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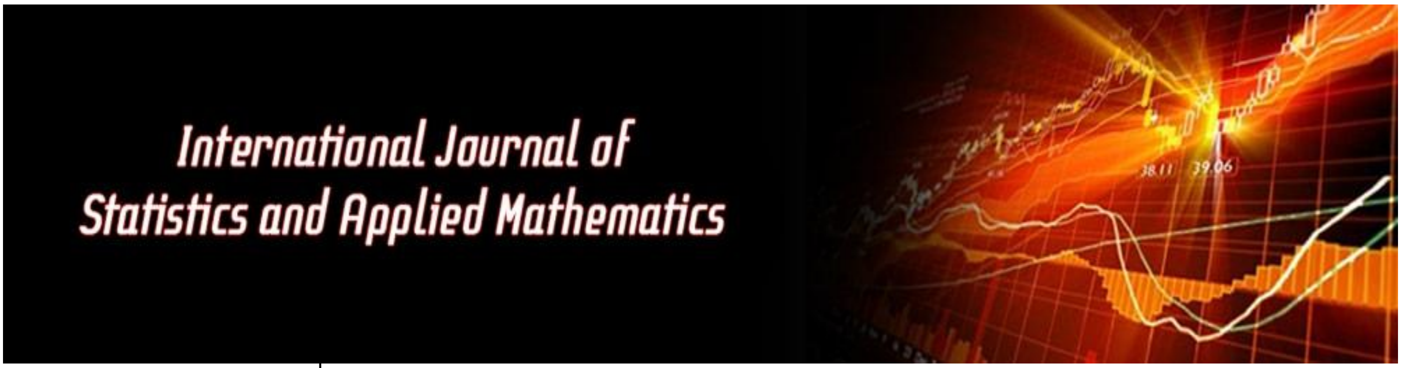

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Control charts and Cusums under linear trend with Rayleigh distribution

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Abstract
This paper intends to assess the performance of Control Charts and CUSUM charts under linear trend with Rayleigh distribution. A distinct approach, in which the operation of the scheme is regarded as forming a Markovian Chain, is set out. The Run Length properties of Control Charts and CUSUM schemes under conditions of slippage in the mean level by step-change to a new sustained level are well documented. Such control procedures are often used where a genuine out of control signal may result from gradual, rather than step changes. This paper presents, the results of evaluation of run length under linear trend with Rayleigh distribution.

Keywords: Control chart, cumulative sum technique, average run length, run length distribution, linear trend, non-homogeneous Morkov chain, transition matrix

1. Introduction

Cumulative Sum Schemes Control Charts were introduced in 1954 by Page (1954). These charts may be used in several situations where a production process is expected to change at an unknown time from an "in control state" to an "out of control state". As soon as one has evidence that the out of control state has observed one would wishes to stop the production process to take remedial measures. CUSUM schemes have proven to be optimal stopping rules in the sense that to minimize expected run length under the out of control state given that the stopping rules has a fixed expected run length under the in-control state Moustakides (1986) [10].

CUSUM control charts have found an interesting variety of applications since their introduction. Several researchers namely Johnson (1966) [7], Hinkley (1970) [3], Brook and Evans (1972) [1], Ashish and Srivastava (1975) [14], Hawkins (1977) [4], Khan (1978) [8], Koning and Does (1988) [9], Rendtel (1990) [12], Rogerson (2006) [13], Cox (2009) [2] have attempted performance of CUSUM charts under various conditions. In most of the research problems the CUSUM chart performance is mainly assessed based on the Average Run Length or its distribution. In other words, the effectiveness of monitoring procedures like Shewart charts with Action limit only, control charts with Warning lines and CUSUM procedures can be demonstrated when there is a slippage in mean level from a target value. This can be done with the help of ARL or other features of run length distributions. The ARL is usually measured on the assumption of step change i.e. abrupt change from the process average. The main purpose of this paper is to assess the performance of control charts and CUSUM charts under linear trend with non-normal distribution namely, Rayleigh. This distribution and its importance is discussed as and when the CUSUM schemes and other control charts performance are assessed. In the subsequent sections we discuss Shewart chart with action line, control chart with warning line and CUSUM procedures.

