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A common coupled fixed point theorem using E.A. like property in supernova spaces

K Sridevi, MVR Kameswari and DMK Kiran

Abstract
In this paper, we introduce a new space, named Supernova Space and provide an example. We prove a common coupled fixed point theorem for four maps satisfying the E. A. Like property and w-compatibility in this space. We also obtain some corollaries from this result.

Keywords: supernova space, center of supernova space, coupled fixed point, E. A. like property, W-compatible.

1. Introduction
A fixed point theorem was given by Brouwer [5] in 1912, after that the polish mathematician Stephan Banach [3] proved contraction mapping theorem and named Banach fixed point theorem in 1922. It is an important tool in the theory of metric spaces. Many authors studied, extended, generalized and improved this theory to generalized metric spaces like statistical metric spaces, Menger spaces, d-complete topological spaces, F-complete metric spaces, G-Metric spaces, Fuzzy Metric spaces, Quasi metric spaces, partial metric spaces, b-metric spaces, dislocated quasi b-metric spaces, complex valued metric spaces [6, 7, 8, 9, 11, 14, 16].

Bhaskar and Lakshmikantham [4] introduced the concept of coupled fixed points for a given partially ordered set$X$. After that Lakshmikantham and Ciric [10] defined coupled coincidence point and common coupled fixed point for a pair of maps and Samet *et al*. [12, 13] proved coupled fixed point theorems. K. P. R. Satstry *et al*. [15, 17, 18] proved some results on coupled fixed point theorems and G. V. R. Babu *et al*. [2] proved some results on coupled fixed point theorems in partially ordered metric spaces. Sumitra *et al*. [21] proved some results on coupled fixed point theorems in Fuzzy metric spaces.

K. Wadhwa *et al*. [21] introduced the notion of E. A. Like property in fuzzy metric spaces. Abbas *et al*. [1] introduced the notion of w-compatible mapping.

In this paper, we introduce a new space, named $supernova$ space and we extend the definition of E. A. Like property introduced by K. Wadhwa *et al*. [21] to $supernova$ space and prove a common coupled fixed point theorem for four maps satisfying w-compatibility in this new $supernova$ space. This paper accepted and presented in International Conference on Mathematical Sciences and Applications by D. M. K. Kiran [19].

2. Preliminaries:
Definition 2.1: Let X be non-empty set, $s \geq 1$ and $d: X \times X \longrightarrow R^+$ be a function. Consider the following conditions of d

(1.1.1) There exists a unique point x_0 such that$d(x, y) = d(y, x) = 0 \Longrightarrow x = y = x_0$.

(1.1.2)$d(x, y) \leq s[d(x, z) + d(z, y)]\ for\ all\ x, y, z \in X$.

Are satisfied. Then (X, d) is called $supernova$ space and s is called a parameter of(X, d), x_0 is called the centre of the $supernova$.

Example 2.2: Let $X = [0, 1]$ and$d(x, y) = |x - y|^2 + |x|$. Then d is a $supernova$ space with $s = 2$.

Definition 2.3: Let (X, d) be a $supernova$ space. A sequence $\{x_n\}$ in (X, d) is convergent if there exists some point $x \in X$ such that

$$\lim_{n\to\infty} d(x_n, x) = \lim_{n\to\infty} d(x, x_n) = 0$$

In this case, x is the called a limit of $\{x_n\}$ and
we write $x_n \longrightarrow x$ as $n \longrightarrow \infty$.

Definition 2.4: Let (X, d) be a $supernova$ space. A sequence $\{x_n\}$ in (X, d) is said to be a Cauchy sequence if $\lim_{n,m\to\infty} d(x_n, x_m) = \lim_{m,n\to\infty} d(x_m, x_n) = 0$

Note 2.5: if $x_n \longrightarrow x$ and $y_n \longrightarrow y$ as $n \longrightarrow \infty$ then $x = y$ and $x = y = x_0$.

Notation 2.6: Write $\Phi = \{\varphi \mid \varphi : \mathbb{R}^+ \longrightarrow \mathbb{R}^+, \varphi \ is \ continuous, incrasing \ and \ \varphi(t) < t \ for \ t > 0\}$. The following Lemma 1.7 can be easily established.

