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K Sai Swathi
 Research Scholars, Department
 of Statistics, Acharya Nagarjuna
 University, Guntur, Andhra
 Pradesh, India

CH V Aruna
 Research Scholars, Department
 of Statistics, Acharya Nagarjuna
 University, Guntur, Andhra
 Pradesh, India

B Guravaiah
 Research Scholars, Department
 of Statistics, Acharya Nagarjuna
 University, Guntur, Andhra
 Pradesh, India

N viswam
 Research Scholars, Department
 of Statistics, Acharya Nagarjuna
 University, Guntur, Andhra
 Pradesh, India

GVSR Anjaneyulu
 Professor, Department of
 Statistics, Acharya Nagarjuna
 University, Guntur, Andhra
 Pradesh, India

Correspondence
K Sai Swathi
 Research Scholars, Department
 of Statistics, Acharya Nagarjuna
 University, Guntur, Andhra
 Pradesh, India

Change point method with Rayleigh distribution

K Sai Swathi, CH V Aruna, B Guravaiah, N Viswam and GVSR Anjaneyulu

Abstract

This paper intends to assess the performance between CUSUM and change point model of Rayleigh distribution by considering ARLs respectively. Finally we conclude that the performance of change point model is superior to CUSUM chart in the context of Rayleigh distribution.

Keywords: control chart, cumulative sum technique, average run length

1. Introduction

The Statistical Process Control (SPC) aims to detect and diagnose situations where the process is gone out of statistical control. This type of problem involves two aspects namely process and statistical aspects. To have detailed outline of some of its statistical modeling aspects see Crowder *et al.* (1987) [1]. In fact the status of Statistical Quality Control may be described as one in which the process readings appear to follow a common statistical model. One model is that the process is in SQC, the successive process readings X_i 's are independent and sampled from the same distribution. When the process goes out of control, it may behave in different ways. In general we can have two types of causes, those that affect single process readings and then disappear, and the second type of causes or sustainable causes. These causes will continue until they are identified and eliminated. In statistical terminology the isolated causes analogous to an out layer. The Shewart \bar{X} with an R or s-chart is an excellent tool for detecting these special causes, namely isolated or sustainable causes. However Shewart control char is less effective for detecting small changes in the process. Standard tools for detecting sustain changes are the CUSUM and EWMA chart. This paper focuses on another, less familiar; method aimed at detecting sustained changes is the Change Point formulation when the process averages follow non-Normal distribution.

2. Change point method

At first let us discuss Change Point Method when the process readings modeled by two Normal Distributions,

$$\left. \begin{aligned} X_i &\approx N(\mu_1, \sigma_1^2) \text{ for } i = 1, 2, \dots, n, \\ X_i &\approx N(\mu_2, \sigma_2^2) \text{ for } i = \tau + 1, \dots, n. \end{aligned} \right\} \quad (2.1)$$

Here the number of observations 'n' is fixed in any traditional statistical settings, but this 'n' may increase unlimitedly in phase-II Statistical Process Control settings. Here both settings i.e. Phase-I and Phase-II will be discussed, with context indicating which of the two applies. In the case of in control distribution is $N(\mu_1, \sigma_1^2)$, the readings follows this distribution up to an epoch τ , the change point, at which point they shift to another Normal distribution different in mean i.e. $\mu_1 \neq \mu_2$, in variability $\sigma_1 = \sigma_2$ or in both mean and variance. In change point method process readings leads to two statistical tasks namely testing task and estimating task. The testing task is to conform whether there has indeed being a change. If the change is there then the task of estimating τ , the time at which shift occurred. Sometimes we may have to estimate some or all of the parameters μ_1, μ_2, σ_1 and σ_2 . In all discussions [see Hawkins, 2003] we work

on change point has focused on shifts in mean only. i.e. $\mu_1 \neq \mu_2$ but $\sigma_1 = \sigma_2 = \sigma$, throughout this paper this type of frame work is used here.

From this change point method we have three scenarios based on the amount of process knowledge viz.

- All parameters μ_1, μ_2, σ_1 and σ_2 are known exactly priory.
- In control parameters μ and σ are known but μ_2 is unknown.
- All parameters μ_1, μ_2, σ_1 and σ_2 are unknown.
- Throughout our discussions we concentrate on the third scenario namely all parameters are unknown under varying distributions.

