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Some path unions invariance under cordial labeling

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Abstract
In this paper we define some new path unions, obtain different path unions from same graph and show them to be invariant under cordial labelings. i.e. we actually obtain cordial labeling of these different structures. We define bull graph and show that Path union of bull graph G = $p_m(C_3bull)$ all three non isomorphic structures, Path union of C_4 bull graph (G = $p_m(C_4bull)$ all three non isomorphic structures, Path union of C_3^+ all two structures, Path union of C^+_4-e all five structures are cordial graphs.

Keywords: Graph, cordial, labeling, crown, bull, path union, fusion

1. Introduction
The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary, A dynamic survey of graph labeling by J. Gallian and Douglas West.

I. Cahit introduced the concept of cordial labeling. f:V(G)→{0,1} be a function. From this label of any edge (uv) is given by |f (u)-f (v) |. Further number of vertices labeled with 0 i.e v_f (0) and the number of vertices labeled with 1 i.e.v_f (1) differ at most by one. Similarly number of edges labeled with 0 i.e.e_f (0) and number of edges labeled with 1 i.e.e_f (1) differ by atmost one.Then the function f is called as cordial labeling. Cahit has shown that: every tree is cordial; Kn is cordial if and only if $n \leq 3$; $K_{m,n}$ is cordial for all m and n; the friendship graph $C_3^{(t)}$ (i.e., the one-point union of t copies of C_3) is cordial if and only if t 6≡ 2 (mod 4); all fans are cordial; the wheel W_n is cordial if and only if n 6≡ 3 (mod 4).A lot of work has been done in this type of labeling. One may refer dynamic survey by J. Gallian.

We have defined path unions on different graphs inparticular on bull graph. For the same given graph there are many path union P_m (G) structures possible. It depends on which point on G is used to fuse with vertex on P_m. If this point is changed and path union is designed then it may be a different (upto isomorphism) structure. We show that for G = bull on C_3,bull on C_4,C_3^+,C_4^+-e the different path union P_m(G) are cordial. It is called as invariance under cordial labeling. We use the convention that $v_f(0,1)$ = (a,b) to indicate the number of vertices labeled with 0 are a and that numbe of vertices labeled with 1 are b. Further $e_f(0,1)$ = (x,y) we mean the number of edges labeled with o are x and number of edges labeled with 1 are y.

2. Preliminaries
The Bull graph on C_3 is defined in We generalize the definition of bull graph. Let G be a (p,q) graph. Choose any two adjacent vertices of G say u and v. Attach an pendent edge each to u and v. It is denoted by bull(G). Thus bull graph of G has P+2 vertices and q+2 edges. The graph G^+ is obtained by fusing an edge at each vertex of G. We call this graph as G- crown and denote it by G^+. The concept was developed for crown graph which is an cycle Cn to each of it's vertex a pendent edge is attached and denoted by C_n^+.

G^+-e is obtained from G^+ by deleting an pendent edge of G^+. When G = Cn or Kn deleting any pendent edge from G^+ makes no difference. Otherwise one has to specify which edge is deleted. Fusion of vertex. Let u ϵ V (G_1) and vϵV (G_2).By fusing vertex u with v we replace these two vertices by a single vertex say w and all edges incident to u in G_1 and that incident with v in G_2 are incident with w. If u and v are from the same graph G still the same procedure is followed and loops are deleted if any is formed. Some times this is called as u is identified with v.

3. Theorems proved

3.1 Theorem: Path union of bull graph ($G = p_m(bull)$) is cordial graph.

Proof: We define this graph as $V(G)=\{u_{i,1},u_{i,2},u_{i,3},w_{i,1},w_{i,2},v_1,v_2,\dots,v_m\}$ $E(G)=\{e_i=(v_iv_{i+1})/i = 1,2,\dots,m-1\}$ U $\{c_{i,1} =(u_{i,1}u_{i,2}),c_{i,2}=(u_{i,2}u_{i,3}),c_{i,3}=(u_{i,3}u_{i,1})\}$ U $\{x_{i,1}=(u_{i,2}w_{i,1}),x_{i,2}=(u_{i,3}w_{i,2})\}$ There are three non isomorphic structures possible on G. Structure 1, structure 2 and structure 3.

- To obtain Structure 1 graph we fuse vertex $u_{i,1}$ on bull with v_i on path Pm.
- To obtain structure 2 graph we fuse vertex $u_{i,2}$ on bull with v_i on path Pm.
- To obtain structure 3 we fuse vertex $w_{i,1}$ on bull with v_i on path Pm.

