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Strong equitable and inverse strong equitable domination number of some special classes of graphs

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Abstract
Let G = (V, E) be a simple, finite, undirected and connected graph. A non-empty subset D of V(G) is called a strong equitable dominating set of G if for every $v \in$ V-D, there exists atleast one $u \in$ D such that u and v are adjacent, also deg (u) ≥ deg (v) and if for every $v \in$ V-D, there exists a vertex $u \in$ D such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of such a minimal strong equitable dominating set is called a strong equitable domination number and it is denoted by γ_{se}(G).If $D' \subseteq V - D$ is a strong equitable dominating set, then D' is called an inverse strong equitable dominating set. The minimum cardinality of a minimal inverse strong equitable dominating set is called an inverse strong equitable domination number and it is denoted by $\gamma_{se}'(G)$

Keywords: Domination number, Strong domination number, equitable domination number, Strong equitable domination number and Inverse strong equitable domination number

1. Introduction

In 1962, Ore used the name "dominating set" and "domination number". In 1977, Cockayne and Hedetniemi made an interesting and extensive survey of the results known at that time about dominating sets in graphs. The survey paper of Cockayne and Hedetniemi has generated a lot of interest in the study of domination in graphs. Domination has a wide range of applications in radio stations, modeling social networks, coding theory, nuclear power plants problems [10]. One of the fastest growing areas in graph theory is the study of domination and related subset problems such as independence, covering, matching and inverse domination. A non-empty subset D of V (G) in a graph G is a dominating set if every vertex in V-D is adjacent to atleast one vertex in D. The domination number $\gamma(G)$ is the minimum cardinality of a minimal dominating set of G.

The concept of inverse domination was introduced by V.R. Kulli [8]. If a non-empty subset D is called the minimum dominating set, then if V-D contains a dominating set D', then D' is called the inverse dominating set of G and γ' (G) is the inverse domination number.

Sampath Kumar and Pushpalatha introduced the concepts of strong and weak domination in graphs [11]. Let u, v $\in$ V then u strongly dominates v if i) uv $\in$ E and ii) deg (u) ≥ deg(v). A non-empty subset $D \subseteq V$ is a strong dominating set of G if every vertex in V-D is strongly dominated by atleast one vertex in D.

Swaminathan *et al.* [12] introduced the concept of equitable domination in graphs. A non-empty subset D of V(G) is called an equitable dominating set if for every $v \in$ V-D, there exists a vertex $u \in$ D such that $uv \in E(G)$ and $|\deg(u) - \deg(v)| \leq 1$. The minimum cardinality of such an equitable dominating set is called an equitable domination number γ_e (G).

In this paper, we investigated some results on the strong equitable domination number of some special classes of graphs.

2. Preliminaries

Definition 2.1 [3]

The Clebsch graph is a strongly regular quintic graph on 16 vertices and 40 edges. It is also known as the Greenwood- Gleason graph.

Definition 2.2

The Soifer graph is a planar graph on 9 vertices and 20 edges.

Definition 2.3

The Chvatal graph is a 4-regular graph on 12 vertices and 24 edges. It is a Hamiltonian graph.

Definition 2.4 [5]

The Herschel graph is a bipartite undirected graph with 11 vertices and 18 edges.

Definition 2.5

The Fritsch graph is the planar graph on 9 vertices and 21 edges.

Definition 2.6

The Franklin graph is a 3- regular graph on 12 vertices and 18 edges. It is a 3-vertex connected and 3 edge connected perfect graph.

Definition 2.7 [9]

The Moser graph (also called the Moser Spindle graph) is an undirected graph on 7 vertices and 11 edges named after Mathematician Leo Moser and his brother William.

3. Results

Strong equitable domination number of some special graphs

Definition 3.1 [1, 2]

