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Inference in linear model with stochastic prior information constraints and compatibility tests

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Abstract
In the estimation of linear statistical model, generally the information contained in the sample observations on dependent variable may be used. In the applied econometric work, additional information namely prior information about unknown regression coefficients or the unknown error variance may be available for the estimation process. In this situation, the econometrician uses both the sample and other prior information on parameters in the estimation and testing. In this paper a new estimation method has been derived for linear regression model under stochastic prior information restrictions. A compatibility test statistic prior information on parameters of linear regression model.

Keywords: linear model, stochastic prior information constraints, econometrician

1. Introduction
In the estimation of linear statistical model, generally the information contained in the sample observations on dependent variable may be used. In the applied econometric work, additional information namely prior information about unknown regression coefficients or the unknown error variance may be available for the estimation process. In this situation, the econometrician used both the sample and other prior information on parameters in the estimation and testing process.

When there exists prior information about unknown regression coefficients in the linear statistical model, it may be usually incorporated into estimation of the parameters. Generally, the prior information may be interms of stochastic restrictions on parameters. Theil and Goldberzer (1961) [9] and Theil (1963) [10] proposed "Mixed Regression Estimator (MRE)', which is one of well-known such techniques in the situation where prior information is represented by stochastic restriction on regression coefficients in the linear statistical model. If the prior information is unbiased, they the MRE is superior to the Ordinary Least Squares (OLS) estimator. The bias of prior information may arise in some practical situation, where the mixed regression model is applied, and it may have serious effects Yancey, Judge and Bock (1974) [12] have shown that the MRE does not necessarily reduce the Mean Square Error (MSE) is prior information is biased. Ohtani and Honda (1984) [8] have examined the small sample properties of the MRE when the ratio of the variance of the prior restriction errors to the variance of the sample restriction errors to the variance of the sample errors is unknown and when the bias of prior information may exist.

2. Types of Prior Information about Parameters
The various types of non-sample or prior information in addition to the sample information are given by:

I. Information on individual or linear combinations of parameters that is exact in nature and this can be considered as linear equality restrictions to be combined with the sample information;
II. Information on individual or linear combinations of parameters that is stochastic in nature and can be considered as linear stochastic restrictions to be combined with the sample information;

III. Information on individual or linear combinations of parameters that is represented by joint or individual prior probability density function to be combined with the sample information, reflected by a likelihood function;
IV. Information on individual or linear combinations of parameters that is represented by inequality restrictions and can be considered as linear inequality restrictions to be combined with sample information.
V. Information given in the form of general linear equality hypotheses, general linear stochastic hypotheses, general linear inequality hypotheses, and prior odds ratios.

For a classical linear statistical model $Y = X\beta + \in$, such that $\in \sim N\left(0, \sigma^2 I\right)$, the various forms of prior information are given by:

I. Exact prior information: $R\beta = r$
II. Stochastic prior information: $R\beta + v = r$
III. Inequality prior information: $R\beta \geq r$
IV. Prior density function: $P(\beta, \sigma^2)$

3. Prior Information in the Form of Stochastic Restrictions

In applied econometric research, in many situations, the assumption of exact prior information is not appropriate and sometimes this type of information does not exist. If uncertainty exists about the prior information specifications on the regression coefficients of the linear statistical model $Y_{n\times 1} = X_{n\times k}\beta_{k\times 1} + \in_{n\times 1}$, one may generally specify the linear stochastic restrictions or linear stochastic hypotheses as $H_0 : R\beta + v = r$

Where R is a known (q×k) matrix of prior information; r is a known observable (q×1) random vector; and v is a (q×1) unobservable random vector such that $v \sim N\left(\delta, \sigma^2\Omega\right)$, with Ω is known.