2. Shewart Control Chart With Action Lines

In the construction of control charts we are using two sets of limits such as action limits or outer limits and warning limits or inner limits. When action lines point plots outside of this limit, a search for an assignable cause is made and corrective action is taken if necessary. Shewart control chart with only action lines, it is denoted by 'Scheme A' and specified distributional assumptions, the evaluation of run length properties follows Geometric distribution with parameter P_A that is

$$\text{ARL} = \frac{1}{P_A} \quad (2.1)$$

Where P_A is the Probability of action limit for a specified process mean.

3. Shewart Control Chart With Warning Lines

In a Shewart control chart with action and warning lines we take decisions with monitoring procedures depend on preceding observations as well as the most recent value. It is denoted by 'Scheme W'. If one or more points fall between the warning line and the central line or very close to the warning line, we should be suspicious that the process may not be operating properly. One possible action to take when this occurs is to increase the sampling frequency. The use of warning limit can increase the sensitivity of the control chart. The complete run length distribution is obtained by using successive powers of the transition matrix. In particular, the ARL is found to be

$$\frac{1 + P_W - P_A}{P_A + P_W (P_W - P_A)} \quad (3.1)$$

Where P_W is the Probability of a violation of warning line which includes more extreme action line violation P_A. The action line scheme is having only two states one Transient and another one is absorbing.

4. Transition Matrices for Control Chart and Cusums with Rayleigh Distribution

4.1 The Importance of Rayleigh Distribution

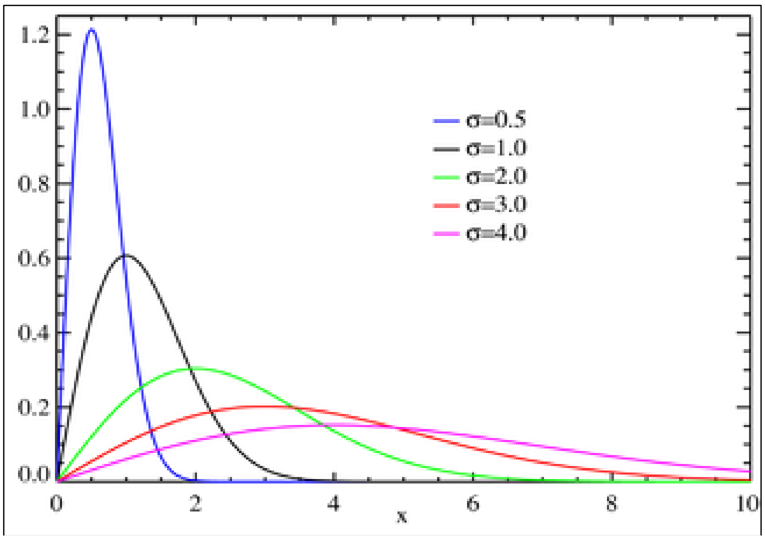
Parametric distributions are often used to model life time and time-to-failure responses. In Probability theory and Statistics, the Rayleigh distribution is a continuous probability distribution.