We further design unit X and unit Y for each structure as shown in figures below.

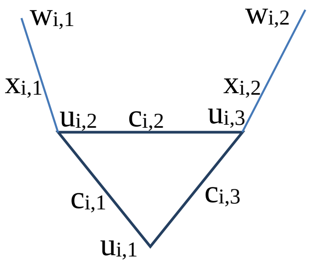

Fig 1: C_3-bull also called as bull graph

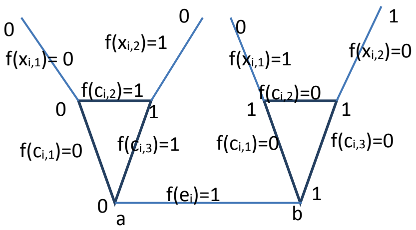

Fig 2: $v_f(0,1)= (5,5), e_f(0,1)=(6,5)$ structure 1 :Type X

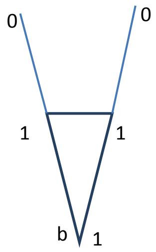

Fig 3: $v_f(0,1)= (3,2)$, $e_f(0,1) = (3,2)$ structure 1

To obtain a path union of larger m>2size, take two copies of type X. By an edge (ab) join the two copies. Here the vertex a is from one copy and b is from other copy. The edge label of (ab) = |1-0| =1. Repeat the process and we will get the path union on even m. For odd m say m = 2t+1 we first obtain the path union on m= 2t and attach type y to the vertex a on P_{2t}(bull) by an edge (ab), the vertex b is from type y. The label of (ab) will be 1.The label number distribution is for m = 2t, we have $v_f(0,1)= (5t,5t)$, $e_f(0,1)=(6t,6t-1)$. For m = 2t+1 we have $v_f(0,1)= (5t+2,5t+3)$,$e_f(0,1)=(6t+3,6t+2)$.It follows that the graph is cordial.

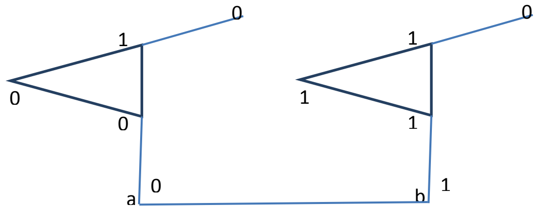

Fig 4: $v_f(0,1)= (5,5), e_f(0,1)=(6,5)$ structure 2 :Type X

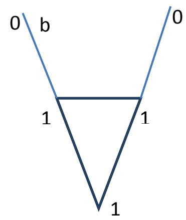

Fig 5: $v_f(0,1)= (3,2), e_f(0,1)=(3,2)$ structure 2 :Type y

The procedure given above is followed for all three structures and we get cordial graph with same label numbers as above.

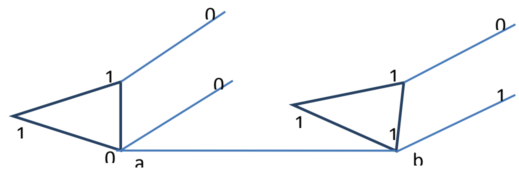

Fig 6: $v_f(0,1)= (5,5), e_f(0,1)=(6,5)$ structure 3 :Type X

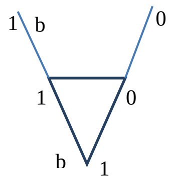

Fig 7: $v_f(0,1)= (3,2), e_f(0,1)=(3,2)$ structure 3 :Type y

3.2 Theorem: Path union of C_3^+ i.e. $G = Pm(C_3^+)$is cordial.

There are two types of structures possible on path union. In one type (structure 1) we use any of the three degree vertex on C_3 crown to fuse with vertex on path P_m. In second type (structure 2) we use any of the pendent vertex of C_3 crown to fuse with vertex on path Pm to obtain G.

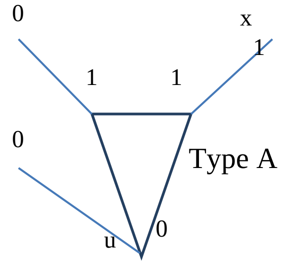

Fig 8: $v_f(0,1)= (3,3)$, $e_f(0,1)=(3,3)$

To design structure 1 we start with A type of copy of graph C_3^+.To obtain P_2(C3+) we attach a copy of type B by an edge to copy of type A of graph. Here new edge is (uv) where u lies on type A graph and v lies on type B graph. and the new edge label is 1.At this stage label number distribution is $v_f(0,1)=(6,6)$, $e_f(0,1)=(6,7)$ and the graph is cordial. To obtain $P_3(C_3^+)$ we choose type B labeling to attach at vertex v of $P_2(C_3^+)$.The new edge is (vv) with label 0.The label numbers are $v_f(0,1)=(9,9)$, $e_f(0,1)=(10,10)$. Then the sequence type A, Type B, type B followed by A,A,B,B….is repeated. The resultant graph is E- cordial.

