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Effect of rotation and non-uniform temperature gradients on the onset of Marangoni convection in a fluid with suspended particles under microgravity condition

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Abstract
The effect of six different non-uniform temperature gradients and rotation on the onset of Marangoni convection in a fluid which has suspended particles in it and confined between an upper free, adiabatic and lower rigid, isothermal boundaries under microgravity condition has been discussed using Linear Stability Analysis. The Eigen values are obtained by using the single term Galerkin Technique followed by matrix method. The microrotation is assumed to be vanished at the boundaries. The effect of Taylor number and different micropolar fluid parameters on the onset of stationary convection has been analyzed by neglecting the body force. Also the comparative influence of six various non-uniform temperature profiles on the onset of Marangoni convection has been studied.

Keywords: Micropolar Fluid, microgravity, non-uniform temperature gradients, Marangoni number, Taylor number

1. Introduction
The fluids with suspended particles which have microstructure are known as the micropolar fluids. Eringen [1] was the first person to develop the theory of such fluids. The particles or the elements which are present in the micropolar fluid possess both translation and rotational motion. Physically, these fluids may be thought of as fluids with dumb-bell shaped particles or molecules. Muddy fluids like crude oils, paint, toothpaste, blood, polymeric fluid suspension etc are the examples for micropolar fluids. So far there are very less researches done on the effect of rotation and non-uniform temperature gradients in a micropolar fluid under microgravity condition. Microgravity condition is the condition where the effect of gravity is neglected as it is very small [2].
Marangoni convection plays a vital role in the field of science and technology because of its applications. The existence of spontaneous interfacial convection was first reposited by Thomson [3] and Yong *et al.* [4]. By observing the spreading of an oil droplet on a water surface, Marangoni was able to conclude that the fluid with lower surface tension spreads on a fluid with higher surface tension and so the effect was named after him. Due to wide applications of Marangoni effect in day to day life people have been studying the science of Marangoni convection for a few decades; there are still many characteristics and properties of Marangoni convection that have not been fully understood. As space exploration became a more popular topic, more attention has been paid to the problems encountered in space where drops of immiscible fluid were found to have a great influence on numerous phenomenon such as heat and mass transfer, reaction rates and physical behaviour of fluids [5].
The most effective mechanism to control convection is by rotating the fluid along vertical axis or by maintaining a suitable non-uniform temperature profile. The literature incorporating this particular problem is actually concerned with the non-uniform basic temperature profiles, microrotation and a corresponding study for micropolar fluids. The effect of rotation on the onset of convection is missing in spite of understanding its importance in controlling the convection in many problems in the areas like Science and Technology.

The effect of non-uniform temperature gradients in micopolar fluids is analyzed by Siddheshwar and Pranesh [6], Pranesh and Riya [7]. Marangoni convection in Micropolar fluids and conducting fluids has been investigated by many authors (Rudraiah [8], Hashim [9], Murthy and Ramana Rao [10] and others). The effect of non-uniform temperature gradients on the onset of Marangoni convection has been studied in detail by Rudraiah and Siddheshwar [11] and Siddheshwar and Pranesh [2]. Also the effect of coriolis force on the onset of Marangoni convection has been studied by Hashim and Sarma [12] and Nanjundappa *et al.* [13] for various Newtonian and Non-newtonian fluids.

The aim of this work is to study the effect of rotation and different non-uniform basic temperature gradients on the onset of Marangoni convection in a layer of Non-newtonian fluid. The critical Marangoni number is determined by matrix method using the Galerkin Technique.

2. Mathematical Formulation

Consider a layer of a micropolar fluid confined between two, infinite horizontal boundaries of depth ‘h’. A Cartesian coordinate system is taken with the origin in the lower boundary and z-axis vertically up-wards. Let ΔT be the temperature difference between lower and upper boundaries of the fluid.

