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New results on l-cordial labeling of graphs

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Abstract
In this paper we obtain new families of L- cordial graphs. It includes Flag of Cn given by FL(C_n), C_n^+, bull(C_n),shell graph S_n, S_n^+, S_n^{+t}.we have shown that all of them are l- cordial.

Keywords: L-cordial, shell, graph, flag graph, crown

1. Introduction
Bapat [2] introduces the following new labeling. A graph G(V,E) has a L-cordial labeling if there is a bijection f from E(G) to {1,2,...,|E|} that assigns 0 to a vertex v if the largest label on the edges incident to v is even number and assigns 1 to v otherwise and this assignment satisfies the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1. A graph that admits an L-cordial labeling is called as L-cordial graph. He shows that stars, path, cycles, and triangular snakes are L-cordial graphs. The graphs considered here are simple, connected and finite and un directed. For definitions and terminology we depend on J. Gillian [5], Harary [6], D. West [7].We also use $v_f(0,1) =(x,y)$ to indicate number of vertices labeled with 0 are x in number and number of vertices with 1 are y in number. In all figures below the number are the edge labels.

2. Preliminaries
1. A shell graph Sn is obtained from a cycle Cn by taking n-3 concurrent chords from any vertex to all other non-adjacent vertices. It has n vertices and 2n-3 edges.
2. Fl(G) is a graph obtained from (p,q) graph g by fusing an edge with any vertex of G. The resultant graph has p+1 vertices and q+1 edges. This is flag graph of G.In this paper we discuss Fl(C_n).
3. G^+ is a crown graph which is same as $G \square K_2$.Initially crown was defines for C_n. It is obtained by fusing an edge each to every vertex of G. If G is a (p,q) graph then G^+ has 2p vertices and q+p edges. In this paper we discuss crown of shell graph.
4. When t distinct edges are fused at each vertex of a (p,q) graph G we get G^{+t}. It has p + tp vertices and q + tp edges. Note that when t=1 we get crown graph.
5. Fusion of vertex. Let G be a (p,q) graph. let $u \neq v$ be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges [6]. If $u \in G_1$ and $v \in G_2$, where G_1 is (p_1,q_1) and G_2 is (p_2,q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with v.
6. A bull graph bull(G) was initially defined for a C_3-bull.It has a copy of G with an pendent edge fused with any two adjacent vertices of G. For G ia a (p,q) graph, bull(G) has p+2 vertices and q+2 edges.

3. Results proved
3.1 Theorem: G = FL(C_n) is L-cordial.

Proof: Define a function $f : E(G) \rightarrow \{0,1\}$ as follows:

Case 1: n is even number, $n = 2x$

Label the pendent edge as n+1. The edge adjacent to pendent edge is labeled as 1 and there on the consecutive edges joining degree 2 vertices are labeled as 2, 3, onwards. The last edge labeled is the one adjacent to pendent edge. The vertex label number distribution is $v_f(0,1)=(x,x+1)$ for n is even number.

Case 2: n is odd number $n= 2x+1$, $x =1, 2,$.

Label the pendent edge as n+1. The edge adjacent to pendent edge is labeled as 1 and there on the consecutive edges joining degree 2 vertices are labeled as 2, 3, onwards upto n-2 edges. From remaining last two edges the one adjacent to pendent edge is labeled as n-1 and remaining edge is labeled as n. The labelnumber distribution is $v_f(0,1)=(x,x)$

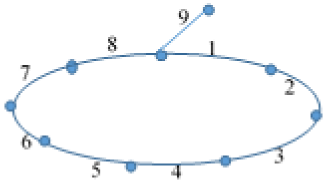

Fig 3.1: $FL(C_8)$: $e_f(0,1) = (4,5)$:case 1

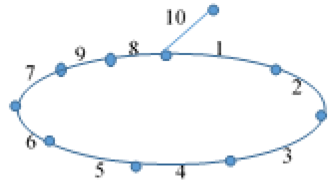

Fig 3.2: $FL(C_9)$: $e_f(0,1) = (5,5)$: case 2

3.2 Theorem: C_n^+ is L-cordial.

Proof: The ordinary labeling is as follows; For cycle Cn define as $(v_1, c_1, v_2, c_2,.. v_{n-1}, c_n, v_1)$. The pendent edge at v_i be e_i.

Case 1: n is even number. $n = 2x$, $x= 2, 3, 4...$

Define a function f: $E(G) \rightarrow \{1,2,..2n\}$ as follows. $f(c_i) = i$; $f(e_i)= n+i$. The label number distribution is $v_f(0,1)=(2x,2x+1)$.