In this situation, Theil and Goldberzer (1961) [9] consider the stochastic prior information and combined it with the sample information. They assumed that the disturbance vectors $\in$ and v are independent to each other, where $\in$ associated with sample information on Y and v associated with stochastic prior information about parameters of the linear statistical model.
They have consider both the combined information and specified the linear statistical model in the form:

$$\begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} X \\ R \end{bmatrix}\beta + \begin{bmatrix} \in \\ v \end{bmatrix} \text{Or } Y^* = X^*\beta + \in^*$$

The mean vector and the dispersion matrix of $\in^*$ are given by

$$\in^* \sim N\left[\begin{pmatrix} 0 \\ f \end{pmatrix}, \sigma^2\begin{bmatrix} I_n & 0 \\ 0 & \Omega \end{bmatrix}\right]$$

Several statisticians and econometricians have suggested different estimation procedures for estimating the parameters of the linear statistical model under linear stochastic restrictions about the regression coefficients.

4. Mixed Regression Estimator and Its Properties

Consider the linear regression model

$$Y = X\beta + \in \ Y = X\beta + \in \qquad \text{... (4.1)}$$

Where Y is a n×1 vector of observations. X is a n×k matrix of non-stochastic variables of rank k, $\in$ is a n×1 vector of normal error terms, I_n is an identity matrix of order n, and β is a k×1 vector of unknown regression coefficients and also the prior information about β by previous samples or by inspection.

$$r = R\beta + v \quad v \sim N\left(\mu\delta, \chi\right) \qquad \text{... (4.2)}$$

Where R is a m×k known matrix of rank m (m≤k) r is a m×1 vector of prior estimate of Rβ, v is a m×1 vector of normal error terms, and χ is m×m known positive definite matrix δ and μ are an m×1 unknown constant vector and an unknown scalar representing the bias of prior information and its magnitude, respectively. Note that prior information is unbiased if μ=0. We assume that $\in$ and v are mutually independent.
There are two alternative estimators of the β. One is the mixed regression estimator (MRE) β^* (feasible generalized least squares estimator), which incorporates prior information into estimation of β.

$$\hat{\beta}^* = \left(X^{|}X / \beta^2 + R^{|}\chi^{-1}R\right)^{-1}\left(X^{|}y / \beta^2 + R^{|}\chi^{-1}r\right) \qquad \text{... (4.3)}$$

Where $s^2 = y^{|}\left[I_n - X\left(X^{|}X\right)^{-1}X^{|}\right]y/n-k$

The other is the OLS estimator b, which makes of no prior information.

$$b = X\left(X^{|}X\right)^{-1}X^{|}y \qquad \text{... (4.4)}$$

5. Mean Square Error of the MRE

Since χ is positive definite, there exists a non-singular matrix A such that $A\chi A^{|} = I_m$.
Pre multiplying equation (4.2) by A, we have

$$Ar = AR\beta + Av \quad Av \sim N\left(\mu A\delta, I_m\right)$$

Putting h=Ar, H=AR, u=Av and $\delta^*=A\delta$
We can rewrite equation (4.2) as

$$h = H\beta + u \quad u \sim N\left(\mu\delta^*, I_m\right) \qquad \text{... (4.5)}$$

Thus equation (4.2) is transformed into (4.5) we now re-parameterize equation (4.1) and (4.5). Since $X^{|}X$ is positive definite and since $H^{|}H$ is positive definite (if m<k, $H^{|}H$ becomes positive semi-definite) there exist a non-singular matrix T such that

$$T^{|}H^{|}HT = \Lambda_k,\ T^{|}X^{|}XT = I_k$$

Where Λ_k is the k×k diagonal matrix whose diagonal elements, $\lambda_1, \lambda_2 \ldots \lambda_k$ are roots of the equation

$$\left|H^{|}H - \lambda X^{|}X\right| = 0$$

Note that $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_k > 0$ and $\lambda_{m+1} = \ldots = \lambda_{m+k} = 0$ (if m=k $\lambda_i > 0$ $\forall$ i=1, 2... k) hold without loss of generality, using the matrix T, we rewrite the equation (4.1) and (4.5) as

$$y^* = X^*\gamma + \in \quad \in \sim N\left(0, \sigma^2 I_n\right) \qquad \text{... (4.6)}$$

$$h = H^*\gamma + u \quad u \sim N\left(\mu\delta^*, I_m\right) \qquad \text{... (4.7)}$$