Lemma 2.7: Let (X, d) be a $supernova$ space and $\{x_n\}$ be convergent to $x \in X$ and $y \in X$. Then

$$\frac{1}{s} d(x, y) \leq \lim_{n\to\infty} inf \ d(x_n, y) \leq \lim_{n\to\infty} \sup d(x_n, y) \leq sd(x, y)$$

$$\frac{1}{s} d(y, x) \leq \lim_{n\to\infty} inf \ d(y, x_n) \leq \lim_{n\to\infty} \sup d(y, x_n) \leq sd(y, x)$$

Definition 2.8 [4]**:** An element $(x, y) \in X \times X$ is called a coupled fixed point of a mapping $P : X \times X \longrightarrow X$ if$P(x, y) = x \ and \ P(y, x) = y$.

Definition 2.9 [10]**:** An element $(x, y) \in X \times X$ is called a coupled coincidence point of a mapping $P : X \times X \longrightarrow X \ and \ R : X \longrightarrow X$ if$P(x, y) = Rx \ and \ P(y, x) = Ry$.

Definition 2.10 [10]**:** An element $(x, y) \in X \times X$ is called a coupled fixed point of the mappings $P : X \times X \longrightarrow X \ and \ R : X \longrightarrow X$ if$P(x, y) = Rx = x \ and \ P(y, x) = Ry = y$.

Definition 2.11 [1]**:** Let X be a non-empty set. The mappings $P : X \times X \longrightarrow X \ and \ R : X \longrightarrow X$ are called w – compatible if $R\big(P(x, y)\big) = P(Rx, Ry) \ and \ R\big(P(y, x)\big) = P(Ry, Rx)$ whenever there exist $x, y \in X$ such that$P(x, y) = Rx \ and \ P(x, y) = Ry$.
Now we extend the definition as E.A.Like property due to K. Wadhwa [21] to $supernova$ spaces.

Definition 2.12: Let (X, d) be a $supernova$ space and $P : X \times X \longrightarrow X \ and \ R : X \longrightarrow X$ be mappings. The pair (P, R) is said to satisfy common E.A.Like property if there exist sequence $\{x_n\}, \{y_n\}$ in X such that

$$\lim_{n\to\infty} P(x_n, y_n) = \lim_{n\to\infty} Rx_n = t$$

$$\lim_{n\to\infty} P(y_n, x_n) = \lim_{n\to\infty} Ry_n = t$$

$t \in R(X) \ or \ P(X \times X)$.

Definition 2.13: Let (X, d) be a $supernova$ space and $P, Q: X \times X \longrightarrow X$ and $R, S: X \longrightarrow X$ be mappings. The pairs (P, R) and (Q, S) satisfy common E. A. Like property, if there exist sequences $\{x_n\}, \{y_n\}, \{z_n\}, \{w_n\}$ in X such that

$$\lim_{n\to\infty} P(x_n, y_n) = \lim_{n\to\infty} Rx_n = \lim_{n\to\infty} Q(z_n, w_n) = \lim_{n\to\infty} Sz_n = t$$

$$\lim_{n\to\infty} P(y_n, x_n) = \lim_{n\to\infty} Ry_n = \lim_{n\to\infty} Q(w_n, z_n) = \lim_{n\to\infty} Sw_n = t$$

Where$t \in R(X) \cap S(X) \ or \ P(X \times X) \cap Q(X \times X)$.

Note 2.14: In this case, we note that $t = x_0$ is the center of the(X, d).

3. Main Results

Theorem 3.1: Let (X, d) be a $supernova$ space with centre x_0 and parameter $s \geq 1$ and $P, Q: X \times X \longrightarrow X$ and $R, S: X \longrightarrow X$ be mapping satisfying

(2.1.1)

$$d(P(x,y),Q(u,v)) \le \varphi\left(\frac{1}{s^2} max \left\{\begin{array}{c} d(Rx,Su), d(Ry,Sv), \frac{1}{2}d(Rx,P(x,y)), \\ \frac{1}{2}d(Ry,P(y,x)), \frac{1}{2}d(Su,Q(u,v)), \frac{1}{2}d(Sv,Q(v,u)), \\ d(Rx,Q(u,v)), d(Ry,Q(v,u)), d(Su,P(x,y)), d(Sv,P(y,x)) \end{array}\right\}\right)$$

(2.1.2)

$$d(Q(x,y),P(u,v)) \le \varphi\left(\frac{1}{s^2} max \left\{\begin{array}{c} d(Sx,Ru), d(Sy,Rv), \frac{1}{2}d(Sx,Q(x,y)), \\ \frac{1}{2}d(Sy,Q(y,x)), \frac{1}{2}d(Ru,P(u,v)), \frac{1}{2}d(Rv,P(v,u)), \\ d(Sx,P(u,v)), d(Sy,P(v,u)), d(Ru,Q(x,y)), d(Rv,Q(y,x)) \end{array}\right\}\right)$$

$for\ all\ x,y,u,v \in X\ and\ for\ some\ \varphi \in \Phi$
(2.1.3) the pairs (P,R) and (Q,S) satisfy common E. A. Like property and
(2.1.4) the pairs (P,R) and (Q,S) are$w - compatible$.
Then the centre x_0 is the common unique coupled fixed point of P,Q,R,S in$X \times X$.