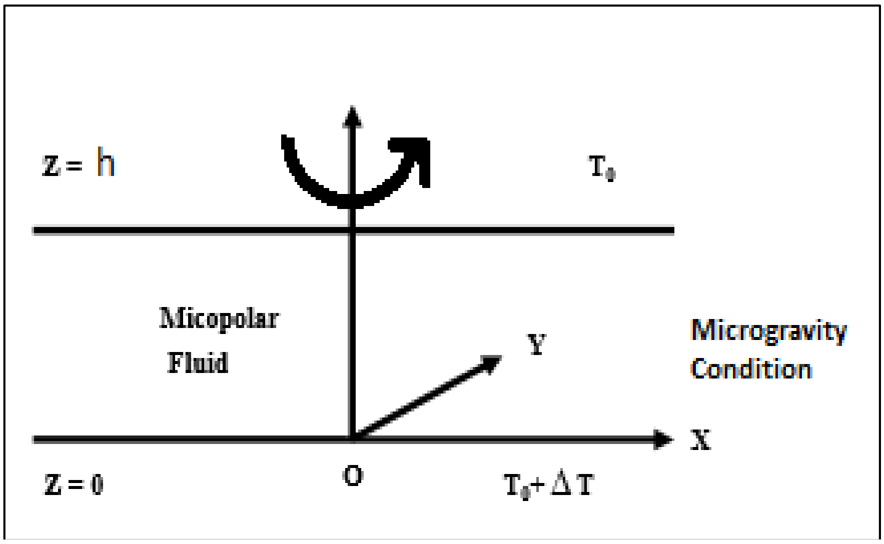

Fig 1: Physical Configuration

3. Governing Equations

The governing equations of this particular problem are as follows:

Equation of Continuity

$$\nabla \cdot \vec{q} \;=\; 0 \tag{1}$$

Equation of Conservation of Linear Momentum

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + (\vec{q}\cdot\nabla)\vec{q} + (2\vec{\Omega}\times\vec{q})\right] \;=\; -\nabla p + (2\zeta+\eta)\nabla^2\vec{q} + \zeta\nabla\times\vec{\omega} \tag{2}$$

Equation of Conservation of Angular Momentum

$$\rho_0 I\left\{\frac{\partial \vec{\omega}}{\partial t} + (\vec{q}\cdot\nabla)\vec{\omega} - \vec{\omega}\times\vec{\Omega}\right\} \;=\; (\lambda'+\eta')\nabla(\nabla\cdot\vec{\omega}) + \eta'\nabla^2\vec{\omega} + \zeta(\nabla\times\vec{q} - 2\vec{\omega}) \tag{3}$$

Equation of Energy

$$\frac{\partial T}{\partial t} + \left[\vec{q} - \frac{\beta}{\rho_0 C_v}(\nabla\times\vec{\omega})\right]\cdot\nabla T \;=\; \chi\nabla^2 T \tag{4}$$

where, $\vec{q}$ is the velocity, $\vec{\omega}$ is the spin, T is the temperature, p is the pressure, ρ_0 is the reference density of the fluid at reference temperature $T=T_0$, ζ is the coupling viscosity co-efficient or vortex viscosity, η is the shear kinematic viscosity co-efficient, I is the moment of inertia, $\lambda'$and $\eta'$are the bulk and shear spin viscosity coefficients respectively, β is the Micropolar heat conduction co-efficient, C_v is the specific heat, χ is the thermal conductivity, $\vec{\Omega}$ is the angular velocity and t is the time.

The boundary condition on temperature is given as T_b = T_0 + ΔT at z = 0 and T_b =T_0 at z = h. The interface at the upper boundary has a temperature dependent surface tension σ(T) (Pearson [14]) given by,

$$\sigma(T) = \sigma_0 - \sigma_1(T - T_0)$$

where, $\sigma_1 = \left(\frac{d\sigma}{dT}\right)_{T_0}$ $and\ \sigma_0 = \sigma(T_0)$

4. Basic State

The basic steady state is assumed to be quiescent and we consider the solution of the form:

$$\vec{q} = \vec{q}_b = (0,0,0), \vec{\omega} = \vec{\omega}_b = (0,0,0), T = T_b(z), p = p_b(z), \vec{\Omega} = \Omega_b \hat{k}, \frac{-h}{\Delta T}\left(\frac{dT_b}{dz}\right) = f(z) \tag{5}$$

where, f(z) is the non-dimensional basic temperature gradient satisfying the condition that $\int_0^1 f(z).dz = 1$.

The different non-uniform temperature gradients are shown in the below table:

Table 1: The six non-uniform temperature gradients

Model	Basic temperature gradients	f(z)
1.	Linear	1
2.	Heating from below	$\begin{cases} \epsilon^{-1} & 0 \le z \le \epsilon \\ 0 & \epsilon < z \le 1 \end{cases}$
3.	Cooling from above	$\begin{cases} 0 & 0 \le z < 1-\epsilon \\ \epsilon^{-1} & 1-\epsilon < z \le 1 \end{cases}$
4.	Step function	$\delta(z-\epsilon)$
5.	Inverted Parabolic	2(1-z)
6.	Parabolic	2z

where ϵ is the thermal depth and δ is the Dirac function. This type of basic temperature gradients arise due to radiation, sudden heating or cooling etc. [2].