The Rayleigh probability density function is given by

$$f(x;\sigma) = \frac{x}{\sigma^2} e^{-x^2/2\sigma^2}, \; x \geq 0,$$

For parameter $\sigma > 0$,

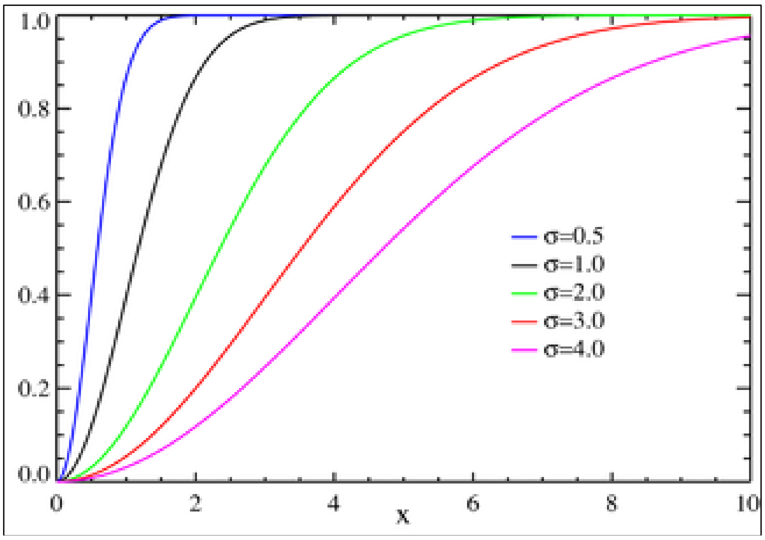
Probability density function



And cumulative distribution function

$$F(x) = 1 - e^{-x^2/2\sigma^2} \text{ For } x \in [0, \infty].$$

Cumulative distribution function



The Mean of Rayleigh distribution is $\sigma\sqrt{\frac{\pi}{2}}$

The Mode of Rayleigh distribution is σ

The Variance of Rayleigh distribution is $\frac{4-\pi}{2}\sigma^2$

Properties

The raw moments are given by: $\mu_k = \sigma^k 2^{k/2}\Gamma(1+k/2)$ Where Γ (2) is the Gamma function.

The mean and variance of a Rayleigh random variable may be expressed as:

$$\mu(X) = \sigma\sqrt{\frac{\pi}{2}} \approx 1.253\sigma \text{, And } \operatorname{var}(X) = \frac{4-\pi}{2}\sigma^2 \approx 0.429\sigma^2 .$$

The mode is σ and the maximum pdf is

$$f_{\max} = f(\sigma;\sigma) = \frac{1}{\sigma}\exp\left(-\frac{1}{2}\right) \approx \frac{0.606}{\sigma}$$

Related distributions

If $R = \sqrt{X^2 + Y^2}$, where $X \sim N(0,\sigma^2)$ and $Y \sim N(0,\sigma^2)$ are independent normal random variables.

If R is a Rayleigh then R^2 has a chi-square distribution with two degrees of freedom: $R^2 \sim x_2^2$

If X has an exponential distribution $X \sim \text{Exponential}(\lambda)$, then $Y = \sqrt{2X\sigma^2\lambda} \sim \text{Rayleigh}(\sigma)$.

If $R \sim \text{Rayleigh}(\sigma)$, then $\sum_{i=1}^{N} R_i^2$ has a Gamma Distribution with Parameters N and $2\sigma^2$: $\left[Y = \sum_{i=1}^{N} R_i^2\right] \sim \Gamma(N, 2\sigma^2)$

Parametric distributions are often used to model life time and time-to-failure responses. In statistical terminology, the Rayleigh distribution is a continuous distribution. For example, the Rayleigh distribution arises when the wind speed is studied with the acceleration of speed towards the orthogonal of two dimensional components. For all these cases we assume that each component is uncorrelated and normally distributed for equal variances. However, the entire phenomena i.e. speed is characterized by a Rayleigh distribution.

4.2 Transition Matrix for Control Chart

The Transition matrices representation for control charts are give below. In case of row labels refer to states at sample (i-1) and column heading to states at sample i. The upper left column partition is the reduced transition matrix after deleting row and column for the absorbing states.

Table 4.1

A. Transition matrix for "Action only" (Shewart) chart		
	clear	**signal**
clear	$1-P_A$	P_A
signal	0	1

Table 4.2

W. Transition matrix for "Action and Warning" control chart			
	clear	**Warning**	**signal**
clear	$1-P_A$	P_W-P_A	P_A
Warning	$1-P_W$	0	P_W
Signal	0	0	1

4.3 Transition Matrix for Cusum Chart with Rayleigh Distribtuion

Brooks and Evans (1972) [1] show that CUSUM procedures may be viewed as Markov chains. However for, continuous distributions, it is necessary to consider the discretization for the markov chain representation and the various states then corresponds to values of the CUSUM at any step. For an instance consider a scheme with decision interval H and reference value K are designed to detect upward shift from a target value. A set of (m + 1) states can be interpreted as the CUSUM values of

$$\leq 0, 0 \text{ to } \frac{H}{m}, \frac{H}{m} \text{ to } \frac{2H}{m}, \text{etc} \text{ -------} (m-1)\frac{H}{m} \text{ to } < H, \text{-------} \geq H \quad (4.3.1)$$

The last of these states that is violation of the decision interval can be thought of as an absorbing barrier.

