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A multivariate generalized arch model for time series analysis

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Abstract
In conventional Time series models, the variance of the error term is assumed to be constant. Under certain situations the assumption of constant variance is inappropriate. The residuals derived from an Autoregression or an Autoregressive Moving Average (ARMA) model or standard regression model with conditional variance of study variable in the time series analysis provides an usual Autoregressive Conditional Heteroscedastic (ARCH) model.
Eollersler (1986) extended ARCH model by developing a technique that allows the conditional variance to be an ARMA model. The Generalized ARCH (p,q) or GARCH (p,q) model allows for both autoregressive and moving average components in the heteroscedastic variance. The application of GARCH (p,q) model often enhances parsimony, because compared with a pure ARCH model, less parameters are needed for the description of the data.
This research paper specifies GARCH models in a multivariate setting

Keywords: Autoregressive Moving Average, heteroscedastic

1. Introduction
Time series is a stretch of values on the same scale indexed by a time like parameter. The basic data and parameters are functions. Time series take on a dazzling variety of shapes and forms indeed there are as many time series as there are functions of real numbers. Concepts related to time series include: Longitudinal data, growth curves, repeated measures, economic models, multivariate analysis, signal processing and system analysis.
A Multivariate Time Series Model provides an adequate unrestricted approximation to the reduced form of an unknown structural specification of a simultaneous equations model. Zellner and palm (1974) and Zellner (1979) have shown that any structural model can be written in the form of a Multivariate time series model.
Methods of time series analysis may be divided into linear and nonlinear; univariate & multivariate methods. In context of statistics, econometrics, quantitative finance, seismology, meteorology and geophysics, the primary goal of time series analysis is “forecasting” while in the context of Datamining, pattern recognition and machine learning, time series analysis can be used for clustering classification, query by content, anomaly detection as well as forecasting.

2. Vector Autoression (Var) Models
Sims (1980) suggested Vector Auto regression (VAR) models for forecasting macro time series. VAR assumes that all the variables are endogenous for instance, consider the following three macro series; money supply, interest rate and output. Vector of three independent variables as a (AR) function of its lagged values. If the number of lags (x) and number of equations (g) increase, then the degrees of freedom problem becomes more difficult. Generally, the number of parameters to be estimated becomes $g + xg^2$
For small samples, individual parameters may not be estimated. So, only simple VAR model can be considered for a small sample. The system of equations has the same set of variables in each equation SUR on the system is equivalent to OLS on each equation.
Under Normality of the disturbances, MLE as well as likelihood ratio test can be performed.

One important application of LR tests in the context of VAR is in determining the choice of lags to be used. In this case, the Log-likelihood for restricted model with m lags and the unrestricted model with q > m lags.

LR test is asymptotically distributed as $\chi^2_{(q-m)g^2}$. In case of sample size T (large) estimate the large number of parameters (qg^2+g) for the unrestricted model. VAR model have been used to test the hypothesis that some variables do not Granger cause some other variables.

2.1 A Simple Vector Auto Regression (VAR)

Consider the simple form as

$$y_{1t} = m_1 + a_{11}y_{1,t-1} + a_{12}y_{2,t-1} + \in_{1t}$$

$$y_{2t} = m_2 + a_{21}y_{1,t-1} + a_{22}y_{2,t-1} + \in_{2t}$$

$$y_t = m + Ay_{t-1} + \in_t \qquad \ldots (2.1)$$

Each variable is expressed as a linear combination of lagged values of itself. The VAR equations may be expanded to consider deterministic time trends and other exogeneous variable.

Where as in univariate case, the behaviour of y's will depend on the properties of the A matrix.

Let the eigen values and eigen vectors of a matrix A be

$$\wedge = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix} \quad c = \begin{bmatrix} \vdots & \vdots \\ c_1 & c_2 \\ \vdots & \vdots \end{bmatrix}$$

Provided that the eigen values are distinct. The eigen vectors are linearly independent and C will be non-singular.