2.1 Implementation of change point method

The formulae below mentioned indicate that the $T_{max,n}$ values are computationally burdensome. For the implementation of the method, construct two arrays of values of

$$S_n = \sum_{i=1}^n X_i \text{ And } W_n = \sum_{i=1}^n (X_i - \bar{X}_n)^2$$

There is no need to store the running mean $\bar{X}_n = \frac{S_n}{n}$, but it will be calculated ‘on the fly’. The observation of the two new table entries can be calculated from the numerically stable recursions, when a new observation is added

$$S_n = S_{n-1} + X_n \text{ And } W_n = W_{n-1} + \frac{[(n-1)X_n - S_{n-1}]^2}{n(n-1)}$$

The two sample statistic T_{jn} are calculated for every possible split point $1 \leq j \leq n$ after finding $T_{max,n}$. It is easy and more convenient to find T_{jn}^2 . The variance explained by a split at point j can be shown to be

$$E_{jn} = \frac{(nS_j - jS_n)^2}{nj(n-j)}$$

And the analysis of variance identity

$$V_{jn} = W_n - E_{jn} \text{ Reduces to } T_{jn}^2 = \frac{(n-2)E_{jn}}{(W_n - E_{jn})}$$

The Change Point test is known by comparing h_n^2 with maximum of statistics of the allowed j values.

If $T_{max,n}^2 > h_n^2$, leading to the signal of a change point, then it is in significant matter to compute the maximum likelihood estimators

$$\hat{\mu}_1 = \frac{S_j}{j}, \hat{\mu}_2 = \frac{(S_n - S_j)}{n-j} - \beta^2 \text{ And } \hat{\sigma}^2 = \frac{V_{jn}}{n-2}$$

(The customary variance estimator), using the value j leading to the maximum. The estimators are somewhat biased even though the maximum likelihood estimator are calculated by Hinkley (1970) [3].

The maximizing j is that which maximizes E_{jn} , so the searching step need only to evaluate E_{jn} for each j, making further T_{jn}^2 calculation necessary only for the maximizing E_{jn} . Thus, while at process reading number n there are n-1 calculations to be performed, each involves only about the floating point operations, so even if n were in the tens of thousands calculating $T_{max,n}^2$ would still be a trivial calculation.

The ever-growing storage requirement for the two tables might be more inconvenient. It is acceptable to restrict the search for change point to the most recent ‘ ω ’ instance, if this can be limited along with the size of the resulting search. It is done only when one must keep a table of only the ‘ ω ’ most recent S_j and W_j values.

Willsky and Jones (1976) [6] discussed the ‘window’ approach which is different from above method, in that observations more than w time periods into the past are not lost, since they are summarized in the window’s leftmost S and W entry. The lost is the ability to split at these old instants. Appropriate values for the table size W may be in the 500 to 2000 range. It is very large as no interesting structure is lost, but small enough to compute for each new reading to less than 20000 operations.

3. Change point method with rayleigh distribution when the parameters are not know

Consider the model with none of the parameters known; we can test the presence of change point with another general Likelihood ration test. This test is a two sample t-test between before shift and after shift of the sequence, maximised across all possible change points; Worsely (1979) [7]. For a given change point ‘ j ’ where $1 \leq j \leq n-1$, let

$$\bar{X}_{jn} = \sum_{i=1}^j X_{ij} / j$$

Be the mean of the ‘ j ’ observations before shift

$$\bar{X}_{jn} = \sum_{i=j+1}^n X_{ij} / (n - j)$$

Be the mean of the $(n-j)$ observations after shift. The residual sum of squares i.e. V_{jn} is given by

$$V_{jn} = \sum_{i=1}^j (X_i - \bar{X}_j)^2 + \sum_{i=j+1}^n (X_i - \bar{X}_j^*)^2 \tag{3.1}$$

Here we assume that there is a single change point approach at epoch ‘ j ’, \bar{X}_{jn} and \bar{X}_{jn}^* are the Maximum Likelihood Estimators of μ_1 , μ_2 and $\sigma_{jn}^2 = \frac{V_{jn}}{n - 2}$ is the usual pooled estimator of σ^2 . A conventional two sample t statistic for comparing these two segments could be

$$T_{jn} = \sqrt{\frac{j(n - j)}{n}} \frac{\bar{X}_{jn} - \bar{X}_{jn}^*}{\hat{\sigma}_{jn}} \tag{3.2}$$

For a stable process, T_{jn} follows at t-distribution with $n-2$ degrees of freedom. An out of control signal is obtain as soon as

$$T_{\max, n} = \text{Max} \{T_{jn} |, \text{for } 1 \leq j \leq n - 1\} \tag{3.3}$$

Exceeds some critical value, h_n .

