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## MHD free convection flow through a vertical porous plate with thermal diffusivity and radiation in slip boundary condition

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### Abstract

Investigation of steady MHD free convection flow with heat and mass transfer of a viscous, incompressible, electrically conducting, optically thin radiating fluid past an infinite vertical plate with thermal diffusivity through porous medium is carried out. Exact solutions of momentum, energy and concentration equations are obtained in closed form by two term perturbation technique. The influence of a number of emerging non-dimensional parameters namely, magnetic parameter, Prandtl number, thermal radiation parameter and Schmidt number examined in detail. Furthermore, the influence of these parameters on heat transfer rate and skin friction is also investigated. It is observed that the velocity is decreased with increasing moving parameter. Increasing permeability parameter increases the velocity but increasing schmidth number decreases the velocity. The study is relevant to chemical materials processing applications.

**Keywords:** MHD, Moving parameter, suction parameter, radiation parameter, magnetic field

### 1. Introduction

Theoretical/experimental investigation of problems of unsteady hydro magnetic natural convection flow of an electrically conducting fluid through porous and non-porous media has received considerable attention of several researchers during past few decades due to its overwhelming and important applications in many areas of science and engineering which includes geophysics, astrophysics, electronics, aeronautics, metallurgy, chemical and petroleum engineering etc. Keeping in view the importance of this fluid flow, several researchers investigated unsteady hydro magnetic free convection flow of an electrically conducting fluid past bodies with different geometries under different initial and boundary conditions. Effect of variable suction and thermophoresis on steady MHD combined free-forces convection heat and mass transfer flow over a semi-infinite permeable inclined plate in the presence of thermal radiation was investigated by Alam *et al.* [1]. Alam *et al.* [2] were studied numerically the combined free forced convection and mass transfer flow transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion. Chamkha & Khaled [3] investigated coupled heat and mess transfer by natural convection from a vertical. Semi-infinite flat plate embedded in a pours medium in the presence of an extreme magnetic field. Chaudhary and Arpita [4] presented the combines heat and mass transfer effects on MHD convection flow past an oscillating plate embedded in porous medium. Many researchers Chaudhary & Jain [5] contributed in studying the application of viscos-elastic fluid flow of several types past porous medium in channels of different cross-sections. EI-Hakim [6] presented MHD oscillatory flow on free convection radiation through a porous medium with constant suction velocity. Gupta and Sharma [7] attended to study to study MHD flow of viscous fluid past a porous medium enclosed by an oscillating porous plate in slip flow regime. An analytical study of MHD free convective, dissipative boundary layer flow past a porous vertical surface in the presence of thermal radiation, chemical reaction and constant suction was done by M.C. Raju *et al.* [8].

Radioactive heat transfer flow is very important in manufacturing industries for the design of reliable equipment, nuclear power plants, gas turbines and various propulsion devices for aircraft, missiles, satellites, and space vehicles. Mohammad *et al.* [9] presented the effects of viscous dissipation on the slip MHD flow and heat transfer past a permeable surface with convective boundary conditions. Diffusion-Thermo effects on MHD flow past an infinite vertical porous plate in the presence of a radiation and chemical reaction was studied by M. Venkateswarlu *et al.* [10]. Nazibuddin Ahmed and Kshor Kumar Das [11] discussed MHD mass transfer flow past a vertical porous plate embedded in a porous medium in a slip flow regime with thermal radiation and chemical reaction.

Pal and Shicakumara [12] were discussed on mixed convection heat transfer from a vertical heated plate embedded in a sparsely packed porous medium. Prakash *et al.* [13] investigated diffusion thermo & radiation effect on unsteady MHD free convection flow through porous medium past an impulsively started infinite vertical plate with variable temperature and uniform mass diffusion. Rajesh and Varma [14] investigated the effects of thermal radiation flow past an exponentially accelerated infinite vertical plate with mass transfer in the presence of magnetic field. Sanga Patham *et al.* [15] studied the effects of thermal radiation on the natural convective heat and mass transfer of a viscous, incompressible, gray absorbing emitting started moving vertical plate with viscous dissipation. Chemical reaction and radiation absorption effects on free convection flow through porous medium with variable suction in the presence of uniform magnetic field was studied by Sudheer Babu and Satyanarayana [16].