Where $X^* = XT$ $H^* = HT$ and $\gamma = T^{-1}\beta$
Now the MRE for γ is given by

$$\hat{\gamma}^* = \left[\left(X^{*|}X^*/s^{*2}\right) + H^{*|}H^*\right]^{|}\left[\left(X^{*|}y^*/s^{*2}\right) + H^{*|}h\right]$$

$$= \left[\left(I_k/s^{*2}\right) + \Lambda_k\right]^{-1}\left[\left(X^{*|}y^*/s^{*2}\right) + H^{*|}h\right]$$

Where $s^{*2} = y^{|}\left(I_n - X^{*|}X^*\right)y/\left(n-k\right)^3$

Denoting the i^{th} column vectors of X^* and H^* as χ_i^* and h_i^* (i=1, 2... k) respectively, we have the i^{th} element of $\hat{\gamma}^*$,

$$\hat{\gamma}^* = \left[s^{*2}/\left(1+\lambda_i^* s^{*2}\right)\right]\left[\left(X^{*|}y^*/s^{*2}\right) + h_i^{*|}h\right]$$

$$= \gamma_i + \left[1/\left(1+\lambda_i s^{*2}\right)\right]\left(X_i^{*|}\in\right) + \left[s^{*2}/\left(1+\lambda_i s^{*2}\right)\right]\left(h_i^{*|}u\right) \qquad \text{... (4.8)}$$

Where γ_i denotes the i^{th} element of γ. Since $X_i^{*|}\in$, $h_i^{*|}u$ and s^{*2} are pair wise independent.

The MSE of $\hat{\gamma}_i^*$ is

$$\mathrm{MSE}\left(\hat{\gamma}_i^*\right)=\mathrm{E}\left(\hat{\gamma}_i^*-\gamma_i\right)^2$$

$$=\sigma^2\mathrm{E}\left[\left\{1/\left(1+\lambda_i s^{*2}\right)\right\}^2\right]+\left[\lambda_i\mu^2\left(h_i^{*\prime}\delta^*\right)^2\right]\mathrm{E}\left[\left\{s^{*2}/\left(1+\lambda_i s^{*2}\right)\right\}^2\right] \quad \text{... (4.9)}$$

To evaluate the MSE of $\hat{\gamma}^*$ as a whole, we consider the weak MSE of $\hat{\gamma}^*$

$$\mathrm{MSE}\left(\hat{\gamma}^*\right)\mathrm{E}\left(\hat{\gamma}^*-\gamma\right)^{\prime}\left(\hat{\gamma}^*-\gamma\right)$$

$$\mathrm{E}=\sum_{i=1}^{k}\mathrm{E}\left[\left(\hat{\gamma}_i^*-\gamma_i\right)^2\right] \quad \text{... (4.10)}$$

Noting that $\lambda_1\geq\lambda_2\geq\ldots\geq\lambda_k>0$ and $\lambda_{m+1}=\ldots=\lambda_{m+k}=0$ and performing some calculations we obtain

$$\mathrm{MSE}\left(\hat{\gamma}^*\right)=\sum_{i=1}^{m}\left\{\sigma^2\mathrm{E}\left(Z_i^2\right)-2\mathrm{E}\left(Z_i\right)+1)+\left[\left(\lambda_i+\mu^2\theta_i^2\right)\big/\lambda_i^2\right]\mathrm{E}\left(Z_i^2\right)\right\}$$

$$+\sum_{i=m+1}^{k}\left\{\sigma^2+\sigma^4\mu\theta_i^2\left(1+2\big/(n-k)\right)\right\} \quad \text{... (4.11)}$$

Where $Z_i=\lambda_i s^{*2}/\left(1+\lambda_i s^{*2}\right)$ and $\theta_i=h_i^{*\prime}\delta^*$

The fact that $\mathrm{E}\left(s^{*4}\right)=\sigma^4\left(1+2\big/(n-k)\right)$ is used in evaluation of the second term in (4.11).