Hawkins *et al.* (2003) [2] showed that, for any type I error, h_n can be computed by using the Bonferroni inequality. They mentioned that the latter is conservative when a process measurement (n) is large. At same time, they provide empirical control limits, $h_{n, \alpha}$ for different type I error, α , and various number of process observations. Also, for large n values ($n \geq 1$), they proposed the following approximation formula for computing $h_{n, \alpha}$

$$h_{n, \alpha} = h_{n, \alpha} \left[0.677 + 0.019 \ln(\alpha) + \frac{1 - 0.1151 \ln(\alpha)}{n - 6} \right] \tag{3.4}$$

Where $\ln(.)$ is the natural log function.

In the case of general change point formulation in which either or both of the parameters μ , σ may shift at the change point τ . Sullivan and Woodall (1996) [5] discuss this formulation and the resultant generalised likely hood ratio test. It provides a single diagnostic to detect shifts in either the mean or the variance or in both. This has the disadvantage of normality assumption. Furthermore, while bounds, approximations and extreme value results are known for the hull distribution of $T_{\max, n}$, there is any hardly sample theory for Sullivan and Woodall statistic. Based on these reasons we will consider for the change point formulation to mean the formulation in the third scenario i.e. none of the process parameters is consider known exactly.

In Phase I, with its static set of data $X_1, X_2 \dots X_n$, traditional fixed sample statistical methods are appropriate. So, for example, it is appropriate to calculate $T_{\max, n}$ for the whole data set and test it against a suitable fractiles of the null distribution of the test statistic for that value of n . If the analysis indicates a lack of control in the Phase I data set, more data will be gathered after process adjustment until a clean data set is achieved.

Phase II data are the process readings gathered subsequently unlike the fixed set of Phase I, they are from a never-ending stream. As each new reading accrues, the SPC check is re-applied. For this purpose, fixed significance level control limits are not

appropriate: rather, concern is with the run lengths, both in-and out-of-control. A convenient summary of the frequency of false alarms is the in-control average run length (ARL), which should be large, and self-contained GLR in which the maximised Likelihood and the Likelihood ratio are used both detection and estimation.

In traditional methods viz. Shewart, CUSUM, EWMA charts, require a Phase I data set to have parameter estimates that can be used in the Phase II calculations. These methods require on to draw a connectional line below the Phase I data and separate the estimated data [Phase I] from the SPC data [Phase II]. On contrast, in change point formulation one does not assume known parameters and hence does not require the estimates produced by a Phase I. Once the preliminaries are complete and the initial process stability has been achieved, the change point allows to go seamlessly into SPC in which, at each instant, all accumulated process readings are analysed and all data is used to test for the presence of a change point. When process remains in control, it also provides on ongoing stream of every improving estimates of the parameters.

The change point model is a schematic approach in which each new observation X_n is added to the data set, the change point statistic $T_{max,n}$ is calculated for the sequence $X_1, X_2...X_n$. If $T_{max,n} > h_n$. Where $\{h_n\}$ is a suitably chosen sequence of control limits, then we conclude that there has been a change in mean. The important point is the choice of the control limits sequence $\{h_n\}$. The ideal would be a sequence $\{h_n\}$, such that the hazard or alarm rate [the conditional probability of a false alarm at any 'n', given that there was no previous alarm] was a constant α , as in case with the Shewart chart. When the hazard rate is constant the in-control ARL would be $\frac{1}{\alpha}$. This approach was used by Margavio *et al.* (1995) [4] in the context of an EWMA chart in context known parameters the false alarm changes point over time. Margavio *et al.* (1995) [4] derived control limits Sequence that would fix the false alarm for EWMA chart to a specified value. The present chapter attempts to obtained distribution of $T_{max,n}$. In fact obtaining distribution of $T_{max,n}$ the sequence is far from being able to provide distributional theory sequence. Hence an attempt is made using simulation to tackle this problem.

