International Journal of Statistics and Applied Mathematics 2018; 3(2): 446-455

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(2): 446-455

www.mathsjournal.com
Received: 19-01-2018
Accepted: 20-02-2018

SD Jilani
Research Scholar (RFSMS),
Department of Statistics,
Acharya Nagarjuna University,
Guntur, Andhra Pradesh, India

A Vasudeva Rao
Professor & Head,
Department of Statistics,
Acharya Nagarjuna University,
Guntur, Andhra Pradesh, India

S Bhanu Prakash
Research Scholar,
Department of Statistics,
Acharya Nagarjuna University,
Guntur, Andhra Pradesh, India

Correspondence
SD Jilani
Research Scholar (RFSMS),
Department of Statistics,
Acharya Nagarjuna University,
Guntur, Andhra Pradesh, India

Linear approximate ml estimation of scale parameter in type II generalized half-logistic distribution under type-II censoring

SD Jilani, A Vasudeva Rao and S Bhanu Prakash

Abstract
The two-parameter Type II generalized half-logistic distribution (GHLD) with known shape parameter is considered here and the ML method does not yield explicit estimator for scale parameter even in complete samples. Therefore, in this paper, we have constructed two linear estimators for scale parameter; one is obtained by making linear approximations to the intractable terms of the maximum likelihood equation of the scale parameter and another is obtained using percentile estimation method. We call these two estimators as linear approximate MLE (LAMLE) and percentile estimator (PCE) respectively. To investigate the performance of LAMLE and PCE, a Monte Carlo simulation is made to compare them with the MLE. Further, we have constructed nearly unbiased LAMLE and nearly unbiased PCE and compared them with MLE through a simulation study. Finally, a numerical example is presented to illustrate the construction of the new estimators developed here.

Keywords: Type II generalized half-logistic distribution, Type-II censoring, Linear approximate maximum likelihood estimator, Percentile estimator, nearly unbiased linear approximate maximum likelihood estimator, nearly unbiased percentile estimator

Introduction

Consider a series system of θ components with individually and identically distributed (iid) individual lifetimes, for example, $F(x)$. The reliability function of such a system is given by $\left[1-F(x)\right]^{\theta}$; hence, the distribution function of the lifetime random variable of a series system is $1-\left[1-F(x)\right]^{\theta}$. Taking $F(x)$ as the standard half logistic model, Kantam *et al.* (2014) [7, 11] proposed a new model called the standard Type-II generalized half logistic distribution (GHLD), whose pdf, cdf, and hazard rate function are respectively given by

$$f(x;\theta)=\frac{2^{\theta}\theta e^{x}}{\sigma\left(1+e^{x}\right)^{\theta+1}}; \qquad 0\le \mathrm{x}<\infty,\ \theta>0\ . \tag{1.1}$$

$$F(x;\theta)=1-\frac{2^{\theta}}{\left(1+e^{x}\right)^{\theta}}\ ; \qquad 0\le \mathrm{x}<\infty,\ \theta>0\ . \tag{1.2}$$

$$h(x,\theta)=\frac{\theta}{1+e^{-x}}, \qquad 0\le \mathrm{x}<\infty,\ \theta>0 \tag{1.3}$$

Where θ is called as the shape parameter. Here, it may be noted that the above standard Type II GHLD is same as the standard Type I GHLD proposed by Olapade (2014) [9]. However, Olapade (2014) [9] derived standard Type I GHLD with a different theoretical argument namely if Y follows an exponential distribution with parameter θ, then X=log ($2e^{Y}$-1) is a standard Type I GHLD with parameter θ.

If we introduce a scale parameter in the above standard distribution, then the resultant distribution may be called as two-parameter Type II GHLD, whose pdf and cdf are respectively given by

$$p(x;\theta,\sigma)=\frac{2^{\theta}\theta e^{x/\sigma}}{\sigma\left(1+e^{x/\sigma}\right)^{\theta+1}};\qquad 0\le x<\infty,\ \theta>0\ and\ \sigma>0. \tag{1.4}$$

$$P(x;\theta,\sigma)=1-\frac{2^{\theta}}{\left(1+e^{x/\sigma}\right)^{\theta}}\ ;\qquad 0\le x<\infty,\ \theta>0\ and\ \sigma>0. \tag{1.5}$$

Kantam *et al.* (2014) [7, 11] have studied some distributional characteristics and estimated the parameters based on a complete sample using ML and approximate ML methods. Rosaiah *et al.* (2014) [11] constructed reliability test plan for the Type II GHLD to determine the termination time of the experiment based on the given sample size, producer's risk and termination number. Devendra Kumar *et al.* (2015) [3] have obtained moments and MLEs of the parameters of upper record values and their confidence intervals assuming the underlying model is three-parameter Type II GHLD. Phillip *et al.* (2016) [10] used the above model as survival model to a breast cancer model and estimated the parameters using ML method.

Vasudeva Rao *et al.* (2017) [13] constructed a new linear estimator for scale parameter of Type I generalized logistic distribution based on Type-II censored samples, by making linear approximations to the intractable terms of the likelihood equation using least squares (LS) method, a new approach of linearization. They called this estimator as linear approximate maximum likelihood estimator (LAMLE). Further, they constructed unbiased LAMLE and have shown that it is just as efficient as the BLUE. They also discussed interval estimation of scale parameter.

ML estimation method does not yield explicit estimator for scale parameter of two-parameter Type II GHLD even in complete samples. Hence, in this paper, with an aim of providing explicit estimator for the scale parameter, we have suggested two linear estimators for scale parameter; one is constructed by making linear approximations to the intractable terms of the ML equation of the scale parameter and another is constructed based on percentile estimation method. This paper is organized as follows. Throughout the paper, we assume the shape parameter θ is known. In Sec. 2, we have constructed the linear estimator of σ based on Type-II censored samples, by making linear approximations to the intractable terms of the ML equation of σ and we call this estimator as linear approximate MLE (LAMLE). Sec. 3 is devoted for construction of other linear estimator based on percentile method and we call this estimator as percentile estimator (PCE). In Sec. 4, based on a Monte Carlo simulation study, we compare both LAMLE and PCE with the corresponding MLE based on bias and mean square error (MSE). In Sec. 5, we have constructed nearly unbiased LAMLE and nearly unbiased PCE and compare them with the corresponding MLE through a simulation study. An illustrative example is presented in Sec. 6. Finally, in Sec.7, we summarise the conclusions of the study.