5. Linear Stability Analysis

Let the basic state be disturbed by an infinitesimal thermal perturbation. We now have,

$$\vec{q} = \vec{q}_b + \vec{q}', \vec{\omega} = \vec{\omega}_b + \vec{\omega}', T = T_b(z) + T', p = p_b(z) + p', \vec{\Omega} = \overrightarrow{\Omega_b} + \vec{\Omega}' \tag{6}$$

where primes denote perturbed quantities and subscript 'b' indicates basic state value.

On substituting equation (6) in equations (1) to (4) and using basic state equations, we get the perturbed state equations as:

$$\nabla . \vec{q}' = 0 \tag{7}$$

$$\rho_0\left[\frac{\partial \vec{q}}{\partial t} + 2\left(\vec{\Omega}_b \times \vec{q}'\right)\right] = -\nabla p + (2\zeta + \eta)\nabla^2 \vec{q}' + \zeta \nabla \times \vec{\omega}' \tag{8}$$

$$\rho_0 I\{\frac{\partial \vec{\omega}'}{\partial t} - \vec{\omega}' \times \vec{\Omega}_b\} = (\lambda' + \eta')\nabla(\nabla \cdot \vec{\omega}') + \eta' \nabla^2 \vec{\omega}' + \zeta(\nabla \times \vec{q}' - 2\vec{\omega}') \tag{9}$$

$$\frac{\partial T'}{\partial t} + \left[\vec{q}' - \frac{\beta}{\rho_0 C_v}(\nabla \times \vec{\omega}')\right] \cdot \nabla T' = \left(\frac{\Delta T}{h}\right)[\vec{\omega}' - \frac{\beta}{\rho_0 C_v}(\nabla \times \vec{\omega}')]\, f(z) + \chi \nabla^2 T' \tag{10}$$

Operating curl once on equations (8) and (9) we get

$$\rho_0\left(\frac{\partial {\phi'}_r}{\partial t} - 2\vec{\Omega}_b \frac{\partial \overrightarrow{\omega'}}{\partial z}\right) = (2\zeta + \eta)\nabla^2 {\phi'}_r + \zeta\left(\frac{\partial \psi_r'}{\partial z} - {\nabla_1}^2 {\omega'}_z\right) \tag{11}$$

$$\rho_0 I\left(\frac{\partial \Omega_z}{\partial t} + \vec{\Omega}_b \psi_r'\right) = \eta' \nabla^2 \Omega_z + \zeta(-\nabla^2 \vec{\omega}' - 2\Omega_z) \tag{12}$$

Operating curl twice on equations (8) and (9) we get,

$$\rho_0\left(\frac{\partial(\nabla^2 \overrightarrow{\omega'})}{\partial t} + 2\vec{\Omega}_b \frac{\partial(\nabla \times \vec{q}')}{\partial z}\right) = (2\zeta + \eta)\nabla^4 w' + \zeta \nabla^2 (\nabla \times \overrightarrow{\omega'}) + \zeta\left(-\nabla^2 \phi_r' - 2\left(-{\nabla_1}^2 \omega_z' + \frac{\partial \psi_r'}{\partial z}\right)\right) \tag{13}$$

$$\rho_0 I\left(\frac{\partial}{\partial t}\left(-{\nabla_1}^2 \omega_z' + \frac{\partial \psi_r'}{\partial z}\right) - \vec{\Omega}_b \frac{\partial \Omega_z}{\partial t}\right) = (\lambda' + \eta')\nabla^2\left(-{\nabla_1}^2 \omega_z' + \frac{\partial \psi_r'}{\partial z}\right) + \zeta\left(-\nabla^2 \phi_r' - 2\left(-{\nabla_1}^2 \omega_z' + \frac{\partial \psi_r'}{\partial z}\right)\right) \tag{14}$$

where, $\phi_r = \nabla \times \vec{q}$, $\omega_z = \nabla \times \vec{\omega}$, $\psi_r = (\frac{\partial \omega_x}{\partial x} + \frac{\partial \omega_y}{\partial y})$