In fact the change point does not depend on the parameters estimation from Phase I so it is possible to star testing for a change point with the third process reading. Table 3.1 is. Obtained by simulation of 10 million sequence of length 200 using Rayleigh distribution. This table shows the control limits for α value of 0.05, 0.02, 0.01, 0.005, 0.002 and 0.001, corresponding in control ARLs of 20, 50, 100, 200, 500 and 1000 for different 'n' values ranging from 3 to 200.

Table 3.1

| CUTOFFS $h_{n,\alpha}$ FOR SAMPLE SIZE 'n' AND HAZARD RATE ' α ' STARTING AT SAMPLE 3 | | | | | | |
|--|----------|----------|----------|----------|----------|----------|
| n | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
| 3 | 26.4517 | 61.65674 | 118.3213 | 239.6466 | 611.6192 | 1158.239 |
| 4 | 8.148804 | 11.68613 | 16.77718 | 23.96427 | 39.21047 | 55.75856 |
| 5 | 5.891949 | 6.953319 | 8.9649 | 11.48376 | 16.82633 | 21.60409 |
| 6 | 5.104095 | 5.451373 | 6.673182 | 8.110302 | 11.4155 | 14.03367 |
| 7 | 4.719309 | 4.747417 | 5.63837 | 6.646386 | 9.192542 | 11.05301 |
| 8 | 4.497061 | 4.348107 | 5.063179 | 5.851018 | 8.02302 | 9.523984 |
| 9 | 4.355226 | 4.094638 | 4.702189 | 5.358792 | 7.314035 | 8.612264 |
| 10 | 4.258586 | 3.921439 | 4.456973 | 5.027352 | 6.843208 | 8.013689 |
| 11 | 4.189662 | 3.796806 | 4.280913 | 4.790654 | 6.510132 | 7.593683 |
| 12 | 4.138848 | 3.703635 | 4.149243 | 4.614154 | 6.263356 | 7.284337 |
| 13 | 4.100447 | 3.631928 | 4.047656 | 4.478133 | 6.073976 | 7.047961 |
| 14 | 4.070882 | 3.575467 | 3.967333 | 4.370557 | 5.924579 | 6.862067 |
| 15 | 4.0478 | 3.530193 | 3.90256 | 4.283685 | 5.804082 | 6.712457 |
| 16 | 4.029593 | 3.493347 | 3.849476 | 4.212323 | 5.705113 | 6.589751 |
| 17 | 4.015129 | 3.462993 | 3.805383 | 4.15286 | 5.622586 | 6.487513 |
| 18 | 4.003588 | 3.437734 | 3.768344 | 4.102711 | 5.552882 | 6.401188 |
| 19 | 3.994364 | 3.416538 | 3.73693 | 4.059982 | 5.493362 | 6.327467 |
| 20 | 3.987002 | 3.398627 | 3.710068 | 4.02325 | 5.442055 | 6.263887 |
| 22 | 3.976538 | 3.370409 | 3.666892 | 3.963679 | 5.358423 | 6.160113 |
| 24 | 3.970215 | 3.349692 | 3.634158 | 3.917862 | 5.293551 | 6.079398 |
| 26 | 3.966777 | 3.334327 | 3.608908 | 3.881917 | 5.242126 | 6.01518 |
| 28 | 3.965401 | 3.322883 | 3.589178 | 3.853269 | 5.200642 | 5.963144 |
| 30 | 3.965522 | 3.31438 | 3.573619 | 3.83015 | 5.166698 | 5.92034 |
| 35 | 3.970065 | 3.301886 | 3.547307 | 3.789134 | 5.104778 | 5.841425 |
| 40 | 3.97808 | 3.297231 | 3.532486 | 3.763587 | 5.064119 | 5.78858 |
| 45 | 3.987793 | 3.297093 | 3.524515 | 3.747407 | 5.036458 | 5.751721 |
| 50 | 3.998296 | 3.299678 | 3.52085 | 3.737231 | 5.017231 | 5.725286 |
| 60 | 4.019917 | 3.309271 | 3.521035 | 3.727685 | 4.994256 | 5.691687 |
| 70 | 4.041075 | 3.321638 | 3.526596 | 3.726261 | 4.983276 | 5.673205 |
| 80 | 4.061185 | 3.33498 | 3.53474 | 3.729106 | 4.97892 | 5.663231 |
| 90 | 4.080104 | 3.348487 | 3.544119 | 3.7343 | 4.978461 | 5.658384 |
| 100 | 4.09785 | 3.361779 | 3.554036 | 3.74081 | 4.980391 | 5.656776 |
| 125 | 4.137614 | 3.393147 | 3.57909 | 3.759533 | 4.990664 | 5.660564 |
| 150 | 4.171928 | 3.421475 | 3.602962 | 3.778972 | 4.004237 | 5.669669 |
| 175 | 4.201978 | 3.446974 | 3.625098 | 3.797784 | 4.01868 | 5.680807 |
| 200 | 4.228651 | 3.470029 | 3.645496 | 3.815569 | 4.033051 | 5.692607 |

