New results on 1-cordial labeling of graphs

Mukund V Bapat

Abstract
In this paper we obtain new families of L- cordial graphs. It includes Flag of Cn given by FL(Cn), Cn, bull(Cn),shell graph Sn, Sn, Sn+t. we have shown that all of them are 1- cordial.

Keywords: L-cordial, shell, graph, flag graph, crown

1. Introduction
Bapat [2] introduces the following new labeling. A graph G(V,E) has a L-cordial labeling if there is a bijection f from E(G) to {1,2,...,|E|} that assigns 0 to a vertex v if the largest label on the edges incident to v is even number and assigns 1 to v otherwise and this assignment satisfies the condition that the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1. A graph that admits an L-cordial labeling is called as L-cordial graph. He shows that stars, path, cycles, and triangular snakes are L-cordial graphs. The graphs considered here are simple, connected and finite and un directed. For definitions and terminology we depend on J. Gillian [5], Harary [6], D. West [7]. We also use \( v_0(0,1) = (x,y) \) to indicate number of vertices labeled with 0 are x in number and number of vertices with 1 are y in number. In all figures below the number are the edge labels.

2. Preliminaries
1. A shell graph Sn is obtained from a cycle Cn by taking n-3 concurrent chords from any vertex to all other non-adjacent vertices. It has n vertices and 2n-3 edges.
2. Fl(G) is a graph obtained from (p,q) graph g by fusing an edge with any vertex of G. The resultant graph has p+1 vertices and q+1 edges. This is flag graph of G. In this paper we discuss Fl(Cn).
3. G' is a crown graph which is same as G\( \square K_2 \).Initially crown was defines for Cn. It is obtained by fusing an edge each to every vertex of G. If G is a (p,q) graph then G' has 2p vertices and q+1 edges. In this paper we discuss crown of shell graph.
4. When t distinct edges are fused at each vertex of a (p,q) graph G we get G'\. It has p + tp vertices and q + tp edges. Note that when t=1 we get crown graph.
5. Fusion of vertex. Let G be a (p,q) graph. let u \( \neq v \) be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1 vertices and at least q-1 edges [10].
6. A bull graph bull(G) was initially defined for a C3-bull.It has a copy of G with an pendent edge fused with any two adjacent vertices of G. For G ia a (p,q) graph, bull(G) has p+2 vertices and q+2 edges. Sometimes this is referred as u is identified with v.

3. Results proved
3.1 Theorem: G = FL(Cn) is L-cordial.
Proof: Define a function \( f : E(G) \rightarrow \{0,1\} \) as follows:

**Case 1:** \( n \) is even number, \( n = 2x \)
Label the pendent edge as \( n+1 \). The edge adjacent to pendent edge is labeled as 1 and there on the consecutive edges joining degree 2 vertices are labeled as 2, 3, onwards. The last edge labeled is the one adjacent to pendent edge. The vertex label number distribution is \( v_f(0,1) = (x,x+1) \) for \( n \) is even number.

**Case 2:** \( n \) is odd number \( n = 2x+1 \), \( x = 1, 2, \ldots \)
Label the pendent edge as \( n+1 \). The edge adjacent to pendent edge is labeled as 1 and there on the consecutive edges joining degree 2 vertices are labeled as 2, 3, onwards. From remaining last two edges the one adjacent to pendent edge is labeled as \( n-1 \) and remaining edge is labeled as \( n \). The label number distribution is \( v_f(0,1) = (x,x) \)

3.2 Theorem: \( C_n^+ \) is L-cordial.

Proof: The ordinary labeling is as follows; For cycle \( C_n \) define as \((v_1, c_1, v_2, c_2, \ldots, v_{n-1}, c_n, v_1)\). The pendent edge at \( v_i \) be \( e_i \).

**Case 1:** \( n \) is even number. \( n = 2x \), \( x = 2, 3, 4, \ldots \)
Define a function \( f : E(G) \rightarrow \{1,2,\ldots,2n\} \) as follows. \( f(c_i) = i; f(e_i) = n+i \). The label number distribution is \( v_f(0,1) = (2x,2x+1) \).

**Case 2:** \( n \) is odd number. \( n = 2x+1 \), \( x = 1, 2, 3, 4, \ldots \)
Define a function \( f : E(G) \rightarrow \{1,2,\ldots,2n\} \) as follows. \( f(c_i) = 2i; f(e_i) = 2i-1 \); \( i = 1,2,\ldots,n \). The vertex label number distribution is \( v_f(0,1) = (2x+1,2x+1) \).

3.3 Theorem: bull \( (C_n) \) is l-cordial.

Proof: Take a cycle \( C_n = (v_1, c_1, v_2, c_2, \ldots, v_{n-1}, c_n, v_1) \) and \((v_1w_1)\) and \((v_2w_2)\) as pendent edges.