A non-empty subset D of V(G) is called a Strong equitable dominating set of a graph G if for every $v \in V-D$, there exists a vertex $u \in D$ such that $uv \in E(G)$ and

i) $\deg(u) \geq \deg(v)$

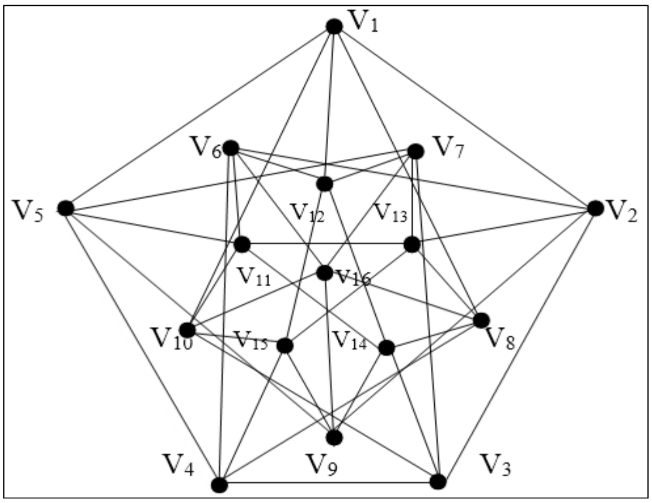
ii) $|\deg(u) - \deg(v)| \leq 1$

The minimum cardinality of the Strong equitable dominating set is the strong equitable domination number and it is denoted by γ_{se} (G).

In this section, we obtained the strong equitable domination number of some special classes of graphs.

Theorem: 3.2

Let G be a Clebsch graph, then its strong equitable domination number is 4. ie, γ_{se} (G) = 4



G: Clebsch Graph

Let V (G) = {v_1, v2 …v16} be the vertex set of a Clebsch graph G. The Clebsch graph is a strongly regular quintic graph on 16 vertices and 40 edges and deg (G) =5.

Let D = {v_1, v3, v12, v16} be the minimum dominating set such that $\gamma(G)$ =4.

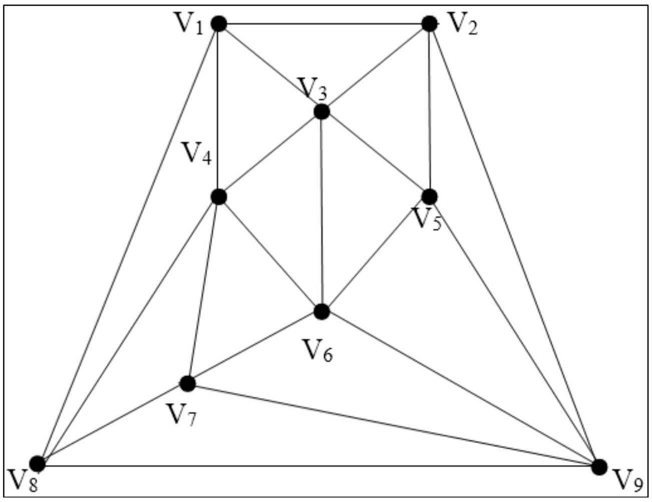
Every vertex in V-D is adjacent to atleast one vertex in D. Then u strongly dominates v, where u$\in$D, v$\in$V-D, uv$\in$E (G) and deg (u) ≥ deg(v). Also, each vertex is of degree 5. It satisfies the equitable domination condition.

Therefore, $|\deg(u) - \deg(v)|$ =0

Thus, γ_{se} (G) = 4

Observation: 3.3
For the Clebsch graph G,
Eccentricity: e (v_1) = e (v_2) = e (v_3) =………. = e (v_{16}) = 2
Radius: r (G) = 2
Diameter: diam (G) = 2

Theorem: 3.4
Suppose G is a Soifer graph with 9 vertices, then γ_{se} (G) = 2



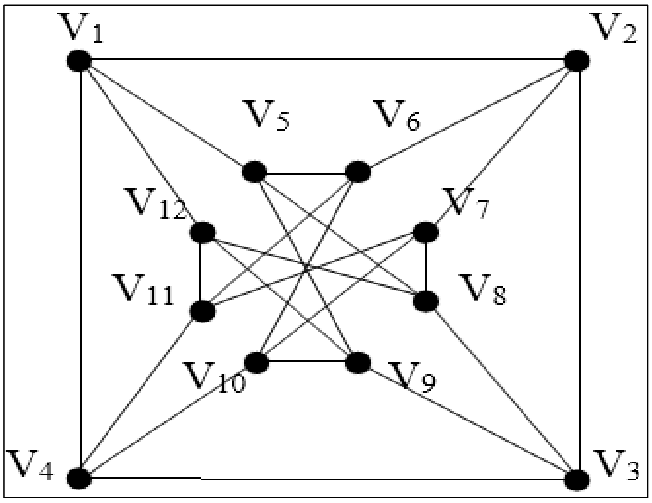
G: Soifer graph

Proof
Let G be a Soifer graph with 9 vertices and 20 edges.
The vertex set of G is V (G) = {v_1, v_2 … v_9}.out of the 9 vertices, only two vertices are enough to cover all the vertices of G. so, the minimum dominating set is D = {v_4, v_9} and γ (G) = 2.