To compare the performance of the two estimators $\hat{\gamma}^*$ and OLSE of γ denoted by $\hat{\gamma}$, we need the weak MSE of $\hat{\gamma}$

$$\mathrm{MSE}\ (\hat{\gamma}) = \mathrm{K}\sigma^2 \quad \text{... (4.12)}$$

Define MSE ($\hat{\gamma}^*$)-MSE ($\hat{\gamma}$) as D (μ^2) we get from (4.11) and (4.12).

$$D\left(\mu^2\right)=\mathrm{MSE}\left(\hat{\gamma}^*\right)-\mathrm{MSE}\left(\hat{\gamma}\right)$$

$$=\sum_{i=1}^{m}\left\{\sigma^2\mathrm{E}\left(Z_i^2\right)-2\mathrm{E}\left(Z_i\right)+1)+\left[\left(\lambda_i+\mu^2\theta_i^2\right)\big/\lambda_i^2\right]\mathrm{E}\left(Z_i^2\right)\right\}$$

$$+\sum_{i=m+1}^{k}\sigma^4\mu^2\theta_i^2\left(1+2\big/(n-k)\right)$$

Since D (μ^2) is a linear function of μ^2
MSE ($\hat{\gamma}^*$) < MSE ($\hat{\gamma}$)

$$\text{iff } \mu^2<-\frac{\sum_{i=1}^{m}\left\{\sigma^2\left[\mathrm{E}\left(Z_i^2\right)-2\mathrm{E}\left(Z_i\right)\right]+\mathrm{E}\left(Z_i^2\right)\big/i\right\}}{\sum_{i=1}^{m}\theta_i^2/\lambda_i^2)\mathrm{E}\left(Z_i^2\right)+\sum_{i=m+1}^{k}\sigma^4\theta_i^2\left(1+2\big/(n-k)\right)} \quad \text{... (4.13)}$$

And vice versa.

Because the denominator of the R.H.S. in (4.13) is positive, the direction of the inequality in (4.13) solely depends on the sign of the numerator.

$$D(0)=\sum_{i=1}^{m}\left\{\sigma^2\left[\mathrm{E}\left(Z_i^2\right)-2\mathrm{E}\left(Z_i\right)\right]+\mathrm{E}\left(Z_i^2\right)\big/\lambda_i\right\}$$

If D (0) ≥ 0 then OLSE > MRE $\forall$ $\mu \neq 0$

If D (0) < 0, however $\exists$ a unique μ^2 below which MRE > OLSE and above which MRE < OLSE. Although it is difficult to prove analytically whether D (0) ≥ 0 or D (0) < 0.

By assuming some conditions on convergence

$$\left[\lim_{n\to\infty}\left(\frac{X^{\prime}X}{n}\right)=Q_1,\ \lim_{n\to\infty}\left(\frac{H^{\prime}H}{n}\right)=Q_2,\ \text{and so on}\right]$$

Ohtani and Honda (1984) [8] have shown that $Z_i \to \hat{\lambda}_i\sigma^2/\left(1+\hat{\lambda}_i\sigma^2\right)$ in probability since s^{*2} is the consistent estimator of σ^2, where $\hat{\lambda}_i$ is the i^{th} root of the equation $\left|Q_2-\hat{\lambda}Q_1\right|=0$. Thus

$$\lim_{n\to\infty}\left\{\sigma^2\left[\mathrm{E}\left(Z_i^2\right)-2\mathrm{E}\left(Z_i\right)\right]+\mathrm{E}\left(Z_i^2\right)\big/\lambda_i\right\}=-\hat{\lambda}_i\sigma^4\left(1+\hat{\lambda}_i\sigma^2\right)>0 \qquad \text{... (4.14)}$$

$\therefore$ D (0) becomes negative asymptotically.