## 2. Formulation of the problem

We consider two dimensional steady, laminar free convection flow of an incompressible viscous fluid past an infinite vertical porous plate. The flow considered in the presence of temperature gradient dependent heat source with radiation and the Soret number.

In Cartesian coordinate system, we take x-axis along the plate in the direction of flow and y-axis normal to it.

Under the above assumptions the governing equations of continuity, momentum, energy and concentration are:

Continuity Equation

$$\frac{\partial v}{\partial y} = 0 \quad \dots (2.1)$$

Momentum Equation

$$v \frac{\partial u}{\partial y} = \vartheta \frac{\partial^2 u}{\partial y^2} + g\beta'(T - T_\infty) + g\beta^*(C - C_\infty) - \frac{\sigma B_0^2}{\rho} u - \frac{\vartheta}{K} u \quad \dots (2.2)$$

Energy Equation

$$v \frac{\partial \theta}{\partial y} = \frac{k}{\rho C_p} \frac{\partial^2 \theta}{\partial y^2} - \frac{1}{\rho C_p} \frac{\partial q_r}{\partial y} + \frac{Q}{\rho C_p} \frac{\partial \theta}{\partial y} \quad \dots (2.3)$$

Concentration Equation

$$v \frac{\partial C}{\partial y} = D_m \frac{\partial^2 C}{\partial y^2} + D_L \frac{\partial^2 \theta}{\partial y^2} \quad \dots (2.4)$$

The boundary conditions to the problem are:

$$u = U_0 + L_1 \left( \frac{\partial u}{\partial y} \right), T = T_w, C = C_w \text{ at } y \rightarrow 0$$

$$u \rightarrow 0, T \rightarrow T_\infty, C \rightarrow C_\infty \text{ as } y \rightarrow \infty \quad \dots (2.5)$$

where  $\beta'$  and  $\beta^*$  are the volumetric coefficient of thermal and concentration expansion,  $\vartheta$  is kinematic viscosity,  $K(t)$  is the permeability of the porous medium,  $u$  and  $v$  are the components of the velocity along x-axis and y-axis directions,  $k$  is thermal conductivity,  $\rho$  is the density of the fluid,  $\sigma$  is the electrical conductivity of the fluid,  $B_0$  is the uniform magnetic field,  $T$  is the temperature of the fluid,  $C_p$  is the specific heat at constant pressure,  $Q$  is the coefficient of heat source parameter,  $T_w$  is the temperature of the wall of the fluid at the plate,  $T_\infty$  is the temperature of the fluid far away from the plate.

Consider the fluid which is optically thin with the relatively low density and radioactive heat flux:

$$\frac{\partial q_r}{\partial y} = 4(T - T_\infty)I \quad \dots (2.6)$$

Where  $I$  is the absorption coefficient at the plate. Permeability  $K(t)$  of the porous medium is considered in the following form:

$$K(t) = K_0 \quad \dots (2.7)$$

For introducing the following dimensionless quantities and variable:

$$u^* = \frac{u}{v_0}, y^* = \frac{yv_0}{\vartheta}, G_r = \frac{g\beta^*\vartheta(T - T_\infty)}{v_0^3}, M^2 = \frac{\sigma B_0^2 \vartheta}{v_0^2},$$

$$G_m = \frac{g\beta^*\vartheta(C_w - C_\infty)}{v_0^3}, Q = \frac{Q\vartheta}{v_0^2(T_w - T_\infty)\rho C_p}, \theta = \frac{(T - T_\infty)}{(T_w - T_\infty)},$$