Since it is an irregular graph, $\Delta(G) = 5$ and $\delta(G) = 4$. Here the vertex v_4 strongly dominates the other vertices v_1,v_3,v_6,v_7,v_8 and the vertex v_9 strongly dominates the vertices v_2,v_5,v_6,v_7,v_8. Hence the dominating set D = {v_4,v_9} satisfies deg(u) ≥ deg(v), where u$\in$D, v$\in$V-D, uv$\in$E(G) and it is the strong dominating set satisfying the condition $|\deg(u) - \deg(v)| \leq 1$ Hence, γ_{se} (G) = 2.

Observation: 3.5
The Soifer graph G has
Eccentricity: e (v_1) = e (v_2) =……..= e (v_9) = 2
Radius: r (G) = 2
Diameter: diam (G) = 2

Theorem: 3.6
If the graph G is a Chvatal graph, then γ_{se} (G) = 4



G: Chvatal graph

Proof

By the definition, the Chvatal graph G is a 4- regular graph with 12 vertices and 24 edges. The vertices of Chvatal graph G are v_1, v_2, v_3 …v_{12}. From this 12 vertices, the set D= {v_1, v_3, v_6, v_{10}} is the minimum dominating set of G and γ (G) = 4.

The vertices of D are adjacent to the vertices of V-D and $u \in D$ strongly dominates $v \in$ V-D such that deg (u) ≥ deg (v). We also found that $|\deg(u) - \deg(v)| = 0$

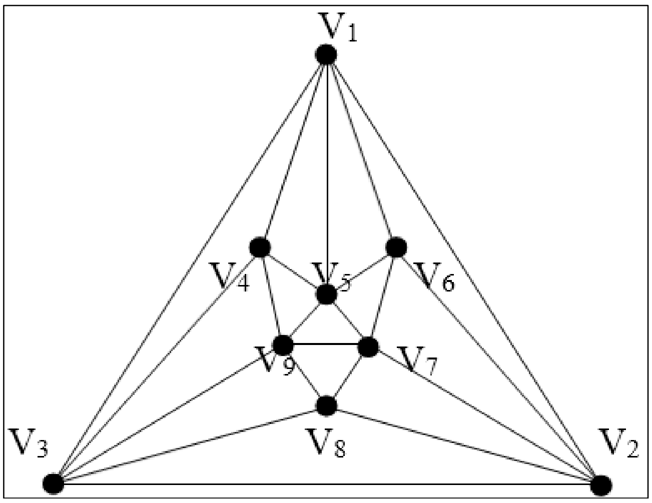
Thus, the strong equitable domination number of Chvatal graph G is 4. ie, γ_{se} (G) = 4

Observation: 3.7

For the Chvatal graph G,
Eccentricity: e (v_1) = e (v_2) =………= e (v_{12}) = 2
Radius: r (G) = 2
Diameter: diam (G) = 2

Theorem: 3.8

For a Fritsch graph, γ_{se} (G) = 2



G: Fritsch graph

Proof

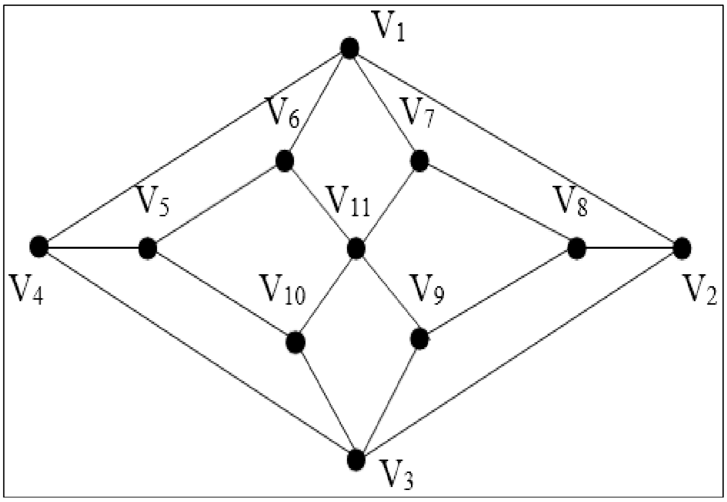
The Fritsch graph G is a planar graph on 9 vertices and 21edges.
Let v_1, v_2…v_9 be the vertices of a Fritsch graph G. The set D={v_1,v_7} is the dominating set and the vertex v_1 strongly dominates the vertices v_2,v_3,v_4,v_5,v_6 and the vertex v_7 strongly dominates the vertices v_2, v_5, v_6, v_8, v_9. Clearly, these two vertices strongly dominate all the other vertices of G. Therefore, deg (u) ≥ deg (v).