2. Linear Approximate ML Estimation of Scale Parameter

Let $X_{r+1:n} < X_{r+2:n} < ... < X_{n-s:n}$ is a Type-II censored sample, where r is the number of left most observations censored and s is the number of right most observations censored from a planned sample of size n drawn from the Type II GHLD given in Eq. (1.5). The log-likelihood function of the sample is

$$\log L = -(n-r-s)\log\sigma + r\log F(z_{r+1:n}) + \sum_{i=r+1}^{n-s}\log f(z_{i:n}) + s\log[1 - F(z_{n-s:n})] \tag{2.1}$$

Where $f(z)$, $F(z)$ are as given in Eqs. (1.1), (1.2) and $z_{i:n} = x_{i:n}/\sigma.$ the maximum likelihood (ML) equation of σ, after simplification, is given by

$$\frac{\partial\log L}{\partial\sigma}=\frac{-1}{\sigma}\left[n-r-s+r\theta\, G(z_{r+1:n})+\sum_{i=r+1}^{n-s}z_{i:n}-(\theta+1)\sum_{i=r+1}^{n-s}H(z_{i:n})-s\theta\, H(z_{n-s:n})\right]=0 \tag{2.2}$$

Where $G(z_{r+1:n})=\dfrac{H(z_{r+1:n})[1-F(z_{r+1:n})]}{F(z_{r+1:n})}$ and $H(z_{i:n})=\dfrac{z_{i:n}}{1+e^{-z_{i:n}}},\ i=r+1,...,n-s$ (2.3)

The ML equation (2.2) does not yield explicit solution for σ because of the presence of the intractable terms $G(z_{r+1:n})$ and $H(z_{i:n})$. hence, we have to employ some iterative method, such as Newton-Raphson method, to obtain the MLE of σ which requires a starting solution near the global maximum.

We, therefore, with an aim of providing an explicit estimator of σ, propose the following linear approximations to the intractable terms given in Eq. (2.3)

$$G(z_{r+1:n}) \approx \alpha_{r+1}+\beta_{r+1}\, z_{r+1:n}\quad\text{and}\quad H(z_{i:n}) \approx \gamma_i+\delta_i\, z_{i:n},\quad i=r+1,...,n-s \tag{2.4}$$

For evaluating the intercepts (α's, γ's) and the slopes (β's, δ's) of the above linear approximations, we have adopted two different linear approximation methods; the first one is based on Taylor series approximation method, which is suggested by Balakrishnan (1989) [1] while studying the approximate ML estimation in scaled Rayleigh distribution and the second one is based on least squares (LS) method, a new approach, suggested by Vasudeva Rao *et al.* (2017) [13] recently while studying the linear approximate ML estimation in scaled Type I generalised logistic distribution.
Upon using the above linear approximations in Eq. (2.2), we get the following approximate ML equation

$$\frac{\partial \log L}{\partial \sigma} \cong \frac{\partial \log \tilde{L}}{\partial \sigma} = \frac{-1}{\sigma}\Bigg[n-r-s+r\theta\ (\alpha_{r+1}+\beta_{r+1}z_{r+1:n}) + \sum_{i=r+1}^{n-s} z_{i:n} - (\theta+1)\sum_{i=r+1}^{n-s}\ (\gamma_i+\delta_i z_{i:n}) - s\theta\ (\gamma_{n-s}+\delta_{n-s}\ z_{n-s:n}) \Bigg] = 0 \tag{2.5}$$

Substituting $z_{i:n} = x_{i:n}/\sigma$ and solving for σ, we get a linear estimator of σ, which we denote as $\tilde{\sigma}$ and is given by

$$\tilde{\sigma} = \sum_{i=r+1}^{n-s} l_i x_{i:n} \tag{2.6}$$

Where

$$l_i = \begin{cases} \dfrac{-r\theta\ \beta_i + (\theta+1)\delta_i - 1}{\Delta} & if\ i = r+1 \\ \dfrac{(\theta+1)\delta_i - 1}{\Delta} & if\ r+1 < i < n-s \\ \dfrac{(s\theta+\theta+1)\delta_i - 1}{\Delta} & if\ i = n-s \end{cases} \tag{2.7}$$

$$\Delta = n-r-s+r\theta\ \ \alpha_{r+1} - (\theta+1)\sum_{i=r+1}^{n-s}\gamma_i - s\theta\ \gamma_{n-s}$$

We call the above estimator $\tilde{\sigma}$ as linear approximate ML estimator (LAMLE) of σ. The mean and variance of the LAMLE $\tilde{\sigma}$ are respectively given by

$$E(\tilde{\sigma}) = \sigma\sum_{i=1}^{n-s} l_i a_{i:n}\ \ and\ \ V(\tilde{\sigma}) = \sigma^2 \sum_{i=1}^{n-s}\sum_{j=1}^{n-s} l_i l_j b_{ij:n} \tag{2.8}$$

Where $a_{i:n} = E(Z_{i:n})$ and $b_{ij:n} = Cov(Z_{i:n}, Z_{j:n})$. Thus, for computing the mean and variance of $\tilde{\sigma}$, we need the means, where
$a_{i:n} = E(Z_{i:n})$ and $b_{ij:n} = Cov(Z_{i:n}, Z_{j:n})$. Thus, for computing the mean and variance of $\tilde{\sigma}$, we need the means, variances and covariances of order statistics $Z_{1:n} < Z_{2:n} < ... Z_{n:n}$ of standard Type II GHLD, those are not available in the literature. Consequently, we are unable to compute the exact mean and exact variance of the LAMLE. However, we have simulated only the means of order statistics of standard Type II GHLD based on 4000 Monte-Carlo runs and tabulated them for *n*=5, 10, 15 &20. And θ=0.5, 1.5(0.5)5.0 in Table 1. Using these simulated means of order statistics, we can compute approximate mean of $\tilde{\sigma}$ for the specified values of n and θ. When sample size *n* is large, we may use the following approximation for $a_{i:n}$.

Table 1: Different estimates of σ based on the above sample size n=10 with various choices of r=0(1)2 and s=0(1)2. The figures given in brackets are the standard errors of the respective estimates

r	**s**	$\hat{\sigma}$ **(SE)**	$\tilde{\sigma}$ **(SE)**	σ^* **(SE)**
0	0	9.916 (3.357)	9.774 (2.727)	9.964 (2.780)
0	1	10.706 (3.750)	10.625 (3.180)	10.851 (3.247)
0	2	11.715 (4.355)	11.676 (3.761)	11.925 (3.841)
1	0	9.930 (3.361)	9.788 (2.731)	9.978 (2.783)
1	1	10.721 (3.755)	10.641 (3.184)	10.866 (3.251)
1	2	11.733 (4.361)	11.694 (3.766)	11.943 (3.847)
2	0	10.110 (3.414)	9.975 (2.782)	10.164 (2.834)
2	1	10.926 (3.820)	10.855 (3.247)	11.079 (3.314)
2	2	11.968 (4.441)	11.942 (3.845)	12.190 (3.924)

$$a_{i:n} \approx F^{-1}\left\{E(U_{i:n})\right\} = \log_e\left[2\left\{1-E(U_{i:n})\right\}^{-1/\theta}-1\right] \qquad \text{(from Eq. (1.2))}$$