It can be noted that from table 3.1, as the ‘ α ’ values decreases the control limits $h_{n,\alpha}$ decreases sharply initially, but then stabilizes. Similar type of behaviour can be observed even with Normal distribution; Hawkins (2003) [2].

It may not be reasonable that start testing at the third observation. However there are cases where the process shift occurs at third observation. In practice the particular should gather a modest number of observations to get an initial verification that the Rayleigh distribution was a reasonable fit. Then only the formal change point can be applied. In view of this our main simulation is based on assumption of 9 readings without testing, with testing starting at the 10th observation, however 9 readings itself is hard to believe in a Rayleigh distribution for the Quality variable. Perhaps it is making reasonable compromise between the conflicting desires.

The same 10 million sequence of length 200 are used to find out cut offs points up to $n=200$. The resulting control limits are presented in table 3.2.

Table 3.2

| CUTOFFS $h_{n,\alpha}$ FOR SAMPLE SIZE ‘n’ AND HAZARD RATE ‘ α ’ STARTING AT SAMPLE 10 | | | | | | |
|---|----------|----------|----------|----------|----------|----------|
| n | 0.05 | 0.02 | 0.01 | 0.005 | 0.002 | 0.001 |
| 10 | 4.258586 | 5.021439 | 5.456973 | 6.027352 | 6.843208 | 7.513689 |
| 11 | 3.841319 | 4.546789 | 4.938576 | 5.464631 | 6.220676 | 6.844801 |
| 12 | 3.67287 | 4.333104 | 4.685837 | 5.168124 | 5.858124 | 6.42529 |
| 13 | 3.55255 | 4.180471 | 4.50531 | 4.956333 | 5.599158 | 6.12564 |
| 14 | 3.46231 | 4.065997 | 4.369914 | 4.79749 | 5.404933 | 5.900903 |
| 15 | 3.392123 | 3.976961 | 4.264606 | 4.673946 | 5.25387 | 5.726107 |
| 16 | 3.335973 | 3.905733 | 4.18036 | 4.57511 | 5.133019 | 5.58627 |
| 17 | 3.290033 | 3.847455 | 4.111431 | 4.494245 | 5.034141 | 5.471858 |
| 18 | 3.251749 | 3.79889 | 4.053991 | 4.426857 | 4.951743 | 5.376515 |
| 19 | 3.219355 | 3.757796 | 4.005387 | 4.369836 | 4.882021 | 5.29584 |
| 20 | 3.191589 | 3.722574 | 3.963727 | 4.320962 | 4.82226 | 5.22669 |
| 22 | 3.146469 | 3.665336 | 3.896029 | 4.24154 | 4.725147 | 5.114321 |
| 24 | 3.111375 | 3.620818 | 3.843375 | 4.179768 | 4.649616 | 5.026923 |
| 26 | 3.083301 | 3.585204 | 3.801252 | 4.13035 | 4.58919 | 4.957005 |
| 28 | 3.06033 | 3.556065 | 3.766788 | 4.089917 | 4.539751 | 4.899799 |
| 30 | 3.041189 | 3.531783 | 3.738068 | 4.056223 | 4.498552 | 4.852127 |
| 35 | 3.004885 | 3.48573 | 3.683598 | 3.992321 | 4.420416 | 4.761715 |
| 40 | 2.979259 | 3.453222 | 3.645149 | 3.947214 | 4.365261 | 4.697895 |
| 45 | 2.960204 | 3.429049 | 3.616559 | 3.913672 | 4.324248 | 4.65044 |
| 50 | 2.945479 | 3.410371 | 3.594466 | 3.887754 | 4.292556 | 4.613769 |
| 60 | 2.924211 | 3.38339 | 3.562555 | 3.850316 | 4.24678 | 4.560801 |
| 70 | 2.909588 | 3.364841 | 3.540616 | 3.824577 | 4.215308 | 4.524385 |
| 80 | 2.898918 | 3.351305 | 3.524606 | 3.805795 | 4.192342 | 4.497811 |
| 90 | 2.890788 | 3.340992 | 3.512408 | 3.791485 | 4.174845 | 4.477564 |
| 100 | 2.884388 | 3.332873 | 3.502806 | 3.78022 | 4.16107 | 4.461626 |
| 125 | 2.873094 | 3.318546 | 3.48586 | 3.76034 | 4.136761 | 4.433498 |
| 150 | 2.865721 | 3.309194 | 3.474798 | 3.747362 | 4.120893 | 4.415137 |
| 175 | 2.86053 | 3.302608 | 3.467009 | 3.738224 | 4.10972 | 4.402209 |
| 200 | 2.856677 | 3.29772 | 3.461228 | 3.731441 | 4.101426 | 4.392612 |