6. Inference in Linear Model with Stochastic Prior Information Constriants and Compatibility Test

Consider the classical multiple linear regression model

$$Y_{n\times 1}=X_{n\times k}\beta_{k\times 1}+\in_{n\times 1} \qquad \text{... (5.1)}$$

With $E\left(\in\right)=0$

And $E\left(\in\in^{\prime}\right)=\sigma^2 I_n$ ... (5.2)

Where I_n is unit matrix of order n.

By using only the sample information on Y and X, one may get the unrestricted least squares estimator of β as

$$\hat{\beta}=\left(X^{\prime}X\right)^{-1}X^{\prime}Y$$

Here, $\tilde{\beta}\sim N\left(\beta,\sigma^2\left(X^{\prime}X\right)^{-1}\right)$

An unbiased estimator of σ^2 is given by

$$\hat{\sigma}^2=\frac{\left(Y-X\hat{\beta}\right)^{\prime}\left(Y-X\hat{\beta}\right)}{n-k}=\frac{Y^{\prime}\left[I-X\left(X^{\prime}X\right)^{-1}X^{\prime}\right]Y}{n-k} \qquad \text{... (5.3)}$$

The unrestricted least squares estimator $\hat{\beta}$ is the maximum likelihood estimator and is the best linear unbiased estimator (BLUE) for β.

Now, augment linearly independent prior restrictions which represent additional information or incomplete prior information on β in the regression model (5.1) as

$$r_{q\times 1}=R_{q\times k}\beta_{k\times 1}+v_{q\times 1} \qquad \text{... (5.4)}$$

Where r is a (q×1) vector of known constants;
R is a (q×k) matrix of known constants;
V is a (q×1) unobservable, normally distributed error random vector independent of $\in$ with mean zero and dispersion matrix ψ. i.e., E (v) = 0, E (vv|) = ψ

And v ~ N (0, ψ) ... (5.5)

Combining the sample and stochastic prior information the linear regression model can be written as

$$\begin{bmatrix} Y \\ r \end{bmatrix} = \begin{bmatrix} X \\ R \end{bmatrix} \beta + \begin{bmatrix} \in \\ v \end{bmatrix} \quad \text{... (5.6)}$$

Or $Y^* = X^* \beta + \in^*$ ... (5.7)

Where $Y^* = \begin{bmatrix} Y \\ r \end{bmatrix}, X^* = \begin{bmatrix} X \\ R \end{bmatrix}$ and $\in^* = \begin{bmatrix} \in \\ v \end{bmatrix}$

When ψ is known positive definite matrix, application of GLS estimation to (5.7) gives an estimator for β as

$$\tilde{\beta} = \left[\frac{(X^| X)}{\sigma^2} R^| \psi^{-1} R \right]^{-1} \left[\frac{(X^| Y)}{\sigma^2} R^| \psi^{-1} R \right] \quad \text{... (5.8)}$$

using an unbiased estimator for σ² given in (5.3) $\tilde{\beta}$ can be written as

$$\hat{\tilde{\beta}} = \left[\frac{(X^| X)}{\hat{\sigma}^2} R^| \psi^{-1} R \right]^{-1} \left[\frac{(X^| Y)}{\hat{\sigma}^2} R^| \psi^{-1} R \right] \quad \text{... (5.9)}$$

Suppose ψ may reflect subjective prior information and plausible ψ ψ is a diagonal matrix with unknown elements σ_i^2 , i=1, 2... q. Thus ψ is unknown.
Define the residual vector for the restricted equations as

$$w = r - R\beta^+ \quad \text{... (5.10)}$$

$$\Rightarrow w_i = r_i - R_i^| \beta^+ \text{, i=1, 2... q} \quad \text{... (5.11)}$$