For every $v \in$ V-D = {v_2, v_3, v_4, v_5, v_6, v_8, v_9}, there exists $u \in D$ such that $|\deg(u) - \deg(v)| \leq 1$.

Thus, the minimum strong equitable domination number of a Fritsch graph G is 2. ie, γ_{se} (G) = 2.

Observation: 3.9
The Fritsch graph G possesses the following properties:
Eccentricity: e (v_1) = e (v_2) =………= e (v_9)=2
Radius: r (G) = 2
Diameter: diam (G) = 2

Theorem: 3.10
Let G be a Herschel graph, then γ_{se} (G) = 3

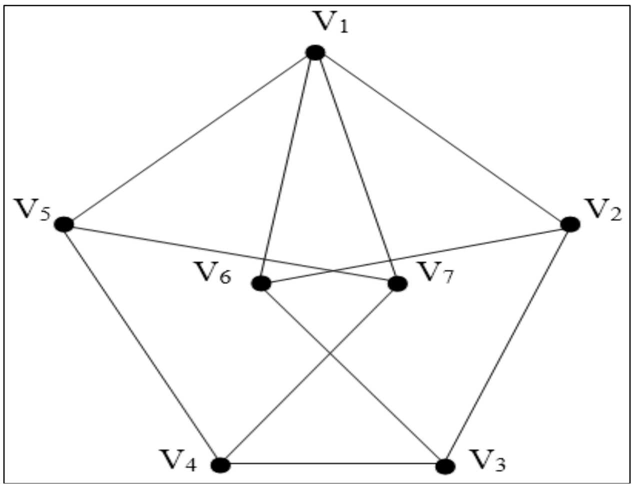


G: Herschel graph

Proof
Suppose V (G) = {v_1, v_2…v_{11}} be the vertex set of the Herschel graph G.
Here, out of these 11 vertices, we need only 3 vertices to dominate all the other vertices of G. so, the minimum dominating set D= {v_1, v_3, v_{11}} and V-D= {v_2, v_4, v_5, v_6, v_7, v_8, v_9, v_{10}}. The degree of the vertices in the set D is 4 and the degree of the vertices in the vertex set V-D is 3.
Hence deg (u) ≥ deg(v), for u$\in$D, v$\in$V-D and uv$\in$E(G) such that $|\deg(u)-\deg(v)|\leq 1$.The dominating set satisfies the equitable domination condition $|\deg(u)-\deg(v)|\leq 1$.
Therefore, the minimum strong equitable domination number is 3. ie, γ_{se} (G) = 3

Observation: 3.11
For the Herschel graph,
Eccentricity: e(v_1) = e(v_2) = e(v_3) = e(v_4) = e(v_6) = e(v_7) = e(v_9) = e(v_{10}) = e(v_{11}) = 3 & e(v_5) = e(v_8) = 4
Radius: r (G) = 3
Diameter: diam (G) = 4

Theorem: 3.12
For a Moser graph G, γ_{se} (G) = 2



G: Moser spindle graph

Proof

Moser spindle graph is an undirected graph with 7 vertices and 11 edges. The vertex v_1 dominates the vertices v_2, v_5, v_6, v_7 and the vertex v_3 dominates the vertices v_2, v_4, v_6.

Clearly, the minimum dominating set D= {v_1, v_3}. Since the degree of the vertices in D is greater than the degree of the vertices in V-D, u ∈ D strongly dominates v ∈ V-D and deg (u) ≥ deg (v). Also, $|\deg(u)-\deg(v)| \leq 1$. Thus γ_{se} (G) = 2