Non-dimensionalizing the perturbed state equations using the following definitions,

$$\left.\begin{aligned}(x^*, y^*, z^*) = \left(\frac{x}{h}, \frac{y}{h}, \frac{z}{h}\right), \nabla = \frac{\nabla^*}{h}, \overrightarrow{q^*} = \frac{\overrightarrow{q'}}{\chi/h}, \overrightarrow{\omega^*} = \frac{\overrightarrow{\omega'}}{\chi/h}, T^* = \frac{T'}{\Delta T},\\ \overrightarrow{\Omega^*} = \frac{\nabla\times\overrightarrow{\omega'}}{\frac{\chi}{h^3}}, {\varphi_r}^* = \frac{{\varphi_r}'}{\frac{\chi}{h^2}}, {\psi_r}^* = \frac{{\psi_r}'}{\frac{\chi}{h^3}}, t^* = \frac{t'}{\frac{h^2}{\chi}}\end{aligned}\right\}\text{ we get,}$$

$$(1 + N_1)\nabla^2 \varphi_r + N_1(D\psi_r - {\nabla_1}^2\omega_z) + \sqrt{Ta}(DW) = 0 \qquad (15)$$

$$\sqrt{Ta}(D\varphi_r) - N_1\nabla^2\Omega_z - (1 + N_1)\nabla^4 W = 0 \qquad (16)$$

$$(1/2)\sqrt{Ta}N_2\psi_r - N_3\nabla^2\Omega_z + N_1\nabla^2 W + 2N_1\Omega_z = 0 \qquad (17)$$

$$N_3\nabla^2(D\psi_r - {\nabla_1}^2\omega_z) - N_1\nabla^2\varphi_r - 2N_1(D\psi_r - {\nabla_1}^2\omega_z) + (1/2)\sqrt{Ta}N_2(D\Omega_z) = 0 \qquad (18)$$

$$N_4(D\psi_r) + N_4(\mathrm{D}^2\omega_z) + N_3\nabla^2\omega_z + N_1\varphi_r - 2N_1\omega_z = 0 \qquad (19)$$

$$\nabla^2 T + f(z)(W - N_5\Omega_z) = 0 \qquad (20)$$

where, * is neglected for simplicity and six non-dimensional parameters obtained are as follows:

$Ta = \frac{4{\Omega_b}^2{\rho_0}^2h^4}{(\zeta+\eta)^2}$ (Taylor Number)

$N_1 = \frac{\zeta}{\zeta+\eta}$ (Coupling Parameter)

$N_2 = \frac{I}{h^2}$ (Inertia Parameter)

$N_3 = \frac{\eta'}{(\zeta+\eta)h^2}$ (Couple Stress Parameter)

$N_4 = \frac{\lambda'+\eta'}{(\zeta+\eta)\,h^2}$ (Stress Parameter)

$N_5 = \frac{\beta}{\rho_0 C_v h^2}$ (Micropolar Heat Conduction Parameter)

6. Normal Mode Analysis

The infinitesimal perturbations W, Ω_z, φ_r, ψ_r, ω_z and T are assumed to be periodic waves and hence these permit a normal mode solution in the form,

$$[W, \Omega_z, \varphi_r, \psi_r, \omega_z, T] = [W(z), \Omega_z(z), \varphi_r(z), \psi_r(z), \omega_z(z), T(z)]\exp[i(lx + my)] \qquad (21)$$

where, l and m are the dimensionless wave number in the x and y directions respectively and σ is the growth rate.
Substituting (21) in equations (15) to (20), noting that the principle of exchange of stabilities is valid and hence assume only the stationary convection. We arrive at the following stability equations.

$$(1 + N_1)(\mathrm{D}^2 - \mathrm{a}^2)\varphi_r + N_1(D\psi_r) + N_1\mathrm{a}^2(\omega_z) + \sqrt{Ta}(DW) = 0 \qquad (22)$$

$$(1 + N_1)(\mathrm{D}^2 - \mathrm{a}^2)^2 W + N_1(\mathrm{D}^2 - \mathrm{a}^2)\Omega_z - \sqrt{Ta}(D\varphi_r) = 0 \qquad (23)$$

$$N_3(\mathrm{D}^2 - \mathrm{a}^2)\Omega_z - \left(\frac{1}{2}\right)\sqrt{Ta}N_2\psi_r - N_1(\mathrm{D}^2 - \mathrm{a}^2)W - 2N_1\Omega_z = 0 \qquad (24)$$

$$N_3(\mathrm{D}^2 - \mathrm{a}^2)(D\psi_r + \mathrm{a}^2\omega_z) - N_1(\mathrm{D}^2 - \mathrm{a}^2)\varphi_r - 2N_1(D\psi_r + \mathrm{a}^2\omega_z) + (1/2)\sqrt{Ta}N_2(D\Omega_z) = 0 \qquad (25)$$

$$N_4(D\psi_r) + N_4(\mathrm{D}^2\omega_z) + N_3(\mathrm{D}^2 - \mathrm{a}^2)\omega_z + N_1\varphi_r - 2N_1\omega_z = 0 \qquad (26)$$