In the usual Markov chain notation with Transition matrix P and reduced matrix are one which is obtained from the deletion of the row and column representing the absorbing barrier. The well-known result for obtaining ARL from an initial zero CUSUM is the sum of the elements in the first row of $(I-R)^{-1}$. While considering the states the degree of discretization has some effect on the accuracy of ARL determination. In the present study 20 states transition matrices were used. Thus for H=5 and K=0.5 with μ at the target value. The transition matrix in general and for the particular study is shown in tables 4.3 and 5.1 respectively.

Table 4.3

	C. Transition matrix for CUSUM scheme H, K (m + 1 states)						
		0	$\frac{H}{m}$	$\frac{2H}{m}$	- -------------	$\frac{(m-1)H}{m}$	≥H
	≤0	$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	--------------	$P_{0,m-1}$	$P_{0,m}$
CUSUM at	$\frac{H}{m}$	$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	--------------	$P_{1,m-1}$	$P_{1,m}$
(i - 1)th sample	$\frac{2H}{m}$	$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	---------------	$P_{2,m-1}$	$P_{2,m}$
	- - - -	- - - -	- - - -	- - - -	- - - -	- - - -	- - - -
	$\frac{(m-1)H}{m}$	$P_{m-1,0}$	$P_{m-1,1}$	$P_{m-1,2}$	---------------	$P_{m-1,m-1}$	$P_{m-1,m}$
	≥H	0	0	0	--------------	0	1

The entries in the above matrix need some explanation. In the first row, all entries corresponding moves from an initial zero CUSUM, and in the first entry, it indicates that a sample i, the CUSUM remains at or below zero. This means the sample value should not exceed the reference value k. Thus

$$P_{0,0} = P(x \leq K) \quad (4.3.2)$$

For a move from state zero to H/m, the ith sample must have a value between K and (K +H/m). So that the subtraction of reference value gives a CUSUM contribution of H/m. After the discretization,

$$P_{0,1} = P(x = K + 2H/m) \quad (4.3.3)$$

5. Run Length Calculation under Linear Trend with Rayleigh Distribution

The method explained for ARL calculations is applicable only under the assumption that the phenomenon or process average undergoes stable distribution. However, in case of liner trend both the distribution functions and transition matrix changes over the time. These changes can be quantifiable for any specific rate of slippage. Here we get a non-homogenous Markov chain and an alternative method of obtaining ARL is essential. This can be easily derived from the procedures for generalising run length distribution under stable conditions.

The entry $P_{0,m}$ in the transition matrix stands for the probability of occurrence of a signal at the first sample instant. That means $P^{i}_{0,m}$ stands for element in column 0, row m of P^{i}. This gives probability of signal at the i^{th} sample. Successive difference between say $P^{i}_{0,m} - P^{i-1}_{0,m}$ gives the probability of signal at the i^{th} sample. For an increasing i, we get probability distribution of run length.

For an illustration, consider W chart with action line at $3.09\sigma_e$ from a target value and warning limit at $1.96\sigma_e$. Let the process changes to a value $1.00\sigma_e$ from the target value, then for Rayleigh variable

$$P_A = 1 - \Phi(3.09 - 1) = 0.015299$$

$$P_W = 1 - \Phi(1.96 - 1) = 0.0.146607$$

The transition matrix in this case is given by

$$P = \begin{bmatrix} 0.853393 & 0.131308 & 0.015299 \\ 0.853393 & 0 & 0.146607 \\ 0 & 0 & 1 \end{bmatrix}$$

The ARL is $\dfrac{1 + P_W - P_A}{P_A + P_W(P_W - P_A)} = 32.7449$

The probability of a signal at the first sample $P_{0,3}$ is 0.015299. Squaring P, we get

$$\begin{bmatrix} 0.840337 & 0.112058 & 0.047605 \\ 0.72828 & 0.112058 & 0.159663 \\ 0 & 0 & 1 \end{bmatrix}$$

The element $P^{2}_{0,3}$ is 0.0476054. Obviously the probability of a signal a sample 2 is 0.032304. Similarly $P^{2}_{0,3}$ is 0.076889, gives 0.0061690is the probability run length for three samples.

