\frac{l^2}{2}\left\{\rho_{XX}(X,t)-\left(\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right)_X\right\}\right]\{(1-R(X,t)h\}+Gh$$

$$\Rightarrow \rho(X,t+h)-\rho(X,t)=-\rho(X,t)R(X,t)h+\frac{l^2}{2}\left[\rho_{XX}(X,t)-\left\{\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right\}_X\right]-\frac{l^2}{2}R(X,t)h\left[\rho_{XX}(X,t)-\left\{\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right\}_X\right]+Gh$$

Dividing both sides by h and Taking limit as $h\to 0$, as $l\to 0$, $\frac{l^2}{2}\to 0$, and assuming $\frac{l^2}{h}\to D$ (constant), we then have

$$\Rightarrow \quad \lim_{h\to 0}\frac{\rho(X,t+h)-\rho(X,t)}{h}$$

$$=\lim_{h\to 0}\left[-\rho(X,t)R(X,t)+\frac{l^2}{2h}\left\{\rho_{XX}(X,t)-\left(\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right)_X\right\}-\frac{l^2}{2}R(X,t)\left\{\rho_{XX}(X,t)-\left(\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right)_X\right\}+G\right]$$

$$\Rightarrow \quad \rho_t(X,t)=\lim_{h\to 0}\left[-\rho(X,t)R(X,t)+\frac{l^2}{2h}\left\{\rho_{XX}(X,t)-\left(\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right)_X\right\}-\frac{l^2}{2}R(X,t)\left\{\rho_{XX}(X,t)-\left(\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right)_X\right\}+G\right]$$

$$\Rightarrow \rho_t(X,t)=-\rho(X,t)R(X,t)+\frac{1}{2}\mathrm{D}\left\{\rho_{XX}(X,t)-\left(\frac{2R_X(X,t)\rho(X,t)}{R(X,t)}\right)_X\right\}+G$$

As we assumed that density of burglars diffuse to spatial location, so that $\rho(X,t)\approx\rho(X)$, $R(X,t)\approx R(X)$, thus the equation reduces to

$$\Rightarrow \rho_t(X,t)=\frac{1}{2}\mathrm{D}\left[\rho_{XX}(X)-\left\{\frac{2R_X(X)\rho(X)}{R(X)}\right\}_X\right]-\rho(X)R(X)+G$$

For simplicity the resulting equation can be written as

$$\Rightarrow \quad \frac{\partial\rho}{\partial t}=\frac{1}{2}\mathrm{D}\left\{\rho_{XX}-\left(\frac{2R_X\rho}{R}\right)_X\right\}-\rho R+G$$

$$\Rightarrow \qquad \frac{\partial \rho}{\partial t} = \frac{1}{2} D \rho_{XX} - D\left(\frac{R_X \rho}{R}\right)_X - \rho R + G \qquad (3)$$

Note that

$$\nabla \cdot \left(\frac{\rho}{R}\nabla R\right) = \frac{\rho}{R}\nabla^2 R + \nabla\frac{\rho}{R}\cdot\nabla R$$

$$= \frac{\rho}{R}R_{XX} + \sum\frac{\partial}{\partial X}\left(\frac{\rho}{R}\right)\cdot\sum\frac{\partial}{\partial X}R$$

$$= \frac{\rho}{R}R_{XX} + \left(\sum\frac{R\rho_X - R_X\rho}{R^2}\right)\sum R_X$$

$$= \frac{\rho}{R}R_{XX} + \left(\frac{R_X\rho_X}{R} - \frac{R_X^2\rho}{R^2}\right)$$

$$= \left(\frac{R_X\rho}{R}\right)_X$$

Also

$$\rho_{XX} = \sum_{i=1}^{n}\frac{\partial^2\rho}{\partial X_i^2} = div(\nabla\rho) = \nabla^2\rho = \Delta\rho$$

where Δ is the discrete spatial laplacian.
Using these notations, we can express the above equation (3) as

$$\frac{\partial \rho}{\partial t} + D\nabla\cdot\left(\frac{\rho}{R}\nabla R\right) = \frac{1}{2} D \nabla^2\rho - \rho R + G \qquad (4)$$

Conclusion

Thus the continuous equation of Risk of victimization field is given by the equation and the continuous equation of density of burglars is given by the equation (4).

References

1. Brantingham PL, Brantingham PJ. Crime pattern theory. In wortley R, Mazerolle L. editors, Environmental criminology and Crime Analysis. Routledge, 2008.
2. Caldwell RG, Black JA. Juvenile Delinquency The Ronald Press Company, New York, 1971.
3. Calvo-Armengol A, Zenou Y. Social Networks and Crime Decisions: The Role of Social Structure in Facilitating Delinquent Behavior, 2003.
4. Cambel M, Ormerod P. Social interaction and the dynamic of crime, 1998.
5. Felson M, Reachel B. Crime and Everyday Life, 2010.