$$(\mathrm{D}^2 - \mathrm{a}^2)T + f(z)(W - N_5\Omega_z) = 0 \qquad (27)$$

where, $\mathrm{a}^2 = \mathrm{l}^2 + \mathrm{m}^2$ and ‘a’ denotes the wave number with its horizontal components as l and m.
Equations (20) to (25) are solved subject to the following boundary conditions:

$$\left.\begin{array}{c} W = DW = T = \Omega_z = 0 \; at \; z = 0, \\ W = D^2W + a^2MT = DT = \Omega_z = 0 \; at \; z = 1 \end{array}\right\} \qquad (28)$$

where, $M = \frac{\sigma_T \Delta T h}{\mu\chi}$ (Marangoni Number)

The condition on Ω_z is the spin-vanishing at the boundaries.

On multiplying equations from (22) to (27) by $\varphi_r, W, \Omega_z, \psi_r, \omega_z$ and T respectively and by integrating with respect to z from 0 to 1, and substituting φ_r = Aφ_{r1}, W = BW_1, Ω_z = CΩ_{z1}, ψ_r = E ψ_{r1}, ω_z = Fω_{z1} and T = GT_1, where A, B, C, E, F, G are the trial functions which satisfy the boundary conditions, we get the following equations.

$$(1 + N_1) < \varphi_{r1}(D^2 - a^2)\,\varphi_{r1} > A + N_1 < \varphi_{r1}\,(D\psi_{r1}) > E + N_1 a^2 < \varphi_{r1}\omega_{z1} > F$$
$$+\sqrt{Ta} < \varphi_{r1}(DW_1) > B = 0 \qquad (29)$$

$$(1 + N_1) < W_1\left(D^2 - a^2\right)^2 W_1 > B + N_1 < W_1\left(D^2 - a^2\right)\Omega_{z1} > C - \sqrt{Ta} < W_1(D\varphi_{r1}) > A = 0 \qquad (30)$$

$$N_3 < \Omega_{z1}(D^2 - a^2)\Omega_{z1} > F - \left(\frac{1}{2}\right)\sqrt{Ta}N_2 < \Omega_{z1}\psi_r > C - N_1 < \Omega_{z1}(D^2 - a^2)\,W_1 >$$
$$- 2N_1 < \Omega_{z1}\Omega_{z1} > = 0 \qquad (31)$$

$$N_3 < \psi_{r1}\left(D^2 - a^2\right)D\,\psi_{r1} > E + N_3 a^2 < \psi_{r1}\left(D^2 - a^2\right)\omega_{z1} > F - N_1 < \psi_{r1}\left(D^2 - a^2\right)\varphi_{r1} > A$$
$$-2N_1 < \psi_{r1}D\psi_{r1} > E - 2N_1 a^2 < \psi_{r1}\omega_{z1} > F + (1/2)\sqrt{Ta}N_2 < \psi_{r1}(D\Omega_{z1}) > C = 0 \qquad (32)$$

$$N_4 < \omega_{z1}(D\psi_{r1}) > E + N_4 < \omega_{z1}(D^2\omega_{z1}) > F + N_3 < \omega_{z1}(D^2 - a^2)\omega_{z1} > F + N_1 < \omega_{z1}\varphi_{r1} > A$$
$$-2N_1 < \omega_{z1}\omega_{z1} > F = 0 \qquad (33)$$

$$< T_1(D^2 - a^2)T_1 > G + < f(z)W_1T_1 > B - N_5 < f(z)\Omega_{z1}T_1 > C = 0 \qquad (34)$$

where, <——> denotes integration with respect to z between 0 and 1.

Equations (27) to (32) can be written as YS = 0 (35)

where,

$$S = \begin{bmatrix} A\ B\ C\ E\ F\ G \end{bmatrix}^{\mathbf{T}}$$

$$Y = \begin{bmatrix}
(1+N_1)\,X_1 & (Ta^{1/2})\,X_4 & 0 & N_1\,X_2 & a^2N_1\,X_3 & 0 \\
-(Ta^{1/2})\,X_8 & (1+N_1)\,(X_{27}+2a^2X_{28}+a^4X_{29}) & N_1\,X_7 & 0 & 0 & -2\,a^2M \\
0 & -N_1\,X_{11} & N_3X_9 - 2N_1X_{10} & -(1/2)(Ta^{1/2})\,N_2X_{12} & 0 & 0 \\
-N_1\,X_{17} & 0 & (1/2)(Ta^{1/2})\,N_2\,X_{18} & N_3X_{13} - 2N_1X_{15} & a^2N_3X_{14} - 2a^2N_1X_{16} & 0 \\
N_1\,X_{23} & 0 & 0 & N_4\,X_{22} & N_4X_{19} - N_3X_{20} + 2N_1X_{21} & 0 \\
0 & X_6 & -N_1\,X_{25} & 0 & 0 & X_{24}
\end{bmatrix}$$