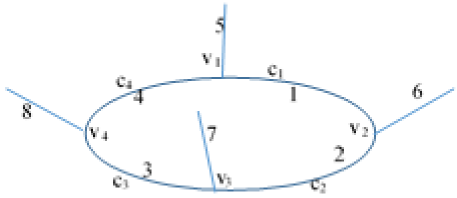

Fig 3.3: C_4^+ : $e_f(0,1) = (4,4)$

Case 2: n is odd number. $n = 2x+1$, $x= 1, 2, 3, 4...$

Define a function f: $E(G) \rightarrow \{1,2,..2n\}$ as follows. $f(c_i) = 2i$; $f(e_i)= 2i-1$; $i = 1,2,..n$. The vertex label number distribution is $v_f(0,1)=(2x+1,2x+1)$.

3.3 Theorem: bull (C_n) is l-cordial.

Proof: Take a cycle $C_n = (v_1, c_1, v_2, c_2, .. v_{n-1}, c_n, v_1)$ and (v_1w_1) and (v_2w_2) as pendent edges.

Define a function f: $E(G) \rightarrow \{1, 2, ..n, n+1, n+2\}$ as follows. For $n= 2x+1$, $f(c_i)= i$ for $i =1, 2, ..n$. $f(v_1w_1)=n+1$ and $f(v_2w_2)= n+2$

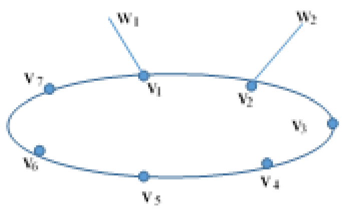

Fig 3.4: ordinary labeling of bull(C_7)

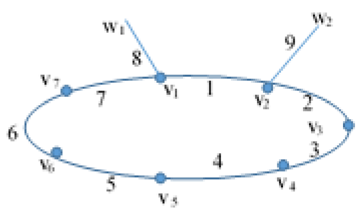

Fig 3.5 Labeled copy of bull(C_7) : $e_f(0,1) = (4,5)$

When n is even number given by 2x we have the vertex label number distribution is $v_f(0,1)=(x+1,x+1)$ when n is odd number $2x+1$, $x= 1,2,..$ $ve_f(0,1)=(x+1,x+2)$ when n is even number 2x, $x=2, 3, ..$

3.4 Theorem: Shel graph S_n is L-cordial.

Proof: Take a cycle $C_n = (v_1, c_1, v_2, c_2, .. v_{n-1}, c_n, v_1)$. The n-3 parallel edges are given by $e_{i-2}=(v_1v_i)$; $i = 3, 4, \ldots n_{i-1}$.

Define a function f : $E(G) \rightarrow \{1,2, 3,..,2n\text{-}3\}$ as follows. $f(e_i) = i$ for $i = 1, 2, .., n\text{-}3$

Case 1: n is odd number, $n = 2x+1$; $x= 2, 3, ...$Then $f(c_i) = (n\text{-}3) +i$ for all $i = 1, 2, 3..n$. The label vertex number distribution is $v_f(0,1)=(x+1,x)$

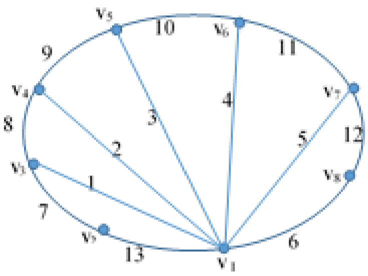

Fig 3.6: Shell S_8 , $e_f(0,1)=(4,4)$

Case 2: n is even number given by $n = 2x$; $x= 2,3,..$ Then $f(c_i) = (n\text{-}3) +i$ for all $i = 2, 3..(n\text{-}1)$; $f(c_1) =2n\text{-}3$, $f(c_n) =n\text{-}2$. The vertex label number distribution is $v_f(0,1)=(x,x)$

4.5 Theorem: Crown of Shell graph, S_n^+ is L-cordial.

Proof: Take a cycle $C_n = (v_1, c_1, v_2, c_2, .. v_{n-1}, c_n, v_1)$. The n-3 parallel edges are given by $e_{i-2}=(v_1v_i)$; $i = 3, 4, \ldots n\text{-}1$. Further the pendent vertices are $\{w_1,w_2, w_3,,\ldots w_n \}$. Corresponding pendent edges are (v_iw_i), $i= 1, 2, ..n$.

Define a function f: $E(G) \rightarrow \{1,2, 3,..,3n\text{-}3\}$ as follows. $f(e_i) = i$ for $i = 1, 2, .., n\text{-}3$
$f(v_iw_i)=n\text{-}3+i$; $i = 1,2 \ldots n$. $f(c_i) = f(v_nw_n) +i$ for all $i = 1, 2, 3..n$. The label number distribution is $v_f(0,1)=(x,x)$ when $n = 2x$. The vertex label number distribution is $v_f(0,1)=(n,n)$ for $n = 2x+1$.