Where $U_{i:n}$ is the i[th] order statistic from U(0,1) and it is well known that $E(U_{i:n}) = i/(n+1)$. Thus, we have

$$a_{i:n} \approx \log_e\left\{2\left[1-i/(n+1)\right]^{-1/\theta}-1\right\} \tag{2.9}$$

The Fisher information of $\tilde{\sigma}$ is given by

$$I = -E\left(\frac{\partial^2 \log \tilde{L}}{\partial\sigma^2}\right) = I_{11}\Big/\sigma^2$$

Where

$$I_{11} = -\left[n-r-s+r\theta\,(\alpha_{r+1}+2\beta_{r+1}a_{r+1:n}) + \sum_{i=r+1}^{n-s} a_{i:n} - \left(\theta+1\right)\right.$$
$$\left.\sum_{i=r+1}^{n-s}(\gamma_i+2\delta_i a_{i:n}) - s\theta\,(\gamma_{n-s}+2\delta_{n-s}\,a_{n-s:n})\right]$$

The asymptotic variance of $\tilde{\sigma}$ can be obtained by inverting the above Fisher information and from which we can compute the standard error of $\tilde{\sigma}$ as

$$\text{S.E.}(\tilde{\sigma}) = \tilde{\sigma}\Big/\sqrt{I_{11}} \tag{2.10}$$

By applying Taylor series method to Eq. (2.4), we get

$$\beta_{r+1} = G'\left(\xi_{r+1:n}\right), \quad \boldsymbol{\beta_{r+1} = G'\left(\xi_{r+1:n}\right)}$$

$$\delta_i = H'\left(\xi_i\right) \quad and \quad \gamma_i = H\left(\xi_i\right) - \xi_i\,\delta_i, \qquad i = r+1,\dots,n-s \tag{2.11}$$

Where, ξ_i is the i[th] population quantile of Type II GHLD, given by

$$\xi_i = F^{-1}(p_i) = \log_e[2(1-p_i)^{-1/\theta}-1], \quad p_i = \frac{i-0.3}{n+0.4} \qquad \text{(From Eq. (1.2) (2.12)}$$

Similarly, by applying least squares method (for details of method pl. see Vasudeva Rao *et al.* (2017) [13] to Eq. (2.4), we get

$$\beta_{r+1} = \left[\sum_{j=1}^{2m+1} z_{r+1,j}G(z_{r+1,j}) - (2m+1)\overline{z}_{r+1}\overline{G(z_{r+1})}\right]\Bigg/\left[\sum_{j=1}^{2m+1} z_{r+1,j}^2 - (2m+1)\overline{z}_{r+1}^{\,2}\right]$$

$$\alpha_{r+1} = \overline{G(z_{r+1})} - \beta_{r+1}\overline{z}_{r+1}, \quad \overline{z}_{r+1} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} z_{r+1,j}, \quad \overline{G(z_{r+1})} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} G(z_{r+1,j})$$

$$\delta_i = \left[\sum_{j=1}^{2m+1} z_{ij}H(z_{ij}) - (2m+1)\overline{z}_i\,\overline{H(z_i)}\right]\Bigg/\left[\sum_{j=1}^{2m+1} z_{ij}^2 - (2m+1)\overline{z}_i^{\,2}\right], i = r+1,\dots,n-s$$

$$\text{and} \quad \gamma_i = \overline{H(z_i)} - \delta_i\,\overline{z}_i, \quad \overline{z}_i = \frac{1}{2m+1}\sum_{j=1}^{2m+1} z_{ij}, \quad \overline{H(z_i)} = \frac{1}{2m+1}\sum_{j=1}^{2m+1} H(z_{ij}) \tag{2.13}$$

$$where \quad z_{ij} = F^{-1}(p_{ij}) = \log_e\left[2\left(1-p_{ij}\right)^{-1/\theta}-1\right],$$

$$p_{ij} = p_i + \frac{j-m-1}{2m(n+0.4)}, \quad j = 1,2,\dots,2m+1$$

Remark: In the above method, we may take any $m \geq 1$. However, Vasudeva Rao *et al.* (2017) [13] have suggested an optimal choice of $m=5$, and we have taken the same choice of m.

Thus, in this section, we construct two LAMLEs for σ based on Type II censored samples using two different linear approximation methods, those are based on the common assumption that the i^{th} standard order statistic $Z_{i:n}$ lies in a narrow neighbourhood of the i^{th} population quantile ξ_i. We denote the LAMLEs of σ computed based on Taylors series method and least squares method respectively by $\tilde{\sigma}_{TS}$ and $\tilde{\sigma}_{LS}$.

3. Percentiles based Estimator of Scale Parameter

Among the most easily obtained estimation methods of the parameters of the Weibull distribution are the graphical approximations to the best linear unbiased estimators. These estimates can be obtained by fitting a straight line to the theoretical points obtained from the distribution function and sample percentile points. This method was originally suggested by Kao (1958, 1959) [5-6] and also see Mann, Schafer and Singpurwalla (1974) [8] and Johnson, Kotz and Balakrishnan (1994) [4]. In case of a Type II GHLD distribution, also it is possible to use same concept to obtain the estimate of σ based on the percentiles because of the structure of its distribution function. From Eq. (1.5), we have

$$P(x;\theta,\sigma) = 1 - 2^{\theta}\left(1+e^{x/\sigma}\right)^{-\theta} \tag{3.1}$$

Which implies

$$x = \sigma \log\left[2\left\{1-P(x;\theta,\sigma)\right\}^{-1/\theta} - 1\right] \tag{3.2}$$

If p_i denotes some estimate of $P(x_{i:n};\theta,\sigma)$, then the estimate of σ can be obtained by minimizing the error sum of squares

$$\sum_{i=r+1}^{n-s}\left[x_{i:n} - \sigma \log\left\{2(1-p_i)^{-1/\theta} - 1\right\}\right]^2$$

If p_i denotes some estimate of $P(x_{i:n};\theta,\sigma)$, then the estimate of σ can be obtained by minimizing the error sum of squares

$$\sigma_{PCE} = \sum_{i=r+1}^{n-s} \omega_i x_{i:n}, \qquad \text{where } \omega_i = \frac{\log\left\{2(1-p_i)^{-1/\theta} - 1\right\}}{\sum_{i=r+1}^{n-s} \log^2\left\{2(1-p_i)^{-1/\theta} - 1\right\}} \tag{3.3}$$

It is possible to use several p_i's as estimator of $P(x_{i:n})$. However, we use $p_i = i/(n+1)$ as the estimator of $P(x_{i:n})$, as $i/(n+1)$ is the expected value of $P(x_{i:n})$. As explained in the above section, since the variances and covariances of standard Type II GHLD are not available in the literature, we cannot compute exact variance and hence, the standard error of σ_{PCE}.