In the case of non-homogenous transition matrix, it is necessary to multiply original P by new transition matrix obtained after allowing a step change in the mean level. We denote the rate of change by Δ and we use ${}^{1}P$, ${}^{2}P$ etc, for the first, second etc samples transition matrices are deduced. In general the Cumulative probability of signal at or before the i^{th} sample is $(0, m)^{th}$ element of the product.

i.e. ${}^{1}P\ {}^{2}P\ {}^{3}P$-------${}^{i}P$

Individual terms of the run length probability distribution are obtained by successive differences of cumulative probabilities.

Reconsider W scheme with A = 3.09, W = 1.96, with 0 shift, we get

P_A =0.000207

P_W =0.019841

We get

$${}^{1}P \begin{bmatrix} 0.980159 & 0.017771 & 0.00207 \\ 0.980159 & 0 & 0.019841 \\ 0 & 0 & 1 \end{bmatrix}$$

At the second sample, the $0.5\sigma_e$ shift i.e. Δ = 0.5 gives

P_A = 0.005628

P_W= 0.053934

We get

$$ {}^{2}P \begin{bmatrix} 0.946066 & 0.048306 & 0.005628 \\ 0.946066 & 0 & 0.053934 \\ 0 & 0 & 1 \end{bmatrix} $$

Also

$$ {}^{1}P\,{}^{2}P \begin{bmatrix} 0.944108 & 0.047347 & 0.008545 \\ 0.927295 & 0 & 0.025357 \\ 0 & 0 & 1 \end{bmatrix} $$

From the above calculations we get the cumulative probabilities of a signal at sample 2 is 0.008545. Subtracting the (0, m)th element of ${}^{1}P$ gives 0.005475. The above computations are presented for the sake of illustration allowing slippages at different levels similar type of calculated and re-designated as ${}^{1}P$ ${}^{2}P$ ${}^{3}P$ -------

For different slippages in mean level, say Δ = 0.01, 0.02, 0.05, 0.1, 0.2 are computed and summarized below.

In case of CUSUM scheme the transition matrix can be obtained by making use of the formulae given in section 4.3.3 these are summarised in the following matrix.

Table 5.1

				Signaling Table For Action And Warning Charts							
											Signal
SHIFT	**P_A**	**P_W**		0.979567322	0.020232231	0.000200447					0.001120045
0	0.000200447	0.02	${}^1P$	0.979567322	0	0.020432678					
				0	0	1		0.97881452	0.020352128	0.000833352	
							${}^1P$ *${}^2P$	0.959006947	0.020352128	0.020640925	0.009329056
0.01				0.979010759	0.020776651	0.00021259		0	0	1	
	0.00021259	0.021	${}^2P$	0.979010759	0	0.020989241					
				0	0	1		0.978235031	0.02088385	0.000881119	
							${}^2P$*${}^3P$	0.957906262	0.02088385	0.021209888	0.016852852
0.02				0.978443039	0.021331584	0.000225378		0	0	1	
	0.000225378	0.022	${}^3P$	0.978443039	0	0.021556961					
				0	0	1		0.976451649	0.022563225	0.000985127	
							${}^3P$*${}^4P$	0.955617693	0.022563225	0.021819083	0.025974939
0.05				0.976671768	0.023060335	0.000267896		0	0	1	
	0.000267896	0.023	${}^4P$	0.976671768	0	0.023328232					
				0	0	1		0.973225054	0.025549373	0.001225573	
							${}^4P$*${}^5P$	0.950776144	0.025549373	0.023674483	0.055767695
0.1				0.973485847	0.026159631	0.000354522		0	0	1	
	0.000354522	0.027	${}^5P$	0.973485847	0	0.026514153					
				0	0	1		0.965846876	0.032326596	0.001826529	
							${}^5P$*${}^6P$	0.940571718	0.032326596	0.027101687	0.077200649
0.2				0.966189411	0.033207053	0.000603536		0	0	1	
	0.000603536	0.034	${}^6P$	0.966189411	0	0.033810589					
				0	0	1		0.935576381	0.059354321	0.005069298	
							${}^6P$*${}^7P$	0.904489885	0.059354321	0.036155794	0.160353622
0.5				0.936141376	0.061431351	0.002427273		0	0	1	
	0.002427273	0.064	${}^7P$	0.936141376	0	0.063858624					
				0	0	1		0.85497677	0.120560962	0.024462269	
							${}^7P$*${}^8P$	0.802326595	0.120560962	0.077112444	0.220349962
1				0.857057081	0.128784994	0.014157925		0	0	1	
	0.014157925	0.143	${}^8P$	0.857057081	0	0.142942919					
				0	0	1					