4. Perfomence of the change point approach with rayleigh distribution

The performance of change point approach in the case of Rayleigh distribution can be assessed with ARL’s of both in control and following shift in a mean. This complication does not arise in the case of Shewart or known parameter CUSUM chart. In this case the response to a shift depends on the number of in control observations preceding it. The reason for this dependence is that the non-centrality parameter of two sample t statistic depends on the sample sizes. Small values in control period leads to a smaller non-centrality parameter and hence low values of shift is considered than a longer in control period.

The above points are illustrated in table 1.3 by considering α value 0.02, 0.01, 0.005 and 0.002. A shift of size $\delta \in \{0, 0.5, 0.6, 0.75, 1.25, 1.5, 1.75, 2, 2.25, 2.5, 3\}$ was introduced $\tau \in \{10, 25, 50, 100, 250\}$.

The values presented in the table 4.1 are the ARL’s of the change point procedures. These were calculated by simulating a data series, adding the shifts to all X_i ’s, $i > \tau$ and counting the number of readings from the occurrence of the shift until the chart is signaled. Any sequence which is signaled before time τ was discovered the appropriate formula h_n is used. So that the in control ARL’s differs from normal.

Table 4.1: The Arl of the Change Point Procedure When Shift Occurs At the ‘Start’ Position with Size ‘ δ ’

| α | start | 0 | 0.5 | 0.6 | 0.75 | 1.25 | 1.5 | 1.75 | 2 | 2.25 | 2.5 | 3 |
|----------|-------|----------|---------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| 0.02 | 10 | 3.289152 | 20.7541 | 36.5089 | 102.6066 | 144.3342 | 42.71179 | 21.04804 | 13.24886 | 9.659798 | 7.452006 | 5.09 |
| | 25 | 5.624377 | 20.6541 | 35.10036 | 96.95557 | 134.0896 | 32.6479 | 11.04804 | 10.98858 | 6.847978 | 5.47006 | 2.839615 |
| | 50 | 2.070861 | 20.3541 | 31.56612 | 89.44565 | 124.332 | 27.14786 | 8.980432 | 9.288579 | 5.729783 | 4.450598 | 11.84368 |
| | 100 | 33.30872 | 21.2541 | 28.33235 | 84.35545 | 113.312 | 22.50896 | 7.080432 | 5.385795 | 3.747978 | 2.77006 | 1.258894 |
| | 250 | 0.638146 | 24.7541 | 22.50036 | 68.95557 | 103.3196 | 18.71959 | 7.004315 | 3.245795 | 2.659978 | 2.750598 | 21.96378 |
| 0.01 | 10 | 4.00041 | 24.3428 | 42.7633 | 121.0012 | 170.1435 | 50.04397 | 24.68882 | 15.50775 | 11.23847 | 8.639493 | 0.977985 |
| | 25 | 3.635815 | 24.4428 | 38.50814 | 110.9017 | 160.4053 | 42.06973 | 15.38824 | 11.57749 | 8.634698 | 5.64493 | 6.287495 |
| | 50 | 4.439894 | 23.7428 | 33.53981 | 102.4963 | 149.1053 | 32.72732 | 8.623937 | 10.27493 | 7.596984 | 4.6993 | 2.13432 |