$$K_0^* = \frac{K_0 v_0^2}{\vartheta^2}, R = \frac{4\theta l}{\rho C_p v_0^2}, S = \frac{4\nu l}{\rho C_p v_0^2}, E_c = \frac{kv_0^3}{\vartheta q C_p}, S_c = \frac{\vartheta}{D}, P_r = \frac{\rho C_p \vartheta}{k}$$

$$h_1 = \frac{L_1 v_0}{\vartheta}, \alpha = \frac{u_0}{v_0}$$

In the equations of (2.2), (2.3), (2.4), we obtain the non-dimensional form of the governing equations are:

$$\frac{\partial u}{\partial y} = G_r T + G_m C + \frac{\partial^2 u}{\partial y^2} - \left[ M^2 + \frac{1}{K_0} \right] u \tag{2.8}$$

$$\frac{\partial \theta}{\partial y} = \frac{1}{P_r} \frac{\partial^2 \theta}{\partial y^2} - R\theta + Q \frac{\partial \theta}{\partial y} \tag{2.9}$$

$$\frac{\partial C}{\partial y} = \frac{1}{S_c} \frac{\partial^2 C}{\partial y^2} + S_0 \frac{\partial^2 T}{\partial y^2} \tag{2.10}$$

And boundary condition (2.5) is reduced to:

$$u = U_0 + h_1 \left( \frac{\partial u}{\partial y} \right), T = 1, C = 1 \text{ at } y = 0$$

$$u \rightarrow 0, T \rightarrow 0, C \rightarrow 0 \text{ as } y \rightarrow \infty \tag{2.11}$$

**3. Solution of problem**

To reduce equation (2.8) to (2.10) to ordinary differential equations, we follow:

$$f(y_1) = f_0(y) + E_c f_1(y) \tag{3.1}$$

Where f stands for u, θ, and C. With the help of equation (3.2), the equation (2.8) to (2.10) reduce to the following differential equations on equating like powers of ε.

$$u_0'' + u_0' - \left( M^2 + \frac{1}{K_0} \right) u_0 = -G_r \theta_0 - G_c C_0 \tag{3.2}$$

$$u_1'' + u_1' - \left( M^2 + \frac{1}{K_0} \right) u_1 = -G_r \theta_1 - G_c C_1 - u_0' \tag{3.3}$$

$$\theta_0'' + (1 + Q)P_r \theta_0' - R P_r \theta_0 = 0 \tag{3.4}$$

$$\theta_1'' + (1 + Q)P_r \theta_1' - \left( R - \frac{n}{4} \right) P_r \theta_1 = -P_r \theta_0' \tag{3.5}$$

$$C_0'' + S_c C_0' - \frac{n}{4} S_c K_c C_0 = -S_c S_0 \theta_0'' \tag{3.6}$$

$$C_1'' + S_c C_1' + S_c C_1 = -S_c S_0 C_0' - S_c S_0 \theta_1'' \tag{3.7}$$

The boundary conditions (2.11) now become:

$$\begin{cases} y = 0 : u_0 = \alpha + h_1 u_0', u_1 = \alpha + h_1 u_1', \theta_0 = 1, \theta_1 = 0, C_0 = 1, C_0 = 0 \\ y \rightarrow \infty : u_0 = 0, u_1 = 0, \theta_0 = 0, \theta_1 = 0, C_0 = 1, C_0 = 0 \end{cases} \tag{3.8}$$