Observation: 3.13

The Moser graph G has

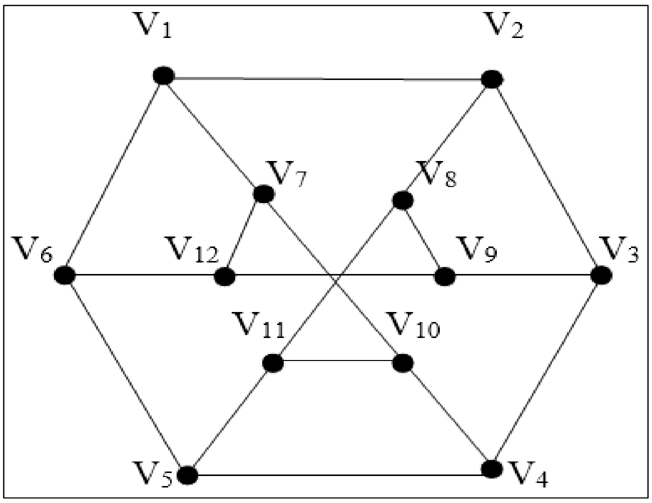
Eccentricity: e (v_1) =e (v_2) =…..e (v_7) =2

Radius: r (G) = 2

Diameter: diam (G) = 2

Observation: 3.14

Let G be a Franklin graph with 12 vertices and 18 edges and it is a 3-regular such that γ_{se} (G) = 4



G: Franklin graph

Observation: 3.15

In a Franklin graph G,

Eccentricity: e (v_1) = e (v_2) =………e (v_{12}) = 3

Radius: r (G) = 3

Diameter: diam (G) = 3

4. Inverse strong equitable domination number of some special graphs

Definition: 4.1 [7]

Let D be the minimum strong equitable dominating set of G. If V-D contains a strong equitable dominating set D', then D' is called the inverse strong equitable dominating set of G with respect to D. The minimum cardinality taken over all the minimal inverse strong equitable domination sets of G is the inverse strong equitable domination number denoted by $\gamma_{se}'(G)$.

Theorem: 4.2

Let G be a Clebsch graph, then $\gamma_{se}'(G)$ = 4

Proof

Let V= {v_1, v_2… v_{16}} be the vertex set of G. We know that, γ_{se} (G) = 4

The necessary and sufficient condition for the existence of atleast one inverse dominating set of G is that G contains no isolated vertices. Let D be the dominating set and there exists another dominating set D' in V-D such that D' satisfies deg(u) ≥ deg(v) and $|\deg(u)-\deg(v)| \leq 1$, for $u \in D'$, $v \in V - D'$, $uv \in E(G)$ .Therefore, the minimum inverse strong equitable domination number of Clebsch graph G is 4. ie, $\gamma_{se}'(G)$ =4

Theorem: 4.3

Suppose G is a Chvatal graph, then its inverse strong equitable domination number is $\gamma_{se}'(G)$ =3

Proof

By the theorem 3.6, the strong equitable domination number of Chvatal graph is γ_{se} (G) = 4

The necessary and sufficient condition for the existence of atleast one inverse strong equitable dominating set of G is that G contains no isolated vertices. Let D be the minimum strong equitable dominating set of G and there exists an inverse strong equitable dominating set D' of G which has three vertices. Clearly u in D' strongly dominates v in V-D' and $|\deg(u) - \deg(v)| \leq 1$.

Then D' is the inverse strong equitable dominating set. Hence, $\gamma_{se}'(G)$ =3

Observation: 4.4

When G is a Fritsch graph, its inverse strong equitable domination number is $\gamma_{se}'(G)$ =2

Theorem: 4.5

For a Franklin graph G, $\gamma_{se}'(G)$ = 3

Proof

Franklin graph is a 3-regular graph with 12 vertices. Let V (G) = {$v_1, v_2 \ldots v_{12}$} be the vertex set of the graph G. The vertex set V can be partitioned into four vertex sets V_1, V_2, V_3 and V_4, each comprising of three vertices and the degree of each vertex being three.

If V_1 is taken as the strong equitable dominating set whose vertices dominate the other three sets giving $\gamma_{se}(G)$=3. Then <V – D> contains another three vertex sets V_2, V_3 and V_4 which dominates the remaining vertices of D. Here V_2 in <V-D> is the inverse strong equitable dominating set. Hence $\gamma_{se}'(G)$ = 3.

Observation: 4.6

The Inverse strong equitable domination number does not exist for the following graphs:

i) Soifer graph
ii) Moser spindle graph
iii) Herschel graph

5. Conclusion

In this paper, we have investigated the strong equitable domination number and the inverse strong equitable domination number of some special classes of graphs. We have also attained the eccentricity, radius and diameter of those graphs.

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