| | | | | | | | | | | | | |
|-------|-----|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|----------|
| | 100 | 2.993735 | 24.2428 | 28.76814 | 91.06063 | 136.0235 | 26.12732 | 6.739369 | 5.57493 | 5.196984 | 3.643002 | 66.93098 |
| | 250 | 7.245406 | 22.4428 | 22.50814 | 78.99627 | 116.4053 | 19.46973 | 5.693686 | 2.549299 | 2.219843 | 3.243002 | 0.814144 |
| 0.005 | 10 | 1.519674 | 27.9315 | 49.01769 | 139.3958 | 195.9529 | 57.37616 | 28.3296 | 17.76664 | 12.81714 | 9.82698 | 2.910413 |
| | 25 | 20479.63 | 29.1315 | 42.97927 | 128.8478 | 176.4987 | 47.79157 | 18.46047 | 13.16641 | 9.421419 | 5.8198 | 10.77791 |
| | 50 | 0.500191 | 28.8315 | 39.1372 | 123.3685 | 145.9573 | 33.39157 | 11.79605 | 9.740653 | 7.464186 | 3.848002 | 1.331927 |
| | 100 | 1.017274 | 29.4315 | 33.90272 | 103.5747 | 119.4042 | 267.9457 | 8.396047 | 8.140653 | 4.764186 | 3.5198 | 32.46803 |
| | 250 | 22.66992 | 25.9315 | 26.48204 | 93.39578 | 95.95734 | 17.34567 | 6.796047 | 4.740653 | 2.791858 | 2.767698 | 0.893502 |
| 0.002 | 10 | 0.716479 | 32.67551 | 57.28555 | 163.7121 | 230.071 | 67.06878 | 33.14246 | 20.75273 | 14.90403 | 11.39675 | 4.355852 |
| | 25 | 5.738785 | 38.67551 | 51.76263 | 143.0114 | 169.6796 | 47.20762 | 22.92455 | 12.77326 | 11.29033 | 8.396752 | 3.706277 |
| | 50 | 2.014705 | 39.67551 | 47.37224 | 135.7042 | 130.1663 | 33.71776 | 14.74551 | 7.75326 | 9.153275 | 5.217525 | 5.234484 |
| | 100 | 39.82295 | 42.67551 | 42.23632 | 115.0114 | 91.13336 | 23.11776 | 11.14551 | 5.725959 | 6.853275 | 3.425248 | 2.721924 |
| | 250 | 0.612897 | 46.67551 | 32.26553 | 95.20859 | 22.07096 | 17.10762 | 7.955123 | 3.025959 | 4.077482 | 2.502481 | 14.10552 |

It is clear from the table that the ARL's are affected by the amount of history is gathered before the shift, with a faster response carrying with more history. These ARL's are also depends α and δ . It can also be observed that α is large or δ is large the ARL tends to be smaller, as one would anticipating.

For different δ values in the range 0 to 3 we present resulting ARLs in figures 4.1 to 4.4