To obtain cordial label of structure 2 we use Type A and type B labeled copies as shown below. Rest of the procedure and result is same. Thus the graph of structure 2 is also cordial.

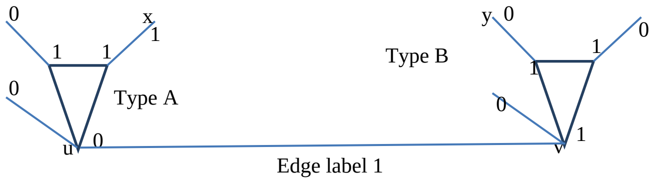

Fig 10: labeled copy of $P_2(C_3^+)$ structure 1

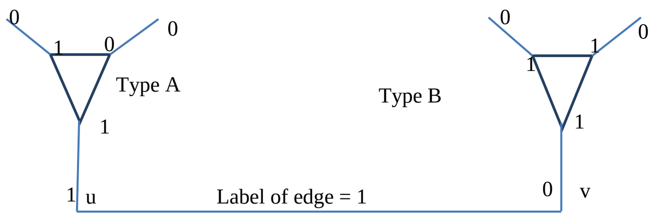

Fig 11: P2(C3+) $v_f(0,1)=(6,6)$, $e_f(0,1)=(6,7)$ structure 2

Final label distribution is: on vertices: vf(0,1) =(3n,3n) and on n is odd number ≥ 3 given by n = 2x+1, distribution on edges $e_f(0,1) = (10+7(x-1),10+7(x-1))$,x=1,2,…and for even n = 2x we have $e_f(0,1) = (10+3(x-1),10+4(x-1))$,x=2,3,…

3.3 Theorem: Path union on bull of C_4 is cordial.

Proof: There are three non-isomorphic structures possible on Pm(C_4 bull).

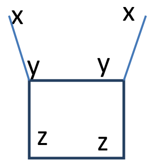

Fig 12: C_4 -bull

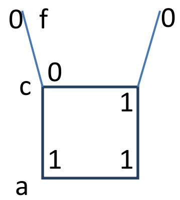

Fig 13: Type A $v_f(0,1)=(3,3)$, $e_f(0,1)=(3,3)$ Structure 1

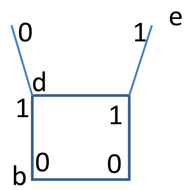

Fig 14: Type B $v_f(0,1)=(3,3)$, $e_f(0,1)=(3,3)$ Structure 1

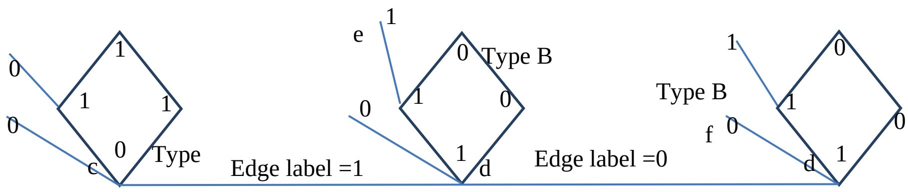

Fig 15: P_3(C_4-bull): $v_f(0,1)=(9,9)$, $e_f(0,1)=(10,10)$ Structure 2

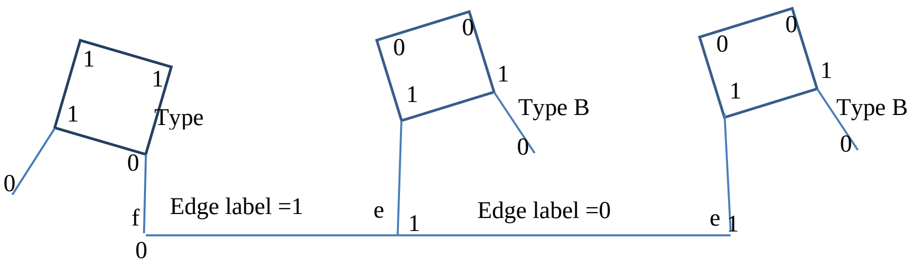

Fig 16: P_3(C_4-bull): $v_f(0,1)=(9,9)$, $e_f(0,1)=(10,10)$ Structure 3

To obtain structure1 we choose the vertex b on type B and followed by type A (vertex a) labelings as shown in figure and the new edge added is (ba) are used to construct the path union. Type A is joined with type A by the edge (aa).This is followed by type A labeling and new edge (aa).After this the sequence repeated is B,A,A,A B….Thus for m ≡ 1(mod 4) type B label is used and for all other m copy type A is used to construct Pm(C_4 bull)