Where R_i is a known (k×1) vector

$$\beta^+ = \left(R^| R \right)^{-1} R^| r \quad \text{... (5.12)}$$

Consider an estimator for ψ as

$$\tilde{\psi} = diag\left[w_1^2, w_2^2, \ldots, w_q^2 \right] \quad \text{... (5.13)}$$

An estimated GLS (EGLS) estimator for β is given by

$$\tilde{\beta}^+ = \left[R^| \tilde{\psi}^{-1} R \right]^{-1} \left[R^| \tilde{\psi}^{-1} R \right] \quad \text{... (5.14)}$$

Now, write the EGLS residuals as

$$w_i = \left[r_i - R_i^| \tilde{\beta}^+ \right] \text{, i=1, 2... q} \quad \text{... (5.15)}$$

$$\Rightarrow r_i = \tilde{r}_i + \tilde{w}_i \text{, i=1, 2... q} \qquad \text{... (5.16)}$$

Where $\tilde{r}_i = R_i^{|}\tilde{\beta}^{+}$

$$\Rightarrow \left(r_i - R_i^{|}\beta\right) = \left(\tilde{r}_i - R_i^{|}\beta\right) + \tilde{w}_i \text{, i=1, 2... q} \qquad \text{... (5.17)}$$

$$\Rightarrow v_i = \tilde{v}_i + \tilde{w}_i \quad \left[\therefore r = R\beta + v \quad or \quad r_i = R_i^{|}\beta + v\right]$$

$$\Rightarrow \text{var}\left(v_i\right) = \text{var}\left(\tilde{v}_i\right) + \text{var}\left(\tilde{w}_i\right)$$

$$\Rightarrow \sigma_i^2 = \text{var}\left(\tilde{v}_i\right) + \text{var}\left(\tilde{w}_i\right)$$

$$\Rightarrow \text{var}\left(\tilde{w}_i\right) = \sigma_i^2 - \text{var}\left(\tilde{v}_i\right)$$

$$\Rightarrow \text{var}\left(\tilde{w}_i\right) = \left[1 - \frac{\text{var}\left(\tilde{v}_i\right)}{\sigma_i^2}\right]\sigma_i^2 \qquad \text{... (5.18)}$$

$$\therefore \sigma_i^2 = \left[1 - \frac{\text{var}\left(\tilde{v}_i\right)}{\sigma_i^2}\right]^{-1} \text{var}\left(\tilde{w}_i\right) \qquad \text{... (5.19)}$$

Consider $\text{var}\left(\tilde{v}_i\right) = \text{var}\left(\tilde{r}_i - R_i^{|}\beta\right)$ [$\therefore$ From (5.17)]

$$= \text{var}\left(R_i^{|}\tilde{\beta}^{+} - R_i^{|}\beta\right)$$

$$\Rightarrow \text{var}\left(\tilde{v}_i\right) = \text{var}\left[R_i^{|}\left(\tilde{\beta}^{+} - \beta\right)\right] \qquad \text{... (5.20)}$$

From the definition of $\tilde{\beta}^{+}$, we have,

$$\text{var}\left(\tilde{\beta}^{+} - \beta\right) = \left[R^{|}\tilde{\psi}^{-1}R\right]^{-1} \qquad \text{... (5.21)}$$

$$\text{var}\left(\tilde{v}_i\right) = R_i^{|}\left[R^{|}\tilde{\psi}^{-1}R\right]^{-1} R_i \qquad \text{... (5.22)}$$