4. Comparison of LAMLE and PCE with MLE

The ML equation (2.2) can be solved iteratively using the well-known Newton-Raphson method, which requires an initial solution of σ near global maxima. Here, LAMLE can be used as such a good initial solution. We denote MLE, thus obtained, as $\hat{\sigma}$. Like in the case of LAMLE, we cannot obtain the expected Fisher information of MLE. However, we can obtain the observed Fisher information as

$$\hat{I} = \left(-\frac{\partial^2 LogL}{\partial \sigma^2}\right)_{\sigma=\hat{\sigma}} \tag{4.1}$$

Where

$$\frac{\partial^2 LogL}{\partial \sigma^2} = \frac{1}{\sigma^2}\Bigg[(n-s-r) + r\theta\left\{G(z_{r+1:n}) + z_{r+1:n}G'(z_{r+1:n})\right\} + 2\sum_{i=r+1}^{n-s} z_{i:n} - (\theta+1) \sum_{i=r+1}^{n-s}\left\{H(z_{i:n}) + z_{i:n}H'(z_{i:n})\right\} - s\theta\left\{H(z_{n-s:n}) + z_{n-s:n}H'(z_{n-s:n})\right\}\Bigg] \tag{4.2}$$

From which the standard error of $\hat{\sigma}$ is obtained as

$$SE(\hat{\sigma}) = \hat{\sigma} \Big/ \sqrt{\hat{I}} \tag{4.3}$$

In Taylor series and LS methods discussed in the above sections, the basic principle is that the intractable terms $G(z_{i:n})$ and $H(z_{i:n})$ are approximated linearly in a narrow neighbourhood of the standard population quantile ξ_i. The solutions of exact and approximate log-likelihood equations tend to each other as $n \to \infty$. Hence, the exact and approximate MLEs are asymptotically identical (Tiku *et al*. 1986) [12]. Therefore, the asymptotic variances of exact and approximate MLEs are same (Bhattacharya, 1985). However, the same cannot be said in small samples. At the same time, the small sample bias and variance of MLE are not mathematically tractable. Therefore, we have resorted to Monte Carlo simulation to compute the bias and MSE of $\hat{\sigma}$, $\tilde{\sigma}_{TS}$, $\tilde{\sigma}_{LS}$ and σ_{PCE} based on 4000 samples of size n=5,10&20 generated from standard Type II GHLD with θ =0.5,1.5&3.0. We tabulate these simulated biases and MSEs in Table 2.

Table 2: Simulated means of order statistics of standard Type-II GHLD with shape parameter θ=0.5, 1.5(0.5)5.0 for sample sizes n=5, 10 & 20.

N	I	0.5	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
5	1	0.63	0.2	0.1831	0.1486	0.125	0.108	0.095	0.0848	0.0766
5	2	1.31	0.5	0.3945	0.3222	0.2725	0.2362	0.2084	0.1866	0.1689
5	3	2.1	0.8	0.6533	0.537	0.4565	0.3973	0.3519	0.3159	0.2866
5	4	3.17	1.3	1.0004	0.8273	0.7069	0.6179	0.5493	0.4947	0.4501
5	5	5.26	2.1	1.6317	1.3563	1.165	1.0234	0.9139	0.8264	0.7548
10	1	0.36	0.1	0.0981	0.0791	0.0663	0.0571	0.0501	0.0446	0.0403
10	2	0.69	0.3	0.1979	0.1603	0.1347	0.1162	0.1022	0.0912	0.0823
10	3	1.04	0.4	0.307	0.2496	0.2103	0.1818	0.1601	0.1431	0.1293
10	4	1.4	0.5	0.4245	0.3464	0.2928	0.2536	0.2238	0.2002	0.1811
10	5	1.81	0.7	0.5595	0.4584	0.3885	0.3374	0.2982	0.2672	0.2421
10	6	2.26	0.9	0.707	0.5812	0.4941	0.43	0.3808	0.3418	0.3101
10	7	2.79	1.1	0.8838	0.7292	0.6217	0.5424	0.4813	0.4327	0.3931
10	8	3.52	1.4	1.1181	0.9259	0.7919	0.6927	0.6162	0.5551	0.5053
10	9	4.54	1.8	1.4327	1.1902	1.0211	0.8958	0.7988	0.7213	0.6579
10	10	6.56	2.6	2.0148	1.677	1.4428	1.2695	1.1354	1.0282	0.9404
20	1	0.19	0.1	0.0507	0.0407	0.0341	0.0293	0.0256	0.0228	0.0206
20	2	0.37	0.1	0.1016	0.0819	0.0685	0.059	0.0517	0.0461	0.0415
20	3	0.55	0.2	0.1529	0.1235	0.1035	0.0891	0.0783	0.0698	0.0629
20	4	0.73	0.3	0.207	0.1675	0.1406	0.1212	0.1065	0.095	0.0858
20	5	0.91	0.3	0.2628	0.213	0.1792	0.1546	0.136	0.1214	0.1096
20	6	1.09	0.4	0.3193	0.2594	0.2185	0.1888	0.1662	0.1484	0.1341
20	7	1.27	0.5	0.3805	0.3098	0.2613	0.226	0.1992	0.178	0.161
20	8	1.46	0.6	0.4424	0.3608	0.3048	0.264	0.2328	0.2083	0.1884
20	9	1.66	0.7	0.5078	0.4151	0.3512	0.3045	0.2688	0.2406	0.2178
20	10	1.88	0.7	0.5812	0.476	0.4034	0.3502	0.3094	0.2773	0.2512
20	11	2.11	0.8	0.6586	0.5405	0.4588	0.3987	0.3527	0.3163	0.2867
20	12	2.36	0.9	0.741	0.6093	0.518	0.4508	0.3992	0.3583	0.325
20	13	2.63	1.1	0.8304	0.6842	0.5826	0.5076	0.45	0.4042	0.367

The efficiencies of $\tilde{\sigma}_{TS}$, $\tilde{\sigma}_{LS}$ and σ_{PCE} comparing the MSE of MLE and are tabulated in Table 2 in the form of percentages after the respective MSEs. Table 2 reveals the following conclusions.

- Since the biases of all the estimators expect σ_{PCE} are negative, ML and LAML methods are underestimating σ, whereas percentile method is over estimating**.**
- When compared with MLE $\tilde{\sigma}_{TS}$, $\tilde{\sigma}_{LS}$ and σ_{PCE} are more biased. However, $\tilde{\sigma}_{LS}$ is slightly less biased than the other two.
- Since, the efficiencies of LAMLEs compared with MLE are 100%, the LAMLEs are as efficient as MLE irrespective of the values of θ, n, r and s values.
- The percentile estimator method is no way comparable with LAML method in terms of both bias and MSE. Moreover the efficiency of σ_{PCE} as compared with MLE is fluctuating w. r. t. θ, n, r and s values.