Then

$$C^{-1}AC = \wedge \quad \text{And } A = C \wedge C^{-1} \qquad \ldots (2.2)$$

Let us consider a new vector of variables z_r as

$$Z_t = c^{-1}y_t \text{ Or } y_t = cZ_t \qquad \ldots (2.3)$$

By pre multiplying equation (2.1) by c^{-1} gives,

$$c^{-1}y_t = c^{-1}m + c^{-1}Ay_{t-1} + c^{-1}\in_t$$

$$\Rightarrow Z_t = m^* + \wedge Z_{t-1} + \eta_t \qquad \ldots (2.4)$$

Where $c^{-1}y_t = Z_t$; $c^{-1}m = m^*$ and $c^{-1}\in_t = \eta_t$ which is a white noise vector. Thus.

$$Z_{1t} = m_1^* + \lambda_1 Z_{1,t-1} + \eta_{1t}$$

$$Z_{2t} = m_2^* + \lambda_2 Z_{2,t-1} + \eta_{2t}$$

Each Z variable follows a separate AR (1) process and is stationary.

I (0), if the eigen values has modulus < 1 is a random walk with drift

I (1), if the eigen value is 1 and is explosive if the eigen value exceeds 1 in numerical value.

2.2 A Three variable Vector Auto Regression (VAR)

By expanding the system of a first order VAR to three variables. Suppose the eigen values of the A matrix are $\lambda_1 = 1$, $|\lambda_2| < 1$ and $|\lambda_3| < 1$. Thus there exists a (3x3) Nonsingular matrix C of eigen vectors A. By defining a three element Z vector as in equation (2.3) follows that

Z_{1t} is I (1); Z_{2t} and Z_{3t} are each I (0). If all 'Y' variables are I (1) then y vector may be expressed as

$$y_t = \begin{bmatrix} \vdots \\ c_1 \\ \vdots \end{bmatrix} Z_{1t} + \begin{bmatrix} \vdots \\ c_2 \\ \vdots \end{bmatrix} Z_{2t} + \begin{bmatrix} \vdots \\ c_3 \\ \vdots \end{bmatrix} Z_{3t}$$

Consider a linear combination of the y variables i.e. I (0), we need to eliminate Z_{1t} element. Let $c^{(2)}$ and $c^{(3)}$ denote the second and third rows of c^{-1}, two co-integrating relations are available in

$$Z_{2t} = c^{(2)} y_t \text{ And } Z_{3t} = c^{(3)} y_t \qquad \dots (2.5)$$

A linear combination of I (0) variables is itself I (0) Thus, any linear combination of the variables in equation (2.5) is also a co-integrating relation with an co-integrating vector. When two or more cointegrating vectors are found there is an infinity of cointegrating vectors.

We consider π matrix then the eigen values are

$$\pi = C(1 - \wedge)C^{-1}$$

$$= \begin{bmatrix} \vdots & \vdots & \vdots \\ c_1 & c_2 & c_3 \\ \vdots & \vdots & \vdots \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & \mu_2 & 0 \\ 0 & 0 & \mu_3 \end{bmatrix} \begin{bmatrix} \cdots & c^{(1)} & \cdots \\ \cdots & c^{(2)} & \cdots \\ \cdots & c^{(3)} & \cdots \end{bmatrix} \qquad \dots (2.6)$$

$$= \begin{bmatrix} \vdots & \vdots \\ \mu_2 c_2 & \mu_3 c_3 \\ \vdots & \vdots \end{bmatrix} \begin{bmatrix} \cdots & c^{(2)} & \cdots \\ \cdots & c^{(3)} & \cdots \end{bmatrix}$$

Thus π splits into the product of a (3x2) matrix of rank two. The matrix contains the two cointegrating vectors with which both cointegrating vectors enter into the Error Correction formulation for each Δy_i the

$$\Delta y_{it} = m_1 - (\mu_2 c_{12}) Z_{2,t-1} - (\mu_3 c_{13}) Z_{3,t-1} + \in_{1t}$$

$$\Delta y_{2t} = m_2 - (\mu_2 c_{22}) Z_{2,t-1} - (\mu_3 c_{23}) Z_{3,t-1} + \in_{2t}$$

$$\Delta y_{3t} = m_3 - (\mu_3 c_{32}) Z_{2,t-1} - (\mu_3 c_{33}) Z_{3,t-1} + \in_{3t}$$

The factorization of π is written

$$\pi = \alpha \beta^1$$

Where α and β are (3x2) matrices of rank two i.e., the rank of π is two and there are two cointegrating vectors. By substitution $\alpha \beta^1$ in.