To construct structure 2 Here the sequence observed is type A then B,B,B then A,B,B,B,A….vertex d on type B and vertex c on type A type is used to add new edge and construct the path union. The new edges added are (cd),(dd),(dd),(dd),. Thus for m ≡ 1(mod 4) type A label is used and for all other m copy type B is used to construct Pm(C_4 bull)

To construct structure 3 Here also the sequence observed is type A then B,B,B, then A,B…is followed. vertex e on type B and vertex f on type A is used to add new edge and construct the path union. Thus for m ≡ 1(mod 4) type A label is used and for all other m copy type B is used to construct Pm(C_4 bull)

The label distribution for all three structures is same and given by $v_f(0,1)$= (3m,3m), $e_f(0,1)$ = (3+7x,3+ 7x) when m = 2x+1,when m ≡ 0(mod 4) we have $v_f(0,1)$= (3m,3m), $e_f(0,1)$ = (14x,14x), m = 4x. For m ≡2(mod 4) we have $v_f(0,1)$= (3m,3m), $e_f(0,1)$ = (6+14x,7+14x),m= 2+14x.Thus the graph is cordial.

3.4 Theorem Let C_4^+ -e be the graph obtained by deleting an pendent edge of C_4^+ Then G =Pm(C_4^+-e) is cordial graph.

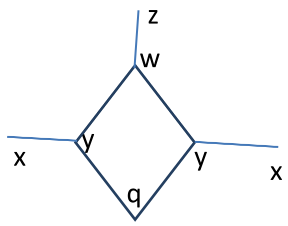

Fig 17: C_4^+ -e, note the points x,y, z, w,q

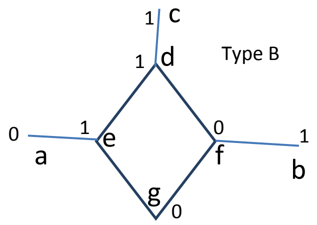

Fig 18: $v_f(0,1)$= (3,4), $e_f(0,1)$=(3,4)

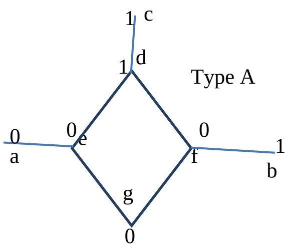

Fig 19: $v_f(0,1)$= (4,3), $e_f(0,1)$=(4,3)

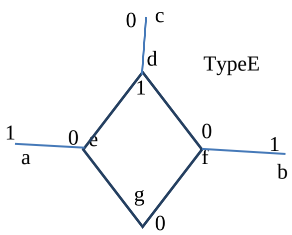

Fig 20: $v_f(0,1)$= (4,3), $e_f(0,1)$=(2,5)

Fig 17 shows that there are five possible different structures of path unions on G. (at five different points x,y,z,w,q.) To construct different structures we start with a path Pm= (v1,v2,v3,…vm).

Structure 1 Fuse the vertex g from different types to verteices on Pm as follows. For m ≡1(m0d 4) use type A For m≡2,3 (m0d 4) use B type of label and for m ≡0 (mod 4) use E type label. Resulting is cordial labeling. The label numbers observed are 1) for m = 2x,x = 1,2,.. we have $v_f(0,1)$= (7x,7x), $e_f(0,1)$=(8(x+1),8(x+1)-1) for m≡2(mod 4) and x is from m = 2x; and for m ≡ o(mod4), write m=2x, $e_f(0,1)$ =(8x-1,8x)

2) for m = 4x+3 (i.e. m ≡3 (mod 4)) we have $v_f(0,1)$=(14x+10,14x+11) and $e_f(0,1)$ = (12+16x,11+16x)

For m = 1+4x (i.e.m≡1(mod 4) we have $v_f(0,1)$=(14x+4,14x+3) and $e_f(0,1)$ = (4+16x,3+16x) The resultant graph is cordial.