where,

$X_1 = < \varphi_{r1}\left(D^2 - a^2\right)\varphi_{r1} >$

$X_2 = < \varphi_{r1}\,(D\psi_{r1}) >$

$X_3 = < \varphi_{r1}\omega_{z1} >$

$X_4 = < \varphi_{r1}(DW_1) >$

$X_6 = < f(z)W_1T_1 >$

$X_7 = < W_1\left(D^2 - a^2\right)\Omega_{z1} >$

$X_8 = < W_1(D\varphi_{r1}) >$

$X_9 = < \Omega_{z1}\left(D^2 - a^2\right)\Omega_{z1} >$

$X_{14} = < \psi_{r1}\left(D^2 - a^2\right)\omega_{z1} >$

$X_{15} = < \psi_{r1}D\psi_{r1} >$

$X_{16} = < \psi_{r1}\omega_{z1} >$

$X_{17} = < \psi_{r1}\left(D^2 - a^2\right)\varphi_{r1} >$

$X_{18} = < \psi_{r1}(D\Omega_{z1}) >$

$X_{19} = < \omega_{z1}(D^2\omega_{z1}) >$

$X_{20} = < \omega_{z1}\left(D^2 - a^2\right)\omega_{z1} >$

$X_{21} = < \omega_{z1}\omega_{z1} >$

$X_{10} = <\Omega_{z1}\Omega_{z1}>$

$X_{11} = <\Omega_{z1}(D^2 - a^2)\, W_1>$

$X_{12} = <\Omega_{z1}\psi_r>$

$X_{13} = <\psi_{r1}(D^2 - a^2)D\, \psi_{r1}>$

$X_{27} = <(D^2W_1)^2>$

$X_{29} = <{W_1}^2>$

$X_{22} = <\omega_{z1}(D\psi_{r1})>$

$X_{23} = <\omega_{z1}\varphi_{r1}>$

$X_{24} = <T_1(D^2 - a^2)T_1>$

$X_{25} = <f(z)\Omega_{z1}T_1>$

$X_{28} = <(DW_1)^2>$

The trial functions satisfying the equation (35) are:

$W_1 = z^2(1 - z^2), T_1 = z(2 - z), \Omega_{z1} = z(1 - z),$

$\phi_{r1} = z(1 - z), \varphi_{r1} = z^2(3 - 2z), \omega_{z1} = z^2(3 - 2z)$

They satisfy all the boundary conditions except the one given by $D^2W + a^2MT = 0$ at z = 1, but the residual from this is included in the residual from the differential equations. Performing the integration, we can calculate the critical Marangoni number M_c which attains its minimum at ${a_c}^2$.

7. Results and Discussion

We consider the effect of rotation, micropolar fluid parameters and non-uniform basic temperature gradients on the onset of Marangoni convection in a micropolar fluid which are represented by Ta, N_1, N_2, N_3, N_4, N_5 and f(z) respectively. The results obtained in the study are depicted by the Figures 2-5 and Table-II.

In the case of the temperature profiles heating from below, cooling from above and step function, the least value of M_c is attained at thermal depth, ϵ = 0.93, ϵ = 0.43 and ϵ = 0.74 respectively.

Fig. 2 is the plot of critical Marangoni number, M_c versus coupling parameter, log N_1 for various non-uniform temperature profiles. With increase in N_1, M_c also increases. Increase in N_1 increases the concentration of microelements in the fluid. These microelements consume most of the energy of the system in order to develop the gyrational velocity. This stabilizes the system. Therefore, onset of convection is delayed with increase in N_1.

Fig. 3 is the plot of critical Marangoni number, M_c versus couple stress parameter, log N_3 for different non-uniform temperature gradients. Clearly M_c decreases with increase in N_3. Increase in N_3 indicates an increase in the couple stress of the fluid. This increase of couple stress causes the microrotation to be decreased and so the energy required for microrotation will be retained in the system making the system more unstable. It advances the convection.