Solving (3.2)-(3.7) under the boundary condition (3.8) and replacing the solutions obtained in (3.1), we have

$$u(y, t) = B_5 e^{-m_3 y} - B_4 e^{-m_2 y} + B_3 e^{-m_1 y} + E_c [B_{16} e^{-m_6 y} - B_{11} e^{-m_5 y} + B_{12} e^{-m_4 y} + B_{13} e^{-m_3 y} - B_{14} e^{-m_2 y} + B_{15} e^{m_1 y}] \dots \tag{3.9}$$

$$\theta(y, t) = e^{-m_1 y} + E_c [-B_6 e^{-m_4 y} + B_6 e^{-m_1 y}] \tag{3.10}$$

$$C(y, t) = (B_2 e^{-m_2 y} - B_1 e^{-m_1 y}) + E_c (B_{10} e^{-m_5 y} + B_7 e^{-m_4 y} + B_8 e^{-m_2 y} - B_9 e^{-m_1 y}) \tag{3.11}$$

**4. Skin Friction**

The expression for the skin friction at the plate is:

$$\tau = \left(\frac{du}{dy}\right)_{y=0} = -B_5m_3 + B_4m_2 - B_3m_1 + E_c(B_{16}m_6 + B_{11}m_5 - B_{12}m_4 - B_{13}m_3 - B_{14}m_2 - B_{15}m_1) \quad \dots (4.1)$$

**5. Nusselt Number**

The expression for the Nusselt number is:

$$Nu = \left(\frac{dT}{dy}\right)_{y=0} = -m_1 + E_c(B_6m_4 - B_6m_1) \quad \dots (5.1)$$

**Table 1:** Effects of various physical parameters on skin friction coefficient for  $P_r = 0.71, Q = 1.0, R = 5.0, S_c = 0.22, S_o = 0.5$ .

$G_r$	$G_c$	$M$	$K_o$	$C_f$	$N_u$
5.0	6.0	5.0	1.0	0.4885	-2.4657
10.0	6.0	5.0	1.0	0.5407	-2.3567
10.0	6.0	5.0	1.0	0.5929	-2.3452
5.0	8.0	5.0	1.0	0.5976	-2.3568
10.0	10.0	5.0	1.0	0.7076	-2.3211
10.0	6.0	6.0	1.0	0.4287	-2.3778
10.0	6.0	7.0	1.0	0.3809	-2.3996
10.0	6.0	5.0	3.0	0.5377	-2.3552
10.0	6.0	5.0	5.0	0.6486	-2.3549

**Table 2:** Effects of various physical parameters on the nusselt number  $G_r = 10, G_c = 6, M = 5, K_o = 1, S_c = 0.22, Q = 1.0, R = 6.0,$  and  $S_o = 0.5$

$P_r$	$Q$	$R$	$C_f$	$N_u$
0.71	1.0	10	0.4885	-2.3652
7.0	1.0	10	0.4443	-3.2170
7.0	1.0	10	0.4181	-4.5461
0.71	3.0	10	0.4413	-3.3312
0.71	5.0	10	0.4208	-4.4323
0.71	1.0	6	0.5174	-1.7833
0.71	1.0	8	0.5013	-2.1874

**Appendix**

$$m_1 = \frac{(1 + Q)P_r + \sqrt{(1 + Q)^2P_r^2 + 4RP_r}}{2}, m_2 = \frac{S_c + \sqrt{S_c^2 + 4S_c}}{2}$$

$$m_{3,6} = \frac{1 + \sqrt{1 + 4\left(M^2 + \frac{1}{K_o}\right)}}{2}, m_4 = \frac{(1 + Q)P_r + \sqrt{(1 + Q)^2P_r^2 + 4\left(R - \frac{n}{4}\right)P_r}}{2}$$

$$m_5 = \frac{S_c + \sqrt{S_c^2 + 4\left(-\frac{n}{4}\right)S_c}}{2}, B_1 = \frac{S_c S_o m_1^2}{m_1^2 - S_c m_1 - S_c}$$

$$B_2 = 1 + B_1, B_3 = \frac{G_c B_1 - G_r}{m_1^2 - m_1 - \left(M^2 + \frac{1}{K_o}\right)}$$