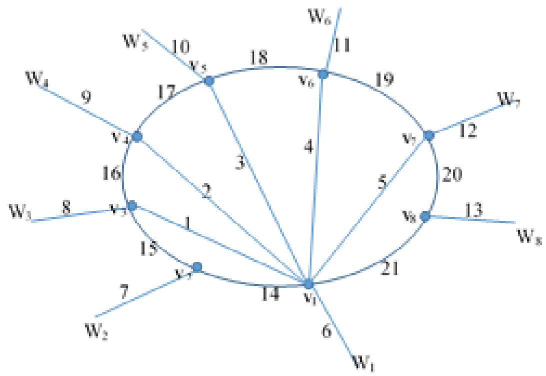

Fig 3.7: Crown Shell S_8 , $e_f(0,1)=(8,8)$

3.6 Theorem: S_n^{+t} is L-cordial.

Proof: Take a cycle $C_n = (v_1, c_1, v_2, c_2, .. v_{n-1}, c_n, v_1)$. The n-3 concurrent edges are given by $e_{i-2}=(v_1v_i)$; $i = 3, 4, \ldots, n\text{-}1$. Further the pendent vertices are $w_{i,j}$ where $i = 1,2,..n$ and $j =1, 2, ..t$. Corresponding pendent edges are $d_{i,j}=(v_i,w_{i,j})$, $j = 1, 2,\ldots,t$. The graph has $(t+2)n\text{-}3$ edges and $n(t+1) = p$ vertices.

Case 1: n is odd number, 2x+1; t is odd number.2y+1

Define a function f: $E(G) \rightarrow \{1,2, 3,..,(t+2)n\text{-}3\}$ as follows.
$f(d_{i,j}) = t(i\text{-}1)+j, j = 1, 2, ..t$;
$f(c_i) = f(d_{n,t})+ i$; $i = 1, 2, ..n$;
$f(ei) = f(c_n)+i$; $i = 1,2, ..(n\text{-}3)$;
The vertices are distributed as $(\frac{p}{2},\frac{p}{2})$.

Case 2: n is odd number, 2x+1 and t is even number say 2y.

Define a function f: $E(G) \rightarrow \{1,2, 3,..,(t+2)n\text{-}3\}$ as follows:
$f(d_{i,j}) = t(i\text{-}1)+j, j = 1, 2, ..t$
$f(c_i)= f(d_{n,t})+ i$; $i = 1, 2, ..n$
$f(ei)= f(c_n)+i$; $i = 1,2, ..(n\text{-}3)$
The vertices are distributed as as $(\frac{p}{2},\frac{p}{2}+1)$.

Case 3: n is even and t is odd number.

Define a function f: E(G) →{1,2, 3,..,(t+2)n-3} as follows.
$f(d_{i,j})$ = t(i-1)+j, j = 1, 2, ..t
$f(c_i)$= $f(d_{n,t})$+ i; i = 1, 2, ..(n-1);
$f(c_n)$= n(t+2)-3
f(ei)= $f(c_{n-1})$+1+i; i= 1,2, ..(n-4);
$f(e_{n-3})$= $f(c_{n-1})$+1;
The vertices are distributed as $(\frac{p}{2}, \frac{p}{2})$.

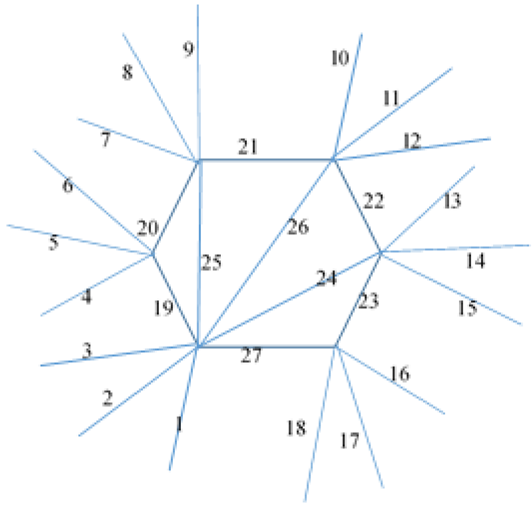

Fig 3.7: S_6^{+3}: $v_f(0,1)$=(12,12)

Case 4: n is even number, 2x and t is even number say 2y

Define a function f: E(G) →{1,2, 3,..,(t+2)n-3} as follows.
$f(d_{i,j})$ = t(i-1)+j,j = 1, 2, ..t
$f(c_i)$= $f(d_{n,t})$+ i; i = 1, 2, ..n
f(ei)= $f(c_n)$+i; i = 1,2, ..(n-3)
The vertices are distributed as $(\frac{p}{2}, \frac{p}{2})$.

4. Conclusions
In this paper we have generalized definition of some established graph and obtained flag graph FL(C_n), Bull graph bull(C_n), crown of shell S_n^+, Shell with t number of pendent edges attached at each point S_n^{+t}.We have discussed L-cordiality and have shown that all these graphs are L-cordial.It is necessary to investigate L-cordial labeling for more graphs.

5. References

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