5. Nearly Unbiased LAMLE and Nearly Unbiased PCE of scale parameter

While comparing LAMLE with MLE in the above section, we observed that even though LAMLE is almost as efficient as MLE, it is biased than MLE. From Eq. (2.8), we may write

$$E\left(\sigma^*\right) = \sigma \ , \ where \ \sigma^* = \left[\sum_{i=r+1}^{n-s} l_i a_{i:n}\right]^{-1} \tilde{\sigma} \tag{5.1}$$

So that σ* is a linear unbiased estimator. Since, we are using simulated $a_{i:n}$'s in place of actual $a_{i:n}$'s, we call σ* as nearly unbiased LAMLE. The asymptotic variance of σ* and hence the standard error of σ* can be obtained as

$$SE(\sigma^*) = \left[\sum_{i=r+1}^{n-s} l_i a_{i:n}\right]^{-1} SE(\tilde{\sigma}) \tag{5.2}$$

Where $SE(\tilde{\sigma})$ is given in Eq (2.10). Similarly, the nearly unbiased percentile estimator of σ, denoted by σ_{UPCE} and given as

$$\sigma_{UPCE} = \left[\sum_{i=r+1}^{n-s} \omega_i a_{i:n}\right]^{-1} \sigma_{PCE} \tag{5.3}$$

Since, the var the performances of σ* and σ_{UPCE} are obtained by comparing them with the corresponding MLE based on bias and variance, those are simulated based on 4000 Monte-Carlo runs. The biases and MSEs of σ* and σ_{UPCE} and $\hat{\sigma}$ are tabulated in Table 3 for Type-II censored samples of size n=5, 10&20 with r=0(1) n/4, s=0(1) n/4 and θ =0.5, 1.5&3.0. From the table we may conclude that

- Since the estimators (except MLE) are nearly unbiased, we get negligible simulated biases for the estimators. We may notice that σ^* and σ_{UPCE} are more biased than $\hat{\sigma}$ when θ <1, whereas they are less biased than $\hat{\sigma}$ when θ >1. Thus, when θ >1, nearly unbiased LAMLE and nearly unbiased PCE are less biased than MLE irrespective of the value of *n* and amount of censoring whether censoring is left or right or doubly.
- In moderate and large samples (n>10), σ^* is performing remarkably with more than 99% efficiency as compared with $\hat{\sigma}$ irrespective of the amount of censoring whether it is left censoring or right censoring or doubly censoring. Even in small samples (n<10), it is performing with a minimum of 95% efficiency.
- Though σ_{UPCE} is as good as σ^* in terms of bias, it is not as efficient as σ^* in terms of MSE.
- For θ >1, except in small samples (n<10), σ^* is not only performing with more than 97% efficiency, but also less biased than MLE. Therefore, unbiased LAMLE is preferable to MLE in these cases.

Remark: In the above, σ* is computed using least squares method. However, we will get the same conclusions even if we use Taylor series method.

Table 3: Relative performance of nearly unbiased LAMLEs σ^* and σ_{UPCE} as compared with the MLE $\hat{\sigma}$ Type II GHLD based on simulated bias and MSE (based on 4000 Monte Carlo runs) from Type II censored samples of size n=5,10&20 with r=0(1)n/4, s=0(1)n/4 and θ =0.5, 1.5, 3.0. The figures given in brackets are the efficiencies (based on MSEs) of the respective estimates as compared with MLE

				$BIAS/\sigma$			MSE/σ^2				
θ	**N**	**R**	**S**	$\hat{\sigma}$	σ^*	σ_{UPCE}	$\hat{\sigma}$	σ^*		σ_{UPCE}	
0.5	5	0	0	-0.0098	-0.0114	-0.0112	0.1494	0.1489	(100.3)	0.1557	(95.9)
0.5	5	0	1	-0.0187	-0.0108	-0.0109	0.1782	0.181	(98.4)	0.1814	(98.2)
0.5	5	1	0	-0.0091	-0.0115	-0.0112	0.1495	0.1488	(100.5)	0.1565	(95.5)
0.5	5	1	1	-0.0179	-0.011	-0.0111	0.1785	0.181	(98.6)	0.1814	(98.4)
0.5	10	0	0	-0.0001	-0.0025	-0.0021	0.0725	0.0722	(100.5)	0.0789	(91.9)
0.5	10	0	1	-0.0013	-0.0031	-0.0033	0.0786	0.0784	(100.3)	0.079	(99.5)
0.5	10	0	2	-0.0039	-0.0042	-0.0035	0.086	0.0861	(99.9)	0.0871	(98.8)
0.5	10	1	0	-0.0000	-0.0025	-0.0021	0.0724	0.0721	(100.4)	0.079	(91.7)
0.5	10	1	1	-0.0013	-0.0032	-0.0032	0.0786	0.0784	(100.2)	0.0791	(99.4)
0.5	10	1	2	-0.0039	-0.0043	-0.0033	0.086	0.0861	(99.8)	0.0871	(98.7)
0.5	10	2	0	0.0001	-0.0025	-0.002	0.0725	0.0722	(100.5)	0.0793	(91.4)
0.5	10	2	1	-0.0011	-0.0032	-0.0032	0.0787	0.0784	(100.3)	0.0793	(99.2)
0.5	10	2	2	-0.0036	-0.0043	-0.0032	0.0861	0.0861	(99.9)	0.0872	(98.7)
0.5	20	0	0	0.0003	-0.004	-0.0047	0.0357	0.0354	(100.7)	0.0393	(90.9)
0.5	20	0	1	0.0002	-0.0036	-0.0043	0.0372	0.037	(100.6)	0.0378	(98.5)
0.5	20	0	2	-0.001	-0.0048	-0.0048	0.0386	0.0384	(100.6)	0.0388	(99.5)
0.5	20	0	3	-0.0013	-0.0037	-0.0037	0.0408	0.0407	(100.3)	0.0411	(99.3)
0.5	20	0	4	-0.0015	-0.0039	-0.0033	0.0427	0.0425	(100.4)	0.0436	(97.9)
0.5	20	0	5	-0.002	-0.0029	-0.0025	0.0452	0.0452	(100.1)	0.0468	(96.5)
0.5	20	1	0	0.0003	-0.004	-0.0047	0.0357	0.0354	(100.7)	0.0393	(90.8)
0.5	20	1	1	0.0002	-0.0036	-0.0043	0.0372	0.037	(100.6)	0.0378	(98.5)
0.5	20	1	2	-0.001	-0.0048	-0.0048	0.0386	0.0384	(100.6)	0.0388	(99.5)
0.5	20	1	3	-0.0013	-0.0037	-0.0037	0.0408	0.0407	(100.3)	0.0411	(99.3)
0.5	20	1	4	-0.0015	-0.0039	-0.0032	0.0427	0.0425	(100.4)	0.0436	(97.9)
0.5	20	1	5	-0.002	-0.0029	-0.0025	0.0452	0.0452	(100.1)	0.0468	(96.5)