Proof: Since the pairs (P,R) and (Q,S) satisfy common E. A. Like property, there exist sequence $\{x_n\},\{y_n\},\{z_n\},\{w_n\}$ in X such that

$$\lim_{n\to\infty} P(x_n,y_n) = \lim_{n\to\infty} Rx_n = \lim_{n\to\infty} Q(z_n,w_n) = \lim_{n\to\infty} Sz_n = t --- (2.1.5)$$

$$\lim_{n\to\infty} P(y_n,x_n) = \lim_{n\to\infty} Ry_n = \lim_{n\to\infty} Q(w_n,z_n) = \lim_{n\to\infty} Sw_n = t --- (2.1.6)$$

For some$t \in R(X) \cap S(X)\ or\ P(X \times X) \cap Q(X \times X)$.

We observe that$t = x_0$, form note after definition 1.4.
Without loss of generality assume that$t \in R(X) \cap S(X)$. Since

$$\lim_{n\to\infty} Rx_n = t \in R(X)\ and\ \lim_{n\to\infty} Rx_n = t \in R(X) --- (2.1.7)$$

There exist $u,v \in X$ such that $t = Ru\ and\ Sv = t$ ----(2.1.8)

Now from Lemma 1.7, (2.1.1), for infinity many$n$, we have,

$$\frac{1}{s}d(t,P(u,u)) \le d(Q(z_n,w_n),P(u,u))$$

$$\le \varphi\left(\frac{1}{s^2} max \left\{\begin{array}{c} d(Sz_n,Ru), d(\ Sw_n,Ru), \frac{1}{2}d(Sz_n,Q(z_n,w_n)), \\ \frac{1}{2}d(Sw_n,Q(w_n,z_n)), \frac{1}{2}d(Ru,P(u,u)), \frac{1}{2}d(Ru,P(u,u)), \\ d(Sz_n,P(u,u)), d(Sw_n,P(u,u)), d(Ru,Q(z_n,w_n)), d(Ru,Q(w_n,z_n)) \end{array}\right\}\right)$$

$$= \varphi\left(\frac{1}{s^2} max \left\{\begin{array}{c} d(Sz_n,t), d(Sw_n,t), s\ max\{d(Sz_n,t), d(t,Q(z_n,w_n))\}, \\ s\ max\{d(Sw_n,t), d(t,Q(w_n,z_n))\}, \frac{1}{2}d(t,P(u,u)), \frac{1}{2}d(t,P(u,u)), \\ d(Sz_n,P(u,u)), d(Sw_n,P(u,u)), d(t,Q(z_n,w_n)), d(t,Q(w_n,z_n)), \end{array}\right\}\right)$$

$$\le \varphi\left(\frac{1}{s^2} max \left\{\begin{array}{c} d(t,t), d(t,t), sd(t,t), sd(t,t), \frac{1}{2}d(t,P(u,u)), \\ \frac{1}{2}d(t,P(u,u)), sd(t,P(u,u)), sd(t,P(u,u)), d(t,t), d(t,t), \end{array}\right\}\right)$$

$$\le \varphi\left(\frac{1}{s^2}\left(s\ d(t,P(u,u))\right)\right) = \varphi\left(\frac{1}{s}\left(d(t,P(u,u))\right)\right)$$

$$\frac{1}{s}d(t,P(u,u)) \le \varphi\left(\frac{1}{s}\left(d(t,P(u,u))\right)\right) ----- (2.1.9)$$

Therefore, $d(t,P(u,u)) = 0$ ----(2.1.10)