$$\Delta y_t = m - \pi y_{t-1} + \in_t$$

i.e. $\Delta y_t = m - \alpha \beta^1 y_{t-1} + \in_t$

$$= \Delta y_t = m - \alpha Z_{t-1} + \in_t \qquad \dots (2.7)$$

Where $Z_{t-1} = \beta^1 y_{t-1}$ contains two cointegrating variables. Suppose that the eigen values are $\lambda_1 = \lambda_2 = 1$ and (λ_3) <1. Then it is possible to find a non-singular matrix p such that $P^{-1}AP = j$ where j is a Jordan matrix

$$J = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \lambda_3 \end{bmatrix}$$

By considering a three element vector $Z_t = P^{-1} y_t$ it follows that Z_1 is I(2), Z_2 is I(1) and Z_3 is I (0) generally all three variables are I(2), then

$$y_t = \begin{bmatrix} \vdots \\ p_1 \\ \vdots \end{bmatrix} Z_{1t} + \begin{bmatrix} \vdots \\ p_2 \\ \vdots \end{bmatrix} Z_{2t} + \begin{bmatrix} \vdots \\ p_3 \\ \vdots \end{bmatrix} Z_{3t}$$

Premultiplying by the second row of p^{-1} namely $P^{(2)}$ gives $P^{(2)} y_t = Z_{2t}$

Similarly $P^{(3)}$ gives $P^{(3)} y_t = Z_{3t}$. Which is I (0)

Thus there are two cointegrating vectors, but only one produces a stationary linear combination of y's. The eigen values are $\lambda_1 = 1,\ \lambda_2 = 1$ and $\lambda_3 = a$. Where the last eigen value is assumed to have modulus less than one. The first two Y variables are random walks with drift, and thus I (1) and the last equation in the VAR connects all three variables. So, that $y^{(3)}$ is I(1).

Then π matrix is

$$\pi = I - A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -1 & -1 & 1-a \end{bmatrix}$$

The rank of π is one, and it may be factorized as

$$\pi = \begin{bmatrix} 0 \\ 0 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 1 & a-1 \end{bmatrix}$$

When the row vector is the cointegrating vector. This result may be seen from

$$Z_t = y_{1t} + y_{2t} + (a-1) y_{3t}$$

$$= y_{1t} + y_{2t} + (a-1)\left(y_{1,t-1} + y_{2,t-1} + a y_{3,t-1} + m_3 + \in_{3t} \right)$$

$$= \Delta\, y_{1t} + \Delta y_{2t} + a Z_{t-1} + (a-1) m_3 + (a-1) \in_{3t}$$

$$= \text{constant } + a Z_{t-1} + v_t$$

Where $v_t = \in_{1t} + \in_{2t} + (a-1) \in_{3t}$

Thus Z_t follows a stable AR (1) process and I (0).

2.3 VAR Higher Order Systems

Consider Second Order System

$$y_t = m + A_1 y_{t-1} + A_2 y_{t-2} + \in_t \qquad \text{… (2.8)}$$

By subtracting y_{t-1} from both sides, we get

$$\Delta y_t = m + (A_1 - I) y_{t-1} + A_2 y_{t-2} + \in_t$$

The process of adding or subtracting (A_1 -I) y_{t-2} on the right side, we get

$$\Delta y_t = m + (A_1 - I) \Delta y_{t-1} - \pi y_{t-2} + \in_t \quad \text{… (2.9)}$$

Where $\pi = I - A_1 - A_2$

Similarly an alternative change in parameter is

$$\Delta y_t = m - A_2 \Delta y_{t-1} - \pi y_{t-1} + \in_t \quad \text{… (2.10)}$$

In second order system, there will be one lagged first difference term on the right hand side.
By continuing the procedure upto Var (p) system defined in equation (2.5) may be change in parameters. i.e,

$$\Delta y_t = m + B_1 \Delta y_{t-1} + B_{p-1} \Delta y_{t-p+1} - \pi y_{t-1} + \in_t \quad \text{… (2.11)}$$

Where the B's are the functions of the A's and $\pi = 1 - A_1 - .... - A_p$. The behaviour of the Y vector depends on the values of λ that solve

$$\left| \lambda^p I - \lambda^{p-1} A_1 - ...... - \lambda A_{p-1} - A_p \right| = 0$$

For explosive roots, consider three possibilities

(i) Rank (π) = k If each root has modulus less than one, π will have full rank and be non-singular. All the 'y' variables in equation (2.5) will be I (0) and unrestricted OLS estimates of equation (2.5) and equation (2.11) will give same inferences about the parameters.