Fig. 4 is the plot of critical Marangoni number, M_c versus micropolar heat conduction parameter log N_5 for different non uniform temperature profiles. It is clear that M_c increases with increase in N_5. As N_5 increases, the amount of heat induced into the system due to these microelements will be more. This reduces the heat transfer from bottom to top which stabilizes the system. Hence the convection is delayed with increase in N_5.

Fig. 5 is the plot of critical Marangoni number, M_c versus Taylor number, $Ta^{1/2}$ for different non-uniform temperature profiles. From the graph it is evident that M_c increases with $Ta^{1/2}$. When rotation is introduced into the system along vertical axis, the motion of the fluid in the horizontal direction will be predominant. This causes a decrease in the vertical motion which in turn makes the system more stable. Hence with the increase in $Ta^{1/2}$ the onset of convection is delayed.

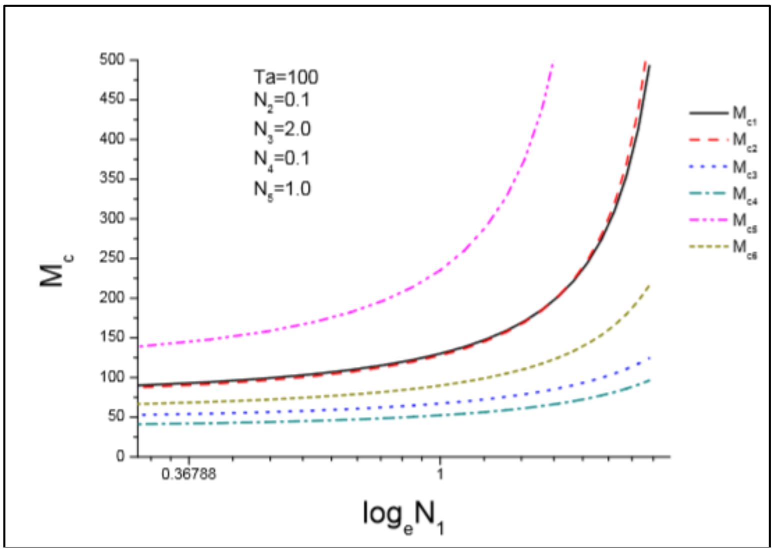

Fig 2: The graph of critical Marangoni number, M_c v/s coupling parameter, N_1 for different non-uniform temperature gradients

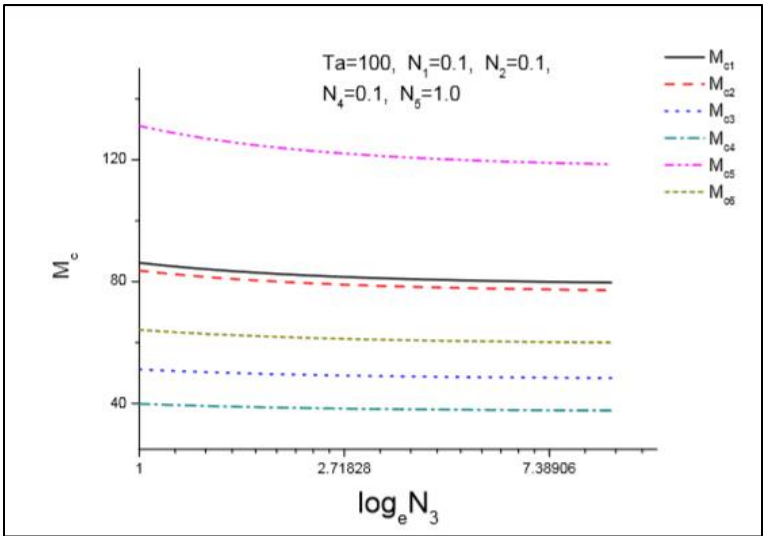

Fig 3: The graph of critical Marangoni number, M_c v/s couple stress parameter, N_3 for different non-uniform temperature gradients