Define a function \( f : E(G) \rightarrow \{1,2,\ldots,n+1,n+2\} \) as follows. For \( n = 2x+1 \), \( f(c_i) = i \) for \( i = 1, 2, \ldots, n \), \( f(v_1w_1) = n+1 \) and \( f(v_2w_2) = n+2 \)

When \( n \) is even number given by \( 2x \) we have the vertex label number distribution is \( v_f(0,1) = (x+1,x+1) \) when \( n \) is odd number \( 2x+1, x = 1, 2, \ldots \) \( v_f(0,1) = (x+1,x+2) \) when \( n \) is even number \( 2x, x = 2, 3, \ldots \)

3.4 Theorem: Shel graph \( S_n \) is L-cordial.

Proof: Take a cycle \( C_n = (v_1, c_1, v_2, c_2, \ldots, v_{n-1}, c_n, v_1) \). The \( n-3 \) parallel edges are given by \( e_{i,2} = (v_1v_i); i = 3, 4, \ldots, n-1 \).
Define a function \( f : E(G) \rightarrow \{1,2,3, ...,2n-3\} \) as follows. \( f(e_i) = i \) for \( i = 1, 2, ..., n-3 \)

Case 1: \( n \) is odd number, \( n = 2x+1; x = 2, 3, ... \) Then \( f(c_{i}) = (n-3) + i \) for all \( i = 1, 2, 3, ... n \). The label vertex number distribution is \( v_i(0,1)=(x+1,x) \)

![Fig 3.6: Shell S8, e(0,1)=(4,4)](image)

Case 2: \( n \) is even number given by \( n = 2x; x = 2, 3, ... \) Then \( f(c_{i}) = (n-3) + i \) for all \( i = 2, 3, ..., (n-1) \); \( f(c_{n}) = 2n-3, f(c_{0}) = n-2 \). The vertex label number distribution is \( v_i(0,1)=(x,x) \) when \( n = 2x \).

4.5 Theorem: Crown of Shell graph, \( S_n^+ \) is L-cordial.

Proof: Take a cycle \( C_n = (v_1, c_1, v_2, c_2, ..., v_n, c_n, v_1) \). The n-3 parallel edges are given by \( e_{i-2}=(v_1,v_i); i = 3, 4, ..., n-1 \). Further the pendent vertices are \( \{w_1,w_2, w_3, ..., w_n\} \). Corresponding pendent edges are \( (v_i,w_i); i = 1, 2, ..., n \).

Define a function \( f : E(G) \rightarrow \{1,2,3, ..., n-3\} \) as follows. \( f(e_i) = i \) for \( i = 1, 2, ..., n-3 \)

![Fig 3.7: Crown Shell S8, e(0,1)=(8,8)](image)

4.6 Theorem: \( S_n^+t \) is L-cordial.

Proof: Take a cycle \( C_n = (v_1, c_1, v_2, c_2, ..., v_n, c_n, v_1) \). The n-3 concurrent edges are given by \( e_{i-2}=(v_1,v_i); i = 3, 4, ..., n-1 \). Further the s t pendent vertices are \( \{w_{ij} \} \) where \( i = 1, 2, ..., n \) and \( j = 1, 2, ..., t \). Corresponding pendent edges are \( d_{i,j}=(v_i,w_{ij}); j = 1, 2, ..., t \). The graph has \( (t+2)n-3 \) edges and \( n(t+1) = p \) vertices.

Case 1: \( n \) is odd number, \( 2x+1; t \) is odd number \( 2y+1 \)

Define a function \( f : E(G) \rightarrow \{1,2,3, ..., (t+2)n-3\} \) as follows.

\( f(d_{ij}) = (t(i-1)+j), j = 1, 2, ..., t \);
\( f(c_{i}) = f(d_{n,t}) + i; i = 1, 2, ..., n \);
\( f(e_i) = f(c_{n}) + i; i = 1, 2, ..., (n-3) \);
The vertices are distributed as \( \binom{P}{n} \binom{P}{2} \).

Case 2: \( n \) is odd number, \( 2x+1 \) and \( t \) is even number say \( 2y \)

Define a function \( f : E(G) \rightarrow \{1,2,3, ..., (t+2)n-3\} \) as follows.

\( f(d_{ij}) = (t(i-1)+j), j = 1, 2, ..., t \);
\( f(c_{i}) = f(d_{n,t}) + i; i = 1, 2, ..., n \);
\( f(e_i) = f(c_{n}) + i; i = 1, 2, ..., (n-3) \);
The vertices are distributed as \( \binom{P}{n} \binom{P}{2} + 1 \).
Case 3: \( n \) is even and \( t \) is odd number.

Define a function \( f: E(G) \rightarrow \{1, 2, 3, \ldots, (t+2)n-3\} \) as follows.

- \( f(d_{i,j}) = t(i-1)+j, \quad j = 1, 2, \ldots,t \)
- \( f(c_i) = f(d_{n,t})+i; \quad i = 1, 2, \ldots,(n-1) \)
- \( f(e_i) = f(c_{n,i})+1+i; \quad i = 1, 2, \ldots,(n-4) \)
- \( f(e_{n,3}) = f(c_{n,1})+1 \)

The vertices are distributed as \( \left( \frac{p}{2}, \frac{p}{2} \right) \).

Case 4: \( n \) is even number, \( 2x \) and \( t \) is even number say \( 2y \)

Define a function \( f: E(G) \rightarrow \{1, 2, 3, \ldots, (t+2)n-3\} \) as follows.

- \( f(d_{i,j}) = t(i-1)+j, \quad j = 1, 2, \ldots,t \)
- \( f(c_i) = f(d_{n,t})+i; \quad i = 1, 2, \ldots,n \)
- \( f(e_i) = f(c_{n,i})+1+i; \quad i = 1, 2, \ldots,(n-3) \)

The vertices are distributed as \( \left( \frac{p}{2}, \frac{p}{2} \right) \).

4. Conclusions

In this paper we have generalized definition of some established graph and obtained flag graph \( FL(C_n) \), Bull graph bull \( (C_n) \), crown of shell \( S_n^t \), Shell with \( t \) number of pendent edges attached at each point \( S_n^t \). We have discussed \( L \)-cordiality and have shown that all these graphs are \( L \)-cordial. It is necessary to investigate \( L \)-cordial labeling for more graphs.

5. References

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