$$B_4 = \frac{G_c B_2}{m_2^2 - m_2 - \left(M^2 + \frac{1}{K_o}\right)}, B_5 = \frac{B_4 \left(\frac{\alpha}{1 + h_1 m_2}\right) - B_3 \left(\frac{\alpha}{1 + h_1 m_1}\right)}{\left(\frac{\alpha}{1 + h_1 m_3}\right)}$$

$$B_6 = \frac{P_r m_1}{m_1^2 - (1 + Q)P_r m_1 - \left(R - \frac{n}{4}\right)P_r}, B_7 = \frac{S_o S_c B_6 m_4^2}{(m_4^2 - S_c m_4 + \frac{n}{4} S_c)}$$

$$B_8 = \frac{S_c B_2 m_2}{(m_2^2 - S_c m_2 + \frac{n}{4} S_c)}, B_9 = \frac{S_o S_c B_6 m_1^2 + S_c m_1 B_1}{(m_1^2 - S_c m_1 + \frac{n}{4} S_c)}$$

$$B_{10} = B_9 - B_8 - B_7, B_{11} = \frac{G_c B_{10}}{m_5^2 - m_5 - \left(M^2 + \frac{1}{K_o}\right)}$$

$$B_{12} = \frac{G_c B_6 - G_c B_7}{m_4^2 - m_1 - \left(M^2 + \frac{1}{K_0}\right)}, B_{13} = \frac{B_5 m_3}{m_3^2 - m_3 - \left(M^2 + \frac{1}{K_0}\right)}$$

$$B_{14} = \frac{G_c B_8 - m_2 B_4}{m_2^2 - m_2 - \left(M^2 + \frac{1}{K_0}\right)}, B_{15} = \frac{-G_r B_6 + G_c B_9 - m_1 B_3}{m_1^2 - m_1 - \left(M^2 + \frac{1}{K_0}\right)}$$

$$B_{16} = \frac{B_{11}(1+h_1 m_5) - B_{12}(1+h_1 m_4) - B_{13}(1+h_1 m_3) + B_{14}(1+h_1 m_2) - B_{15}(1+h_1 m_1)}{(1+h_1 m_6)},$$

$$B_{17} = -m_3 B_5 + m_2 B_4 - m_1 B_3,$$

$$B_{18} = -m_6 B_{16} + m_5 B_{11} - m_4 B_{12} - m_3 B_{13} + m_2 B_{14} - m_1 B_{15}$$