From (5.20),

Now replace $\dfrac{\text{var}\left(\tilde{v}_i\right)}{\sigma_i^2}$ with $\dfrac{R_i^{|}\left[R^{|}\tilde{\psi}^{-1}R\right]^{-1} R_i}{w_i^2}$ and $\text{var}\left(\tilde{w}_i\right)$ with w_i^2 in the equation (5.19) provides an estimator for σ_i^2 as

$$\hat{\tilde{\sigma}}_i^2 = \left[1 - \frac{R_i^{|}\left[R^{|}\tilde{\psi}^{-1}R\right]^{-1} R_i}{w_i^2}\right]^{-1} \tilde{w}_i \qquad \text{... (5.23)}$$

Thus, a two stage estimator for ψ is given by

$$\hat{\tilde{\psi}} = diag\left(\hat{\tilde{\sigma}}_1^2, \hat{\tilde{\sigma}}_2^2, \ldots, \hat{\tilde{\sigma}}_q^2\right) \qquad \text{... (5.24)}$$

Hence, a two stage restricted EGLS estimator for β is given by

$$\hat{\hat{\beta}}^{*}=\left[\frac{X^{|}X}{\sigma^{2}}+R^{|}\hat{\hat{\psi}}^{-1}R\right]^{-1}\left[\frac{X^{|}X}{\sigma^{2}}+R^{|}\hat{\hat{\psi}}^{-1}r\right] \quad ... (5.25)$$

Following Court (1976), a new compatibility test statistic for sample and stochastic prior information is given by

$$\Gamma=\left(r_{i}-R_{i}^{|}\hat{\hat{\beta}}^{*}\right)^{|}\left[\hat{\hat{\psi}}-R\left(\frac{X^{|}X}{\sigma^{2}}+R^{|}\hat{\hat{\psi}}^{-1}R\right)^{-1}R^{|}\right]^{-1}\left(r-R^{|}\hat{\hat{\beta}}^{*}\right) \quad ... (5.26)$$

Here, Γ follows χ^2 distribution with degrees of freedom equal to the rank k of R.

7. Conclusions

In the present research study an attempt has been made to propose some criteria for testing linear restrictions on parameters of linear regression models with nonspherical disturbances. New estimation methods have been suggested for estimating multiple linear regression models under sample and stochastic prior information constraints in the present study. A compatibility test has been presented under stochastic linear constraints.

A new estimation method has been derived for linear regression model under stochastic prior information restrictions. A compatibility test statistic has been given for sample and stochastic prior information on parameters of linear regression model.

8. Reference

1. Bera AK, Testing for the Regression Coefficient Stability, Journal of Quantitative Economics. 1997; 13:17-35.
2. Bock ME, Judge GG, Yancey TA. Some Comments on Estimation in Regression after Preliminary Tests of Significance. 1973; 1:191-200.
3. Court RH. A Note on Theils's Use of Incomplete Prior Information in Regression Analysis. 1976; 18(3):161-162.
4. Farebrother RW. A Survey of Some Recent Econometric Work on Tests for Inequality Constraints. 1988; 4(2):227-238.
5. Firoozi Fathali. A Comparison of the Procedures for Testing Joint Inequality Hypotheses. 1993; 9(1):73-84.
6. Leo Michelis. The distributions of the J and Cox non-nest tests in regression models with Weakly Correlated Regressors, Journal of Econometrics. 1999; 93:369-401.
7. Ohtani K. On Pooling Disturbance Variances when the Goal is Testing Restrictions on Regression Coefficients. 1987; 35:219-231.
8. Ohtani K, Honda Y. Small Sample Properties of the Mixed Regression Estimator. 1984; 26:375-385.
9. Theil H, Goldberger AS. On pure and mixed Statistical estimation in economics, International Economic Review. 1961; 1:65-78.
10. Theil M. On the use of Incomplete Prior Information in Regression Analysis, Journal of the American Statistical Association. 1961; 1963:401-14.
11. Wallace TD. Weaker Criteria and Tests for Linear Restriction in Regression, Econometrica. 1961; 40(1972):689-698.
12. Yancey TA, Judge GG, Bock ME. A Mean Square Error test when Stochastic restrictions are used in regressin, Communication in Statistics. 1974; 3:55-768.