(ii) Rank (π) = r < k This situation will occur if there is a unit root with multiplicity (k-r) and the remaining 'r' roots are numerically less than one.

(iii) Rank (π) =0. This is a special case. It will only occur if A_1+ ……+ A_p= I, in this case π =0 and equation (2.11) shows the VAR should be specified in terms of first differences of the variables.

3. Specification of Generalized Autoregressive Conditional Heteroscedasticity (Garch) Model

Consider the regression model with ARCH (m) disturbances as

$$Y_t = X_t^1 \beta + \in_t \quad \text{... (3.1)}$$

Such that

$$\in_t = \sqrt{h_t} v_t \quad \text{… (3.2)}$$

Where v_t is i.i.d. with zero mean and unit Variance; and

$$h_t = \delta_0 + \delta_1 \in_{t-1}^2 + \delta_2 \in_{t-2}^2 + ... + \delta_m \in_{t-m}^2 \quad \text{… (3.3)}$$

The Generalized ARCH model may be specified as

$$h_t = C_0 + C_1 h_{t-1} + C_2 h_{t-2} + ... + C_r h_{t-r} + \alpha_1 E_{t-1}^2 + \alpha_2 E_{t-2}^2 + ... + \alpha_m E_{t-m}^2 \quad \text{… (3.4)}$$

Hence, $C_0 = [1 - C_1 - C_2 - ... - C_r] \delta_0$

This model is usually denoted by

$$\in_t \sim GARCH(r, m) \quad \text{… (3.5)}$$

4. Specification of multivariate garch (r,m) model

Consider a set of n regression equations for time series involving GARCH (r,m) disturbances as

$$Y_{t_{(nx1)}} = F^{1}_{(nxk)} X_{t_{(kx1)}} + \in_{t_{(nx1)}}$$

Where X_t is a vector of explanatory variables;
$\in_t$ is a vector of white noise errors.
The conditional variance-covariance matrix of the errors is given by

$$H_t = E\left[\in_t \in_t^1 / Y_{t-1}, Y_{t-2}, ..., X_t, X_{t-1}, ...\right]$$

The multivariate generalization of GARCH (r,m) model may be specified as

$$H_t = C + D_1 \; H_{t-1} \; D_1^1 + D_2 \; H_{t-2} D_2^1 + ... + D_r H_{t-r} D_r^1 + A_1 \in_{t-1} \in_{t-1}^1 \; A_1^1 \; + A_2 \in_{t-2} \in_{t-2}^1 \; A_2^1 + ... + A_m \in_{t-m} \in_{t-m}^1 \; A_m^1$$

Where C, D_j and A_j for j = 1, 2 … denote (n x m) matrices of parameters.
One may assume that the conditional correlations among the elements of $\in_t$ are constant over time.
One may write H_t as

$$H_t = \left(\left(h_{ij}^{(t)}\right)\right)$$

Where, $h_{ii}^{(t)} = E\left[\in_{it}^2 / Y_{t-1}, Y_{t-2}, ... X_t, X_{t-1}, ...\right]$

$$h_{ii}^{(t)} = E\left[\in_{it} \in_{jt} \; / Y_{t-1}, Y_{t-2}, ... X_t, X_{t-1}, ...\right]$$

It should be noted that

$$h_{ij}^{(t)} = \rho_{ij} \sqrt{h_{ij}^{(t)}} \sqrt{h_{jj}^{(t)}}$$

Hence ρ_{ij} is the constant correlation coefficient.
The parameters of multivariate GARCH (r, m) model may be estimated by using the maximum likelihood method of estimation.

5. Conclusions

Multivariate Time Series Model Building involves five important steps. The first and foremost step is the Identification. The second step is the Specification of the model. The third step is the Estimation of parameters and testing the Hypotheses. The fourth step is the diagnostic checking and the last step is the Model to be used for Forecasting. An interesting development is the specification of Generalized ARCH models in a multivariate setting in the present study. The present study restricts to the case of series which may be seen as a linear transformation of a white noise error in the multivariate linear time series model.

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