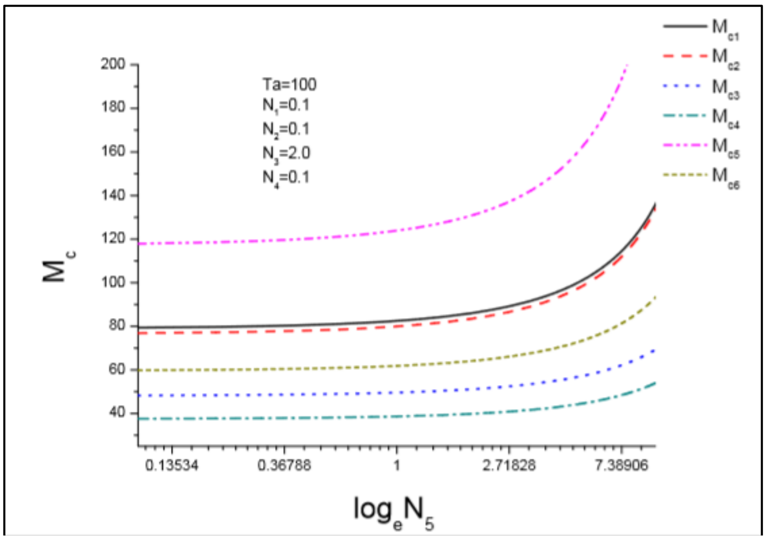

Fig 4: The graph of critical Marangoni number, M_c v/s micropolar heat conduction parameter, N_5 for different non-uniform temperature gradients

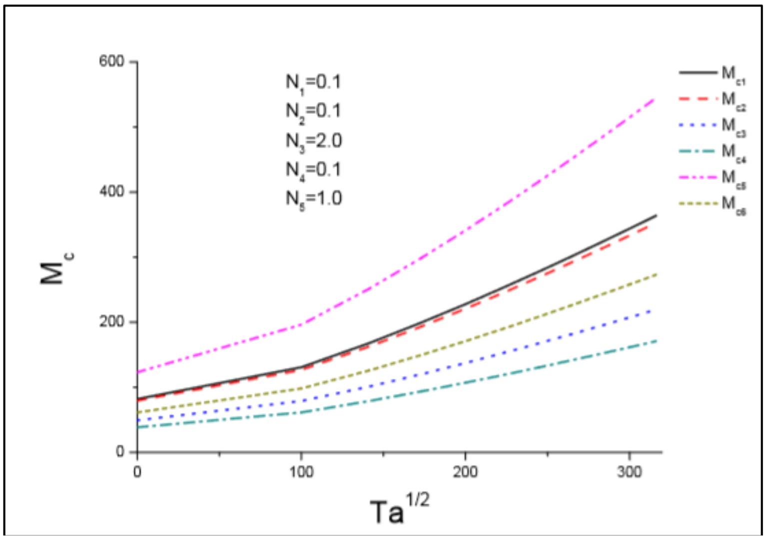

Fig 5: The graph of critical Marangoni number, M_c v/s Taylor number, $Ta^{1/2}$ for different non-uniform temperature gradients

Table 2: Values of Critical Marangoni Number, M_c with Respect to Different Values of Inertia Parameter, log N_2 and Stress Parameter, log N_4 for Different Non-Uniform Temperature Gradients

Non-uniform temperature gradients →	Linear	Heating from below	Cooling from above	Step function	Inverted parabolic	Parabolic
$\log_e N_2$	M_{c1}	M_{c2}	M_{c3}	M_{c4}	M_{c5}	M_{c6}
0	82.469369	79.933459	49.590514	38.648253	123.837081	61.818821
0.405465	82.510408	79.974090	49.608342	38.662095	123.916461	61.845161
0.693147	82.596346	80.059175	49.645664	38.691074	124.082746	61.900309
1.252762	83.248659	80.705056	49.928728	38.910861	125.346036	62.318700
$\log_e N_4$	M_{c1}	M_{c2}	M_{c3}	M_{c4}	M_{c5}	M_{c6}
0	82.495109	79.958942	49.601708	38.656944	123.886844	61.835349
0.693147	82.495446	79.959275	49.601855	38.657058	123.887495	61.835565
1.098612	82.495669	79. 959497	49.601952	38.657134	123.887928	61.835709
1.386294	82.495829	79.959654	49.602049	38.657188	123.888236	61.835852

Table-II is the table of values of the critical Marangoni number, M_c with respect to the inertia parameter, log N_2 and stress parameter, log N_4. From the above entries, it is very clear that as both N_2 and N_4 increase, M_c increases. Thus the parameters N_2 and N_4 stabilise the system. This shows that the onset of convection is delayed.

8. Conclusion

It is observed that for the critical Marangoni number, M_c the following inequality holds for the six models chosen in the study:
$M_{c4} < M_{c3} < M_{c6} < M_{c2} < M_{c1} < M_{c5}$.
i.e., the inverted parabolic temperature is the most stabilizing temperature profile and step function is the most destabilizing temperature profile. Hence by creating conditions for a suitable non-uniform basic temperature gradient, we can also make an appropriate decision in delaying or advancing the convection.
It also indicates that the convection in Non-newtonian fluids may be delayed by rotation and by adding micron sized suspended particles.

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