Again for infinitely many n,

$$\frac{1}{s}d(P(u,u),t) \le d\big(P(u,u),Q(z_n,w_n)\big)$$

$$\le \varphi\left(\frac{1}{s^2}max\left\{\begin{array}{c} d(Ru,Sz_n),d(Ru,Sw_n),\frac{1}{2}d\big(Ru,P(u,u)\big), \\ \frac{1}{2}d\big(Ru,P(u,u)\big),\frac{1}{2}d\big(Sz_n,Q(z_n,w_n)\big),\frac{1}{2}d\big(Sw_n,Q(w_n,z_n)\big), \\ d\big(Ru,Q(z_n,w_n)\big),d\big(Ru,Q(w_n,z_n)\big),d\big(Sz_n,P(u,u)\big),d\big(Sw_n,P(u,u)\big) \end{array}\right\}\right)$$

$$= \varphi\left(\frac{1}{s^2}max\left\{\begin{array}{c} d(t,Sz_n),d(t,Sw_n),\frac{1}{2}d\big(t,P(u,u)\big),\frac{1}{2}d\big(t,P(u,u)\big), \\ s\,max\{d(Sz_n,t),d\big(t,Q(z_n,w_n)\big)\},s\,max\{d(Sw_n,t),d\big(t,Q(w_n,z_n)\big)\}, \\ d\big(t,Q(z_n,w_n)\big),d\big(t,Q(w_n,z_n)\big),d\big(Sz_n,P(u,u)\big),d\big(Sw_n,P(u,u)\big) \end{array}\right\}\right)$$

$$\le \varphi\left(\frac{1}{s^2}max\left\{\begin{array}{c} d(t,t),d(t,t),\frac{1}{2}d\big(t,P(u,u)\big),\frac{1}{2}d\big(t,P(u,u)\big),sd(t,t), \\ sd(t,t),d(t,t),d(t,t),sd\big(t,P(u,u)\big),sd\big(t,P(u,u)\big) \end{array}\right\}\right)$$

$$\le \varphi\left(\frac{1}{s^2}\Big(sd\big(t,P(u,u)\big)\Big)\right) = \varphi\left(\frac{1}{s}\Big(d\big(t,P(u,u)\big)\Big)\right)$$

Therefore

$$\frac{1}{s}d(P(u,u),t) \le \varphi\left(\frac{1}{s}\Big(d\big(t,P(u,u)\big)\Big)\right) - - - - - (2.1.11)$$

Now from (2.1.10) and(2.1.11), we get

$d(P(u,u),t) = 0 - - - -(2.1.12)$

From(2.1.10), (2.1.12) and (1.1.1), we write

$d\big(t,P(u,u)\big) = d(P(u,u),t) = 0 \implies P(u,u) = t = x_0 - - - -(2.1.13)$

Now from Lemma 1.7, (2.1.2), for infinity many$n$, we have,

$$\frac{1}{s}d\big(t,Q(v,v)\big) \le d\big(P(x_n,y_n),Q(v,v)\big)$$

$$\le \varphi\left(\frac{1}{s^2}max\left\{\begin{array}{c} d(Rx_n,Sv),d(Ry_n,Sv),\frac{1}{2}d\big(Rx_n,P(x_n,y_n)\big), \\ \frac{1}{2}d\big(Ry_n,P(y_n,x_n)\big),\frac{1}{2}d\big(Sv,Q(v,v)\big),\frac{1}{2}d\big(Sv,Q(v,v)\big), \\ d\big(Rx_n,Q(v,v)\big),d\big(Ry_n,Q(v,v)\big),d\big(Sx_n,P(x_n,y_n)\big),d\big(Sy_n,P(y_n,x_n)\big) \end{array}\right\}\right)$$

$$= \varphi\left(\frac{1}{s^2}max\left\{\begin{array}{c} d(Rx_n,t),d(Ry_n,t),s\,max\{d(Rx_n,t),d\big(t,P(x_n,y_n)\big)\}, \\ s\,max\{d(Ry_n,t),d\big(t,P(y_n,x_n)\big)\},\frac{1}{2}d\big(Sv,Q(v,v)\big),\frac{1}{2}d\big(Sv,Q(v,v)\big), \\ d\big(t,Q(v,v)\big),d\big(t,Q(v,v)\big),d\big(Sx_n,P(x_n,y_n)\big),d\big(Sy_n,P(y_n,x_n)\big) \end{array}\right\}\right)$$

$$\le \varphi\left(\frac{1}{s^2}max\left\{\begin{array}{c} d(t,t),d(t,t),sd(t,t),sd(t,t),\frac{1}{2}d\big(t,Q(v,v)\big), \\ \frac{1}{2}d\big(t,Q(v,v)\big),sd\big(t,Q(v,v)\big),sd\big(t,Q(v,v)\big),d(t,t),d(t,t) \end{array}\right\}\right)$$

$$\le \varphi\left(\frac{1}{s^2}\Big(sd\big(t,Q(v,v)\big)\Big)\right) = \varphi\left(\frac{1}{s}\Big(d\big(t,Q(v,v)\big)\Big)\right)$$