0.5	20	2	0	0.0003	-0.004	-0.0047	0.0357	0.0354	(100.8)	0.0393	(90.8)
0.5	20	2	1	0.0002	-0.0036	-0.0042	0.0372	0.037	(100.6)	0.0378	(98.4)
0.5	20	2	2	-0.0009	-0.0049	-0.0047	0.0386	0.0384	(100.7)	0.0388	(99.5)
0.5	20	2	3	-0.0013	-0.0037	-0.0036	0.0409	0.0407	(100.4)	0.0412	(99.3)
0.5	20	2	4	-0.0014	-0.004	-0.0032	0.0427	0.0425	(100.4)	0.0436	(97.9)
0.5	20	2	5	-0.0019	-0.0029	-0.0024	0.0452	0.0451	(100.1)	0.0468	(96.6)
0.5	20	3	0	0.0004	-0.004	-0.0046	0.0357	0.0354	(100.8)	0.0394	(90.7)
0.5	20	3	1	0.0003	-0.0036	-0.0042	0.0373	0.037	(100.7)	0.0379	(98.3)
0.5	20	3	2	-0.0009	-0.0049	-0.0047	0.0387	0.0384	(100.7)	0.0389	(99.4)
0.5	20	3	3	-0.0012	-0.0037	-0.0035	0.0409	0.0407	(100.4)	0.0412	(99.2)
0.5	20	3	4	-0.0014	-0.004	-0.0031	0.0427	0.0425	(100.5)	0.0436	(97.9)
0.5	20	3	5	-0.0019	-0.0029	-0.0023	0.0452	0.0451	(100.1)	0.0468	(96.6)
0.5	20	4	0	0.0004	-0.004	-0.0046	0.0357	0.0354	(100.8)	0.0395	(90.5)
0.5	20	4	1	0.0004	-0.0036	-0.0042	0.0373	0.037	(100.7)	0.038	(98.2)
0.5	20	4	2	-0.0008	-0.0048	-0.0047	0.0387	0.0384	(100.7)	0.0389	(99.3)
0.5	20	4	3	-0.0012	-0.0037	-0.0035	0.0409	0.0407	(100.4)	0.0412	(99.2)
0.5	20	4	4	-0.0013	-0.0039	-0.003	0.0427	0.0425	(100.5)	0.0436	(97.9)
0.5	20	4	5	-0.0018	-0.0029	-0.0021	0.0452	0.0452	(100.2)	0.0468	(96.7)
0.5	20	5	0	0.0005	-0.0039	-0.0046	0.0358	0.0355	(100.8)	0.0396	(90.3)
0.5	20	5	1	0.0004	-0.0035	-0.0041	0.0373	0.0371	(100.7)	0.0381	(98)
0.5	20	5	2	-0.0008	-0.0048	-0.0046	0.0387	0.0384	(100.8)	0.039	(99.2)
0.5	20	5	3	-0.0011	-0.0036	-0.0034	0.0409	0.0408	(100.4)	0.0413	(99.1)
0.5	20	5	4	-0.0013	-0.0039	-0.0029	0.0428	0.0426	(100.5)	0.0437	(97.9)
0.5	20	5	5	-0.0018	-0.0028	-0.002	0.0453	0.0452	(100.2)	0.0468	(96.8)
1.5	5	0	0	-0.0297	-0.0112	-0.0111	0.1377	0.1422	(96.9)	0.1439	(95.7)
1.5	5	0	1	-0.0471	-0.011	-0.0115	0.1716	0.1826	(94)	0.1844	(93)
1.5	5	1	0	-0.0295	-0.0114	-0.0111	0.1376	0.142	(96.9)	0.1443	(95.4)
1.5	5	1	1	-0.0468	-0.0113	-0.0117	0.1716	0.1825	(94)	0.1839	(93.3)
1.5	10	0	0	-0.0095	-0.0025	-0.0021	0.0682	0.0692	(98.7)	0.0713	(95.7)
1.5	10	0	1	-0.0138	-0.0033	-0.0033	0.0756	0.077	(98.1)	0.0773	(97.7)
1.5	10	0	2	-0.0185	-0.0047	-0.0035	0.0852	0.0874	(97.5)	0.0897	(94.9)
1.5	10	1	0	-0.0095	-0.0024	-0.0021	0.0682	0.0691	(98.7)	0.0713	(95.6)
1.5	10	1	1	-0.0137	-0.0032	-0.0032	0.0755	0.077	(98.1)	0.0773	(97.6)
1.5	10	1	2	-0.0184	-0.0046	-0.0033	0.0851	0.0874	(97.4)	0.0897	(94.9)
1.5	10	2	0	-0.0094	-0.0024	-0.002	0.0682	0.0691	(98.7)	0.0715	(95.4)
1.5	10	2	1	-0.0136	-0.0032	-0.0031	0.0756	0.077	(98.1)	0.0774	(97.6)
1.5	10	2	2	-0.0183	-0.0046	-0.0032	0.0851	0.0874	(97.4)	0.0896	(95)
1.5	20	0	0	-0.0045	-0.0043	-0.0044	0.0337	0.0338	(99.9)	0.0351	(96.2)
1.5	20	0	1	-0.0054	-0.0039	-0.0042	0.0356	0.0357	(99.6)	0.0358	(99.3)
1.5	20	0	2	-0.0075	-0.0054	-0.0047	0.0373	0.0375	(99.6)	0.0383	(97.5)
1.5	20	0	3	-0.0086	-0.0041	-0.0037	0.0402	0.0405	(99.2)	0.0418	(96.2)
1.5	20	0	4	-0.0091	-0.0045	-0.0033	0.0427	0.0431	(99.2)	0.0454	(94.2)
1.5	20	0	5	-0.0098	-0.0033	-0.0025	0.0461	0.0467	(98.8)	0.0498	(92.6)
1.5	20	1	0	-0.0045	-0.0043	-0.0044	0.0338	0.0338	(99.9)	0.0351	(96.2)
1.5	20	1	1	-0.0054	-0.0039	-0.0042	0.0356	0.0357	(99.6)	0.0358	(99.3)
1.5	20	1	2	-0.0075	-0.0054	-0.0047	0.0373	0.0375	(99.6)	0.0383	(97.5)
1.5	20	1	3	-0.0086	-0.0041	-0.0036	0.0402	0.0405	(99.2)	0.0418	(96.2)
1.5	20	1	4	-0.0091	-0.0045	-0.0033	0.0427	0.0431	(99.2)	0.0454	(94.2)
1.5	20	1	5	-0.0098	-0.0033	-0.0025	0.0461	0.0467	(98.8)	0.0498	(92.6)
1.5	20	2	0	-0.0045	-0.0043	-0.0044	0.0338	0.0338	(99.9)	0.0351	(96.2)
1.5	20	2	1	-0.0054	-0.0039	-0.0041	0.0356	0.0357	(99.7)	0.0358	(99.2)
1.5	20	2	2	-0.0075	-0.0054	-0.0046	0.0373	0.0375	(99.6)	0.0383	(97.5)
1.5	20	2	3	-0.0085	-0.0041	-0.0036	0.0402	0.0405	(99.2)	0.0418	(96.2)
1.5	20	2	4	-0.0091	-0.0044	-0.0032	0.0427	0.0431	(99.2)	0.0454	(94.2)
1.5	20	2	5	-0.0098	-0.0032	-0.0024	0.0461	0.0467	(98.8)	0.0498	(92.7)
1.5	20	3	0	-0.0044	-0.0043	-0.0044	0.0338	0.0338	(99.9)	0.0351	(96.2)
1.5	20	3	1	-0.0054	-0.0039	-0.0041	0.0356	0.0357	(99.7)	0.0359	(99.2)
1.5	20	3	2	-0.0074	-0.0054	-0.0046	0.0373	0.0375	(99.6)	0.0383	(97.5)
1.5	20	3	3	-0.0085	-0.0041	-0.0035	0.0402	0.0405	(99.2)	0.0418	(96.2)
1.5	20	3	4	-0.0091	-0.0044	-0.0031	0.0428	0.0431	(99.2)	0.0454	(94.2)
1.5	20	3	5	-0.0098	-0.0032	-0.0023	0.0461	0.0467	(98.8)	0.0498	(92.7)
1.5	20	4	0	-0.0044	-0.0042	-0.0043	0.0338	0.0338	(99.9)	0.0352	(96.1)
1.5	20	4	1	-0.0053	-0.0038	-0.0041	0.0356	0.0357	(99.7)	0.0359	(99.1)
1.5	20	4	2	-0.0074	-0.0053	-0.0045	0.0373	0.0375	(99.6)	0.0383	(97.5)
1.5	20	4	3	-0.0084	-0.004	-0.0035	0.0402	0.0405	(99.2)	0.0418	(96.2)
1.5	20	4	4	-0.009	-0.0043	-0.003	0.0428	0.0431	(99.2)	0.0454	(94.3)
1.5	20	4	5	-0.0097	-0.0031	-0.0022	0.0462	0.0467	(98.8)	0.0497	(92.9)
1.5	20	5	0	-0.0044	-0.0041	-0.0043	0.0338	0.0339	(99.9)	0.0352	(95.9)
1.5	20	5	1	-0.0053	-0.0037	-0.004	0.0356	0.0357	(99.7)	0.0359	(99.1)