Structure 2 Fuse the vertex b from different types to verteices on Pm as follows. For m ≡1(m0d 4) use type A For m≡2,3 (m0d 4) use B type of label and for m ≡0 (mod 4) use E type label. Resulting is cordial labeling. The label numbers observed are

1) for m = 2x,x = 1,2,.. we have $v_f(0,1)$= (7x,7x), $e_f(0,1)$=(8(x+1),8(x+1)-1)for m≡2(mod 4) and x is from m = 2x; and for m ≡ o(mod4), write m=2x, $e_f(0,1)$ =(8x-1,8x)

2) for m = 4x+3 (i.e. m ≡3 (mod 4)) we have $v_f(0,1)$=(14x+10,14x+11) and $e_f(0,1)$ = (12+16x,11+16x) For m = 1+4x (i.e.m≡1(mod 4) we have $v_f(0,1)$=(14x+4,14x+3) and $e_f(0,1)$ = (4+16x,3+16x) The resultant graph is cordial.

Structure 3 Fuse the vertex f from different types to verteices on Pm as follows. For m ≡1(m0d 4) use type A For m≡2,3 (m0d 4) use B type of label and for m ≡0 (mod 4) use E type label. Resulting is cordial labeling. The label numbers observed are 1) for m = 2x,x = 1,2,.. we have $v_f(0,1)$= (7x,7x), $e_f(0,1)$=(8(x+1),8(x+1)-1)for m≡2(mod 4) and x is from m = 2x; and for m ≡ o(mod4), write m=2x, $e_f(0,1)$ =(8x-1,8x)

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Structure 4 Fuse the vertex d from different types to verteices on Pm as follows. For m ≡1(m0d 4) use type A For m≡2,3 (m0d 4) use B type of label and for m ≡0 (mod 4) use E type label. Resulting is cordial labeling. The label numbers observed are 1) for m = 2x,x = 1,2,.. we have $v_f(0,1)$= (7x,7x), $e_f(0,1)$=(8(x+1),8(x+1)-1)for m≡2(mod 4) and x is from m = 2x; and for m ≡ o(mod4), write m=2x, $e_f(0,1)$ =(8x-1,8x)

2) for m = 4x+3 (i.e. m ≡3 (mod 4)) we have $v_f(0,1)$=(14x+10,14x+11) and $e_f(0,1)$ = (12+16x,11+16x)

For m = 1+4x (i.e.m≡1(mod 4) we have $v_f(0,1)$=(14x+4,14x+3) and $e_f(0,1)$ = (4+16x,3+16x) The resultant graph is cordial.

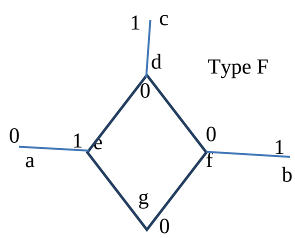

Fig 21: $v_f(0,1)$= (4,3), $e_f(0,1)$=(2,5)

Structure 5 Fuse the vertex c from different types to verteices on Pm as follows. For m ≡1(m0d 4) use type A For m≡2,3

(m0d 4) use B type of label and for m ≡0 (mod 4) use F type label. Resulting is cordial labeling. The label numbers observed are 1) for m = 2x,x = 1,2,.. we have $v_f(0,1)$= (7x,7x), $e_f(0,1)$=(8(x+1),8(x+1)-1)for m≡2(mod 4) and x is from m = 2x; and for m ≡ o(mod4), write m=2x, $e_f(0,1)$ =(8x-1,8x)
2) for m = 4x+3 (i.e. m ≡3 (mod 4)) we have $v_f(0,1)$=(14x+10,14x+11) and $e_f(0,1)$ = (12+16x,11+16x)
For m = 1+4x (i.e.m≡1(mod 4) we have $v_f(0,1)$=(14x+4,14x+3) and $e_f(0,1)$ = (4+16x,3+16x) The resultant graph is cordial.

4. Conclusion

So far a specific graph only was called as bull graph. We have generalized the definition of bull graph to any graph G. Then we have designed path union on bull type of graphs. We have further shown all possible structures (upto isomorphism) on path union follow cordial labeling. (invariance under cordiality)

5. References

1. Andar M, Boxwala S, Limaye N. New families of cordial graphs. J. Combin. Math. Combin. Comput. 2005; 53(134):117-154.
2. Andar M, Boxwala S, Limaye N. On the cordiality of the t-ply Pt(u,v), Ars Combin. 2005; 77(135):245-259.
3. Bapat Mukund Ph.D. thesis submitted to university of Mumbai. India, 2004.
4. Cahit I. Cordial graphs: a weaker version of graceful and harmonious graphs, Ars Combin. 1987; 23:201-207.