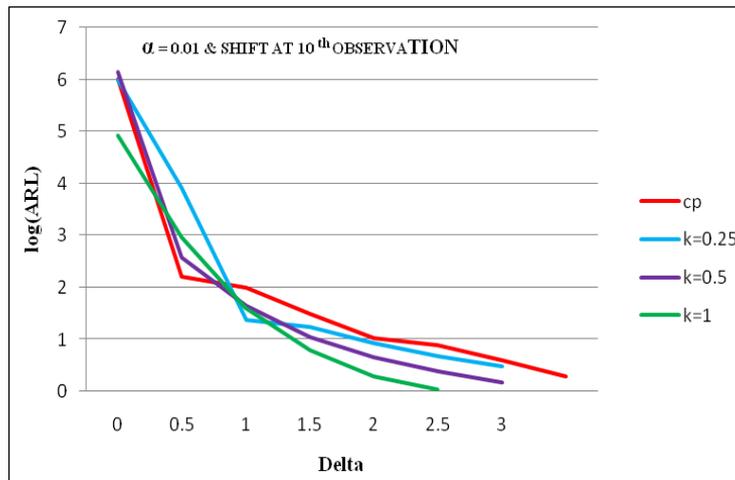


Fig 4.1

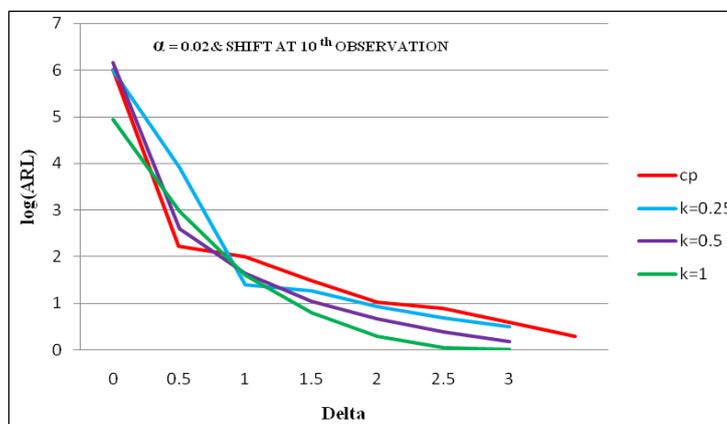


Fig 4.2

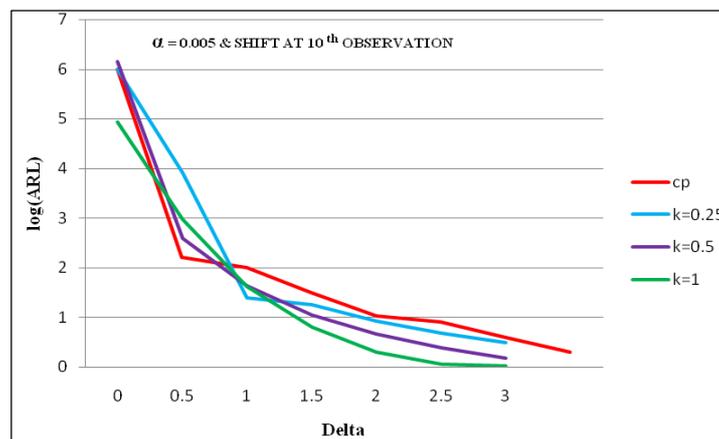


Fig 4.3

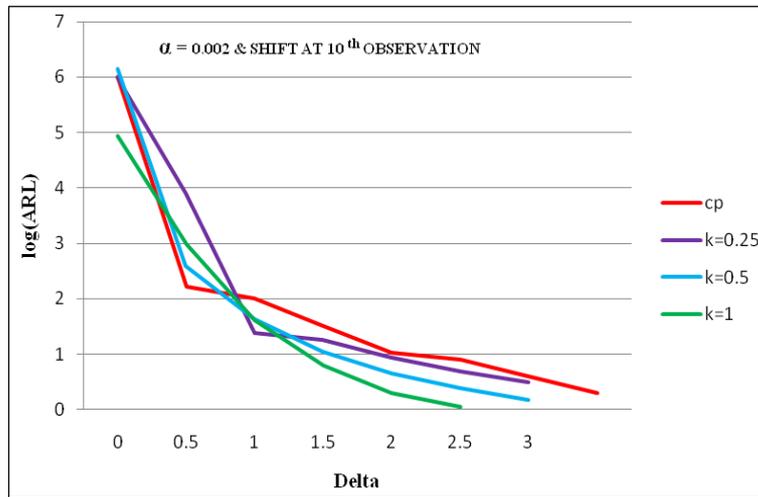


Fig 4.4

From the figures we note that the change point formulation is generalizing the optimal diagnostic for small shifts in the process average it is superior to the CUSUM chart with reference value $k=0.25$, where as in the case of medium size shifts change point model is better than $k=0.5$ CUSUM chart and when $k=1$ CUSUM chart performed well in all cases it is marginal only in the case of normal distribution, Hawkins (2003) [2]. Where as in the case of Rayleigh distribution change point model prove to be strongest diagnostic for detecting shifts in the process average when compare with the CUSUM chart. In other words while CUSUM chart it is never best, the change point method is always best in the case of Rayleigh distribution.

5. Conclusions

In this paper control limits or cut off points $[h_{n,\alpha}]$ are obtained by simulation of 10 million sequence of length 200 with the process readings start at 3 and 10 are obtained in the context of Rayleigh distribution. I.e. when the phenomenon under consideration follows Rayleigh distribution respectively.

This paper also provides a comparative study between change point model and CUSUM chart by considering ARLs in the context of Rayleigh distribution respectively. We found that the performance of change point model is superior to CUSUM chart in the context of Rayleigh distribution, it is found that the change point model is slightly worse than CUSUM chart in detecting small shifts, which is similar case with the normal distribution Hawkins *et al.* (2003) [2]. It would be very interesting to study the behaviour of change point model and comparing its performance with CUSUM chart in the context of non-normal distributions i.e. when the process average follows non-normal distribution.

6. References

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