1.5	20	5	2	-0.0074	-0.0052	-0.0045	0.0374	0.0376	(99.6)	0.0384	(97.5)
1.5	20	5	3	-0.0085	-0.0039	-0.0034	0.0403	0.0406	(99.2)	0.0418	(96.3)
1.5	20	5	4	-0.009	-0.0043	-0.0029	0.0428	0.0432	(99.2)	0.0454	(94.4)
1.5	20	5	5	-0.0097	-0.003	-0.0021	0.0462	0.0468	(98.8)	0.0497	(93.1)
3.0	5	0	0	-0.0378	-0.0115	-0.0113	0.1439	0.1505	(95.6)	0.1517	(94.8)
3.0	5	0	1	-0.0492	-0.0113	-0.0117	0.186	0.1987	(93.6)	0.2002	(92.9)
3.0	5	1	0	-0.0376	-0.0117	-0.0114	0.1437	0.1502	(95.7)	0.1521	(94.5)
3.0	5	1	1	-0.049	-0.0116	-0.0119	0.186	0.1986	(93.7)	0.1999	(93)
3.0	10	0	0	-0.0137	-0.0026	-0.0021	0.0716	0.0731	(98)	0.0748	(95.7)
3.0	10	0	1	-0.0171	-0.0036	-0.0033	0.0811	0.083	(97.7)	0.0837	(96.9)
3.0	10	0	2	-0.02	-0.0052	-0.0035	0.0929	0.0955	(97.3)	0.098	(94.8)
3.0	10	1	0	-0.0137	-0.0025	-0.0021	0.0715	0.073	(98)	0.0748	(95.6)
3.0	10	1	1	-0.0171	-0.0034	-0.0033	0.081	0.083	(97.6)	0.0837	(96.8)
3.0	10	1	2	-0.0199	-0.005	-0.0034	0.0928	0.0954	(97.3)	0.098	(94.7)
3.0	10	2	0	-0.0136	-0.0024	-0.002	0.0716	0.0731	(98)	0.075	(95.5)
3.0	10	2	1	-0.017	-0.0034	-0.0032	0.0811	0.083	(97.6)	0.0838	(96.8)
3.0	10	2	2	-0.0198	-0.0049	-0.0033	0.0929	0.0955	(97.3)	0.098	(94.7)
3.0	20	0	0	-0.0065	-0.0045	-0.0045	0.0355	0.0357	(99.6)	0.0367	(96.9)
3.0	20	0	1	-0.0075	-0.0042	-0.0043	0.0379	0.0381	(99.4)	0.0385	(98.5)
3.0	20	0	2	-0.0093	-0.0059	-0.0049	0.0402	0.0404	(99.4)	0.0415	(96.7)
3.0	20	0	3	-0.01	-0.0045	-0.0038	0.0436	0.0441	(99)	0.0456	(95.7)
3.0	20	0	4	-0.01	-0.0049	-0.0034	0.0467	0.0472	(99.1)	0.0497	(94)
3.0	20	0	5	-0.0101	-0.0036	-0.0026	0.0507	0.0513	(98.8)	0.0546	(92.8)
3.0	20	1	0	-0.0065	-0.0045	-0.0045	0.0355	0.0357	(99.6)	0.0367	(96.9)
3.0	20	1	1	-0.0075	-0.0042	-0.0043	0.0379	0.0381	(99.3)	0.0385	(98.5)
3.0	20	1	2	-0.0093	-0.0058	-0.0049	0.0402	0.0404	(99.3)	0.0415	(96.7)
3.0	20	1	3	-0.01	-0.0045	-0.0038	0.0436	0.0441	(99)	0.0456	(95.6)
3.0	20	1	4	-0.01	-0.0049	-0.0034	0.0467	0.0472	(99.1)	0.0497	(94)
3.0	20	1	5	-0.0101	-0.0036	-0.0026	0.0507	0.0513	(98.8)	0.0546	(92.8)
3.0	20	2	0	-0.0065	-0.0045	-0.0045	0.0355	0.0357	(99.6)	0.0367	(96.9)
3.0	20	2	1	-0.0074	-0.0041	-0.0043	0.0379	0.0382	(99.4)	0.0385	(98.5)
3.0	20	2	2	-0.0093	-0.0058	-0.0048	0.0402	0.0404	(99.3)	0.0416	(96.7)
3.0	20	2	3	-0.01	-0.0044	-0.0038	0.0437	0.0441	(99)	0.0456	(95.7)
3.0	20	2	4	-0.01	-0.0048	-0.0034	0.0468	0.0472	(99.1)	0.0497	(94)
3.0	20	2	5	-0.0101	-0.0035	-0.0025	0.0507	0.0513	(98.8)	0.0546	(92.8)
3.0	20	3	0	-0.0065	-0.0044	-0.0044	0.0356	0.0357	(99.6)	0.0367	(96.9)
3.0	20	3	1	-0.0074	-0.0041	-0.0043	0.0379	0.0382	(99.4)	0.0385	(98.5)
3.0	20	3	2	-0.0092	-0.0057	-0.0048	0.0402	0.0405	(99.3)	0.0416	(96.7)
3.0	20	3	3	-0.0099	-0.0043	-0.0037	0.0437	0.0441	(99)	0.0456	(95.6)
3.0	20	3	4	-0.0099	-0.0048	-0.0033	0.0468	0.0472	(99.1)	0.0497	(94)
3.0	20	3	5	-0.01	-0.0034	-0.0024	0.0507	0.0513	(98.8)	0.0546	(92.9)
3.0	20	4	0	-0.0064	-0.0044	-0.0044	0.0356	0.0357	(99.6)	0.0367	(96.8)
3.0	20	4	1	-0.0073	-0.004	-0.0043	0.0379	0.0382	(99.3)	0.0385	(98.5)
3.0	20	4	2	-0.0092	-0.0057	-0.0048	0.0402	0.0405	(99.3)	0.0416	(96.7)
3.0	20	4	3	-0.0098	-0.0043	-0.0037	0.0437	0.0441	(99)	0.0457	(95.7)
3.0	20	4	4	-0.0099	-0.0047	-0.0032	0.0468	0.0472	(99.1)	0.0498	(94.1)
3.0	20	4	5	-0.01	-0.0034	-0.0023	0.0508	0.0514	(98.8)	0.0546	(93)
3.0	20	5	0	-0.0064	-0.0043	-0.0044	0.0356	0.0358	(99.5)	0.0368	(96.7)
3.0	20	5	1	-0.0073	-0.0039	-0.0042	0.038	0.0382	(99.3)	0.0386	(98.4)
3.0	20	5	2	-0.0092	-0.0055	-0.0047	0.0403	0.0406	(99.3)	0.0417	(96.7)
3.0	20	5	3	-0.0099	-0.0041	-0.0036	0.0438	0.0442	(99)	0.0457	(95.8)
3.0	20	5	4	-0.0099	-0.0046	-0.0031	0.0469	0.0473	(99.1)	0.0498	(94.2)
3.0	20	5	5	-0.01	-0.0032	-0.0022	0.0509	0.0515	(98.8)	0.0546	(93.1)