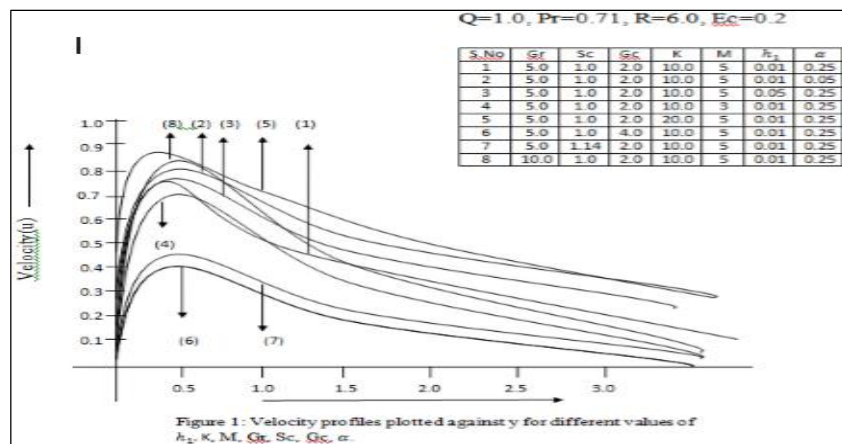
$$B_{19} = m_4 B_6 - m_1 B_6$$

$$B_{20} = -m_2 B_2 + m_1 B_1$$

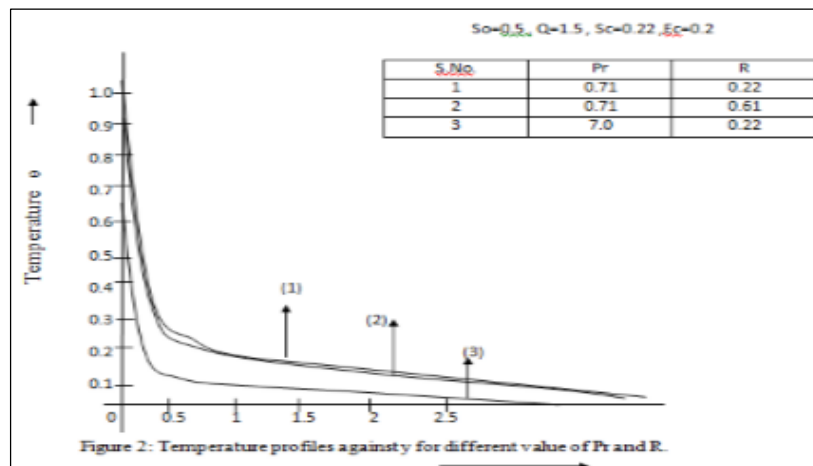
$$B_{21} = -m_5 B_{10} - m_4 B_7 - m_2 B_8 + m_1 B_9$$

**6. Results and Discussion**

In order to get physical insight of the problem, calculations have been made for velocity and temperature profiles, species concentration function, skin friction and Nusselt number for different values of the parameters entered into the problem viz.  $K_0$  (Permeability parameter),  $M$  (Magnetic parameter),  $h_1$  (Velocity slip parameter),  $G_r$  (Grashoff number),  $P_r$  (Prandtl number),  $S_c$  (Schmidt number),  $\alpha$  (Moving parameter),  $R$  (Radiation parameter) and  $G_c$  (Grashoff number for mass transfer). In figure 1, the velocity distribution is plotted against  $y$  for fixed values  $P_r=0.71$  (air as a fluid),  $R=6.0$ ,  $Q=1.0$ , and  $E_c=0.2$ . We observe that velocity increases with the increase in  $K_0$ , but decreases with increase in  $M$ ,  $R$ ,  $S_c$  and  $P_r$ . It is also being observed that the velocity increases rapidly near the plate and then decreases slowly far away from the plate. Temperature distribution, which depends only on  $R$ , is shown in figure 2. It is clear from the figure 2, that the temperature decreases with the increase of  $P_r$  and  $R$ . For the case of the heat sink ( $R < 0$ ) the temperature of the fluid is less as compared to heat source ( $R > 0$ ).



**Fig 1:** Velocity profiles plotted against  $y$  for different values of  $h_2, K, M, Gr, Sc, G_c, \alpha$ .



**Fig 2:** Temperature profiles against  $y$  for different value of  $Pr$  and  $R$ .

The concentration profile (C) is shown in figure 3 against y for different values of  $P_r$  and  $S_c$ . It is concluded that increase in  $S_c$  decreases species concentration. It is also observed that for water ( $P_r=7.0$ ) taking  $S_c=2.7$  species concentration further decreases. Another important parameter viz. the Nusselt number (Nu) at the plate is plotted against radiation parameter (R) in figure 4. It follows that Nusselt number increases when  $P_r$  increases.