Therefore

$\frac{1}{s} d\big(t, Q(v,v)\big) \leq \varphi\left(\frac{1}{s}\Big(d\big(t, Q(v,v)\big)\Big)\right) - - - - - (2.1.14)$

Therefore, $d\big(t, Q(v,v)\big) = 0 - - - - (2.1.15)$

Similarly,

$$\frac{1}{s} d(Q(v,v), t) \leq \varphi\left(\frac{1}{s}\Big(d\big(t, Q(v,v)\big)\Big)\right) - - - - - (2.1.16)$$

$from$ (2.1.15), (2.1.16)

$d(Q(v,v), t) = 0 - - - - (2.1.17)$

From(2.1.15), (2.1.17)and (1.1.1),

$d\big(t, Q(v,v)\big) = d(Q(v,v), t) = 0 \implies Q(v,v) = t = x_0 - - - - (2.1.18)$

From (2.1.8), (2.1.3), (2.1.18)

$P(u,u) = t = Ru - - - - (2.1.19)$

$Q(v,v) = t = Sv - - - - (2.1.20)$

Since the pairs (P,R) and (Q,S) are$w - compatible$, we have

$$R\big(P(u,u)\big) = P(Ru, Ru) = P(t,t) \Rightarrow Rt = P(t,t) - - - - (2.1.21)$$
$$S\big(Q(v,v)\big) = Q(Sv, Sv) = Q(t,t) \Rightarrow St = Q(t,t) - - - - (2.1.22)$$

Consider,

$$d(Rt, St) = d\big(P(t,t), Q(t,t)\big)$$

$$\leq \varphi\left(\frac{1}{s^2} max\left\{\begin{array}{c} d(Rt,St), d(Rt,St), \frac{1}{2} d\big(Rt, P(t,t)\big), \\ \frac{1}{2} d\big(Rt, P(t,t)\big), \frac{1}{2} d\big(St, Q(t,t)\big), \frac{1}{2} d\big(St, Q(t,t)\big), \\ d\big(Rt, Q(t,t)\big), d\big(Rt, Q(t,t)\big), d\big(St, P(t,t)\big), d\big(St, P(t,t)\big) \end{array}\right\}\right)$$

$$= \varphi\left(\frac{1}{s^2} max\left\{\begin{array}{c} d(Rt,St), d(Rt,St), \frac{1}{2} d(Rt,Rt), \\ \frac{1}{2} d(Rt,Rt), \frac{1}{2} d(St,St), \frac{1}{2} d(St,St), \\ d(Rt,St), d(Rt,St), d(St,Rt), d(St,Rt) \end{array}\right\}\right)$$

$$= \varphi\left(\frac{1}{s^2} max\left\{d(Rt,St), d(St,Rt), \frac{1}{2} d(Rt,Rt), \frac{1}{2} d(St,St)\right\}\right)$$

Therefore, $d(Rt, St) \leq \varphi\left(\frac{1}{s^2} max\{d(Rt,St), d(St,Rt), sd(Rt,St), sd(St,Rt)\}\right)$

Therefore, $d(Rt, St) \leq \varphi\left(\frac{1}{s} max\{d(Rt,St), d(St,Rt)\}\right) - - - - (2.1.23)$

Similarly,

$$d(St, Rt) \leq \varphi\left(\frac{1}{s} max\{d(Rt,St), d(St,Rt)\}\right) - - - - (2.1.24)$$

Therefore, $max\{d(Rt,St), d(St,Rt)\} \leq \varphi\left(\frac{1}{s} max\{d(Rt,St), d(St,Rt)\}\right)$

Therefore, $max\{d(Rt,St), d(St,Rt)\} = 0$

Therefore, $d(Rt, St) = d(St, Rt) = 0 - - - - (2.1.25)$

From(2.1.25) and (1.1.1), we get $St = Rt = t----(2.1.26)$

From(2.1.21), (2.1.22) and (2.1.26), we get

$P(t,t) = Rt = t$

$Q(t,t) = St = t$

Therefore, (t,t) is a common coupled fixed point of $P, Q, R, and\ S$.
Let (u,v) be a common coupled fixed point of $P, Q, R\ and\ S$. Then