6. Illustration

We consider the following sorted sample of size 10, which is simulated from Type II GHLD with σ =10 and θ =3. 0.07, 0.50, 2.90, 4.99, 5.02, 6.46, 8.06, 8.93, 9.46, 10.52.

We have computed $\hat{\sigma}$, $\tilde{\sigma}$ and σ^* and for the above sample with different choices of r and s and tabulated below. Here, for computation of $\tilde{\sigma}$ and σ^*, we have used least squares method. Since, σ_{PCE} and σ_{UPCE} are not so efficient, we have not computed them.

The standard errors of $\hat{\sigma}$, $\tilde{\sigma}$ and σ^* are computed from Eqs. (4.3), (2.10) and (5.2) respectively, which are derived based on Fisher information of the estimators. From the above table, we may notice that the standard error MLE is larger than that of LAMLE, perhaps, this is due to the fact that the former is based on observed Fisher information whereas the latter is based on expected Fisher information. Even though, the standard error of $\tilde{\sigma}$ is slightly smaller than that of σ^*, σ^* is better than $\tilde{\sigma}$, since σ^* is the bias reduced estimator.

7. Summary of the conclusions

- Since, ML method does not provide explicit estimator for scale parameter of type II GHLD, in this paper, we have investigated some alternative simple linear estimators namely LAMLE, PCE, nearly unbiased LAMLE and nearly unbiased PCE.
- Based on a Monte Carlo simulation study, we have shown that LAMLE is as efficient as MLE though it is biased than MLE. LAMLE can also be used as starting value to obtain MLE using an iterative method.
- PCE and nearly unbiased PCE are not recommended as they are not as efficient as LAMLE and nearly unbiased LAMLE respectively.
- When shape parameter θ >1, in moderate and large samples for estimation of scale parameter nearly unbiased LAMLE is preferable to MLE because it is not only less biased but also almost as efficient as MLE.
- When shape parameter θ <1, LAMLE may be recommended as an alternative to MLE.

8. References

1. Balakrishnan N. Approximation MLE of scale parameter of the Rayleigh distribution with censoring. IEEE Transactions on Reliability. 1989; (38):355-357.
2. Bhattacharya GK. The asymptotic of maximum likelihood and related estimators based on Type-II censored data. JASA. 1985; (80):398-404.
3. Devendra Kumar, Neetu Jain, Shivani Gupta. The Type I Generalized Half-Logistic Distribution Based on Upper Record Values. Journal of Probability and Statistics. 2015, 1-11.
4. Johnson NL, Kotz S, Balakrishnan N. Continuous Univariate Distribution. 2nd edition, New York, Wiley, 1994, 1.
5. Kao JHK. Computer methods for estimation Weibull parameters in reliability studies. Transaction of IRE-Reliability and Quality control. 1958; 13:15-22.
6. Kao JHK. A graphical estimation of mixed Weibull parameters in reliability studies. Technometrics. 1959; 1:389-407.
7. Kantam RRL, Ramakrishna V, Ravikumar MS. Estimation and testing in type 1 generalized half logistic distribution. Modern applied statistical methods. 2014; 12(1):198-206.
8. Mann Schafer RE, Singpurwalla ND. Methods for Statistical Analysis of Reliability and Life Data. New York, Wiley, 1974.
9. Olapade AK. The type I generalized half logistic distribution. Journal of the Iranian Statistical Society. 2014; 13(1):69-82.
10. Phillip Oluwatobi Awodutiristice, Akintayo Kehinde Olapade, Oladapo Adedayo Kolawole. The Type I Generalized Half Logistic Survival Model. International Journal of Theoretical and Applied Mathematics. 2016; 2(2):74-78.
11. Rosaiah K, Kantam RRL, Rama Krishna V. Type-II Generalized Half Logistic Distribution-An Economic Reliability Test Plan. Journal of Chemical, Biological and Physical Sciences. 2014; 4(1):501-508.
12. Tiku ML, Tan WY, Balakrishnan N. Robust Inference, New York: Marcel Dekker, 1986.
13. Vaudeva Rao A, Sitaramacharyulu P, ChenchuRamaiah M. Linear approximate ML estimation in scaled Type I generalized logistic distribution based on Type-II censored samples. Commun. Statist-Simul. Comput. 2017; 46(3):1682-1702.