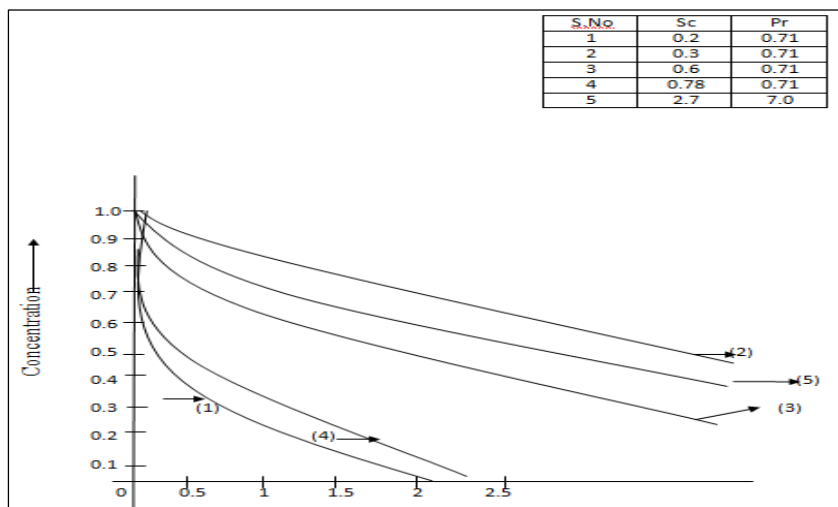


Fig 3: Concentration profiles against y for different value of Pr and Sc.

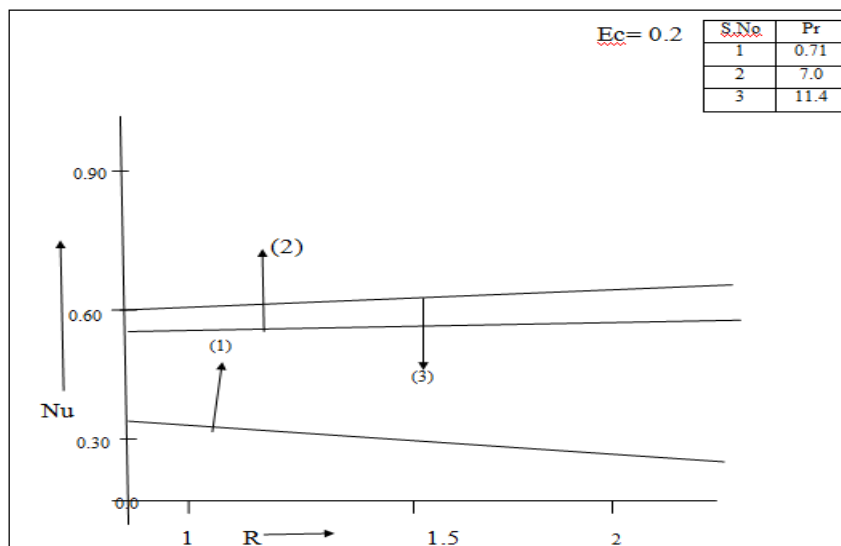


Fig 4: Nusselt number plotted against R for different values of Pr.

We observe from figure 5 that important parameter viz. skin friction ( $\tau$ ) increases when  $Q$ ,  $G_c$  and  $G_r, h_1$  are increased but decreases with increase in  $K_0$ .

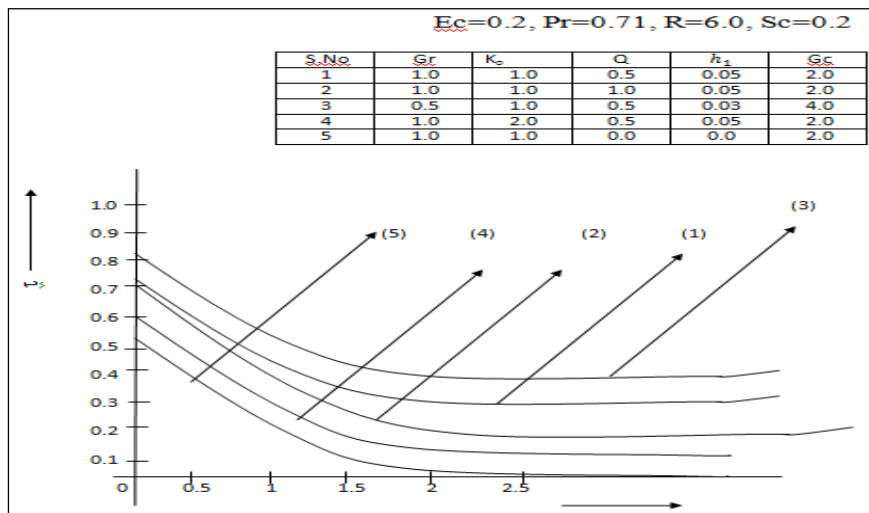


Fig 5: Skin - friction r against M for different value of Gr, K, Q,  $h_1$   $G_c$ .