$d(u,v) = d\big(P(u,v), Q(v,u)\big)$

$$\leq \varphi\left(\frac{1}{s^2} max\left\{\begin{array}{c} d(Ru,Sv), d(Rv,Su), \frac{1}{2}d\big(Ru,P(u,v)\big), \\ \frac{1}{2}d\big(Rv,P(v,u)\big), \frac{1}{2}d\big(Sv,Q(v,u)\big), \frac{1}{2}d\big(Su,Q(u,v)\big), \\ d\big(Ru,Q(v,u)\big), d\big(Rv,Q(u,v)\big), d\big(Sv,P(u,v)\big), d\big(Su,P(v,u)\big) \end{array}\right\}\right)$$

$$= \varphi\left(\frac{1}{s^2} max\left\{\begin{array}{c} d(u,v), d(v,u), \frac{1}{2}d(u,u), \\ \frac{1}{2}d(v,v), \frac{1}{2}d(v,v), \frac{1}{2}d(u,u), \\ d(u,v), d(v,u), d(v,u), d(u,v) \end{array}\right\}\right)$$

$$= \varphi\left(\frac{1}{s^2} max\{d(u,v), d(v,u), d(u,u), d(v,v)\}\right)$$

$$d(u,v) \leq \varphi\left(\frac{1}{s^2} max\{d(u,v), d(v,u), sd(u,v), sd(v,u)\}\right)$$

$$d(u,v) \leq \varphi\left(\frac{1}{s} max\{d(u,v), d(v,u)\}\right)----(2.1.19)$$

Similarly,
$$d(v,u) \leq \varphi\left(\frac{1}{s} max\{d(u,v), d(v,u)\}\right)----(2.1.20)$$

Therefore, $max\{d(u,v), d(v,u)\} \leq \varphi\left(\frac{1}{s} max\{d(u,v), d(v,u)\}\right)$

Therefore, $max\{d(u,v), d(v,u)\} = 0$

Therefore, $d(u,v) = d(v,u) = 0 \Rightarrow u = v = t = x_0$

Therefore, the center of $supernova$ space is unique common coupled fixed point of $P, Q, R\ and\ S$.

Corollary 3.2: Let (X,d) be a $supernova$ space with centre x_0 and parameter $s \geq 1$ and $P: X \times X \longrightarrow X$ and $R: X \longrightarrow X$ be mapping satisfying

(2.2.1)

$$d\big(P(x,y), P(u,v)\big) \leq \varphi\left(\frac{1}{s^2} max\left\{\begin{array}{c} d(Rx,Ru), d(Ry,Rv), \frac{1}{2}d\big(Rx,P(x,y)\big), \\ \frac{1}{2}d\big(Ry,P(y,x)\big), \frac{1}{2}d\big(Ru,P(u,v)\big), \frac{1}{2}d\big(Rv,P(v,u)\big), \\ d\big(Rx,P(u,v)\big), d\big(Ry,P(v,u)\big), d\big(Ru,P(x,y)\big), d\big(Rv,P(y,x)\big) \end{array}\right\}\right)$$

$for\ all\ x,y,u,v \in X\ and\ for\ some\ \varphi \in \Phi.$

(2.2.2) the pairs (P,R) satisfy common E. A. Like property and

(2.2.3) the pairs (P,R) are $w-compatible$.
Then the centre x_0 is the common unique coupled fixed point of P, Q in $X \times X$.

Corollary 3.3: Let (X, d) be a $supernova$ space with centre x_0 and parameter $s = 1$ and and $P, Q, R, S: X \longrightarrow X$ be mappings satisfying

(2.3.1) $d(Px, Qy) \leq \varphi\left(max\left\{d(Rx, Sy), \frac{1}{2}d(Rx, Px), \frac{1}{2}d(Sy, Qy), d(Rx, Qy), d(Sy, Px)\right\}\right)$

(2.3.2) $d(Qx, Py) \leq \varphi\left(max\left\{d(Sx, Ry), \frac{1}{2}\ d(Ry, Py), \frac{1}{2}d(Sx, Qx), d(Ry, Qx), d(Sx, Py)\right\}\right)$

$for\ all\ x, y \in X\ and\ for\ some\ \varphi \in \Phi$

(2.3.3) the pairs (P, R) and (Q, S) satisfy common E. A. Like property and
(2.2.4) the pairs (P, R) are weakly compatible.
Then the centre x_0 is the common unique fixed point of P, Q, R, S in$X$.

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