## 7. Reference

1. Alam MS, Rahman MM, Sattar MA. Effect of variable suction and thermophoresis on steady MHD combined free-forced convective journal of thermal sciences. 2008; 47:758-765.
2. Alam MS, Rahman MM, Sattar MA. Numerical study of the combined free-forced convection and mass transfer flow past a vertical porous plate in a porous medium with heat generation and thermal diffusion, Nonlin. Anal. Model. Control. 2006; 11:331-343.
3. Chamkha AJ. Thermal radiation and buoyancy effects on hydro magnetic flow over an accelerating permeable surface with heat source or sink, int. J Eng. Sci. 2000; 38:1699-1712.
4. Chaudhary RC, Arpita J. combined heat and mass transfer effects on MHD free convection flow past an oscillating plate embedded in porous medium, Rom. Journ. Phys. 2007; 52:502-524.
5. Chaudhary RC, Jain P. Hall Effect on MHD mixed convection flow of a viscoelastic fluid past an infinite vertical porous plate with mass transfer and radiation theoretical and applied mechanics. 2006; 33:281-309.
6. EL-Hakien MA. MHD oscillatory flow on free convection radiation through a porous medium with constant suction velocity, journal of magnetism and magnetic materials. 2000; 220:71-76.
7. Gupta M, Sharam S. MHD flow of viscous fluid through a porous medium bounded by an oscillating porous plate in slip flow regime. Acta ciencia indica. 1991; 17:389-394.
8. Raju MC, Reddy NA, Varma SVK. Analytical study of MHD free convective, dissipative boundary layer flow past a porous vertical surface in the presence of thermal Radiation, Chemical reaction and constant suction, Ain shams Engineering journal, 2014; 5:1361-1369.
9. Yazadi MH, Abdullah S, Hashim I, Sopian K. Effect of viscous dissipation on the slip MHD flow and heat transfer past a permeable surface with convective boundary condition, Energies. 2013; 4:2273-2294.
10. Venkateswarlu M, Ramana G, Reddy V, Lakshami DV. Diffusion-Thermo Effects on MHD Flow past an Infinite Vertical Porous Plate in the Presence of Radiation and Chemical Reaction, International Journal of Mathematical Archive. 2013; 4(9):39-51.
11. Ahmed N, Das KK. MHD mass transfer Flow Past a Vertical porous plate embedded in a porous medium in a slip flow regime with thermal radiation and chemical reaction, open journal of fluid dynamics. 2013; 3:230-239.
12. Pal D, Shiva kumara IS. Mixed convection heat transfer from a vertical heated plate embedded in a sparsely packed porous medium, International Journal of Applied Mechanics and Engineering. 2006; 11:929-939.
13. Prakash J, Bhanumathi D, Kumar AGV, Verma SVK. Diffusion-thermo and radiation effects on unsteady MHD flow through porous medium past an impulsively started infinite vertical plate with diffusion. Transp. Porous Media. 2013; 96:135-151.
14. Rajesh V, Verma SVK. Radiation and mass transfer effects on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. ARPN J Eng. Appl. Sci. 2009; 4:20-26
15. Sangapatnam S, Reddy NB, Vallampati RP. Radiation and mass transfer effects on MHD free convection flow past an impulsively started isothermal vertical plate with dissipation. Thermal Sci. 2009; 13:171-181.
16. Sudheer Babu M, Satya Narayan PV. Effects of the chemical reaction and radiation absorption on free convection flow through porous medium with variable suction in the presence of uniform magnetic field JP Journal of heat and mass transfer. 2009; 3:219-234.