6. Run Length Properties of Standard Control Charts under Linear Trend with Rayleigh Distribution

The following table 6.1 gives the average run length and other properties of the run length distributions for three basic control charts namely A scheme, W scheme and CUSUM scheme. In these tables Δ values ranges from 0 to 1. Here we note that the shift will be detected rapidly when the trend is greater than 1 standard error per sample in all chart methods.

Table 6.1

Run Length Properties of Control Procedures Under Linear Trend				
			SCHEME	
Δ		**A**	**W**	**C**
0	ARL	504.492	431.7027	400.0484
	DEL*ARL	0	0	0
0.01	ARL	494.9281	422.3233	390.669
	DEL*ARL	4.949281	4.223233	3.90669
0.02	ARL	485.5536	413.1359	381.4817
	DEL*ARL	9.711071	8.262718	7.629633
0.05	ARL	458.5292	386.6888	355.0345
	DEL*ARL	22.92646	19.33444	17.75173
0.1	ARL	416.9404	346.1159	314.4616
	DEL*ARL	41.69404	34.61159	31.44616
0.2	ARL	345.2592	276.6742	245.02
	DEL*ARL	69.05184	55.33484	49.00399
0.5	ARL	199.1828	138.9849	107.3307
	DEL*ARL	99.59141	69.49247	53.66535
1	ARL	86.86585	44.40481	12.75057
	DEL*ARL	41.36585	22.74481	12.75057

The entries in the third and fourth column are obtained by using equation (1) and (2) the first column ARL are obtained by using from initial zero CUSUM which is the sum of the elements in the first row of $(I - R)^{-1}$ of respective slippage values. Here CUSUM scheme is operated with H = 5, K = 0.5 under Rayleigh distribution. The following table gives values of ARL and Δ*ARL (displacement ARL) for these basic control charts and the corresponding data for further two CUSUM scheme with H =8, K=0.25 and H=2.5, K=1

Table 6.2: Further Run Length Data for Control Schemes

	Control Charts								Cusum Schemes					
	A=3.09		A=3		A=3.09,W=1.96		A=3,W=2		H=8,K=0.25		H=5,K=0.5		H=2.5,K=1	
Δ	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**
0	504.492	0	405.0888	0	431.7027	0	376.5596	0	483.2373	0	400.0484	0	335.3028	0
0.005	499.6861	2.498431	401.0746	2.005373	426.9887	2.134944	372.5839	1.862919	478.1382	2.390691	395.3345	1.976672	331.327	1.656635
0.01	494.9281	4.949281	397.1004	3.971004	422.3233	4.223233	368.6484	3.686484	473.3802	4.733802	390.669	3.90669	327.3915	3.273915
0.015	490.2174	7.353261	393.1657	5.897485	417.7058	6.265588	364.7527	5.471291	468.6695	7.030042	386.0516	5.790774	323.4959	4.852438
0.02	485.5536	9.711071	389.2701	7.785402	413.1359	8.262718	360.8966	7.217931	464.0057	9.280113	381.4817	7.629633	319.6397	6.392793
0.025	480.9362	12.0234	385.4133	9.635333	408.613	10.21533	357.0794	8.926986	459.3883	11.48471	376.9588	9.42397	315.8225	7.895564
0.03	476.3647	14.29094	381.5949	11.44785	404.1368	12.1241	353.301	10.59903	454.8168	13.6445	372.4825	11.17448	312.0441	9.361323
0.04	467.3578	18.69431	374.0717	14.96287	395.3223	15.81289	345.8587	13.83435	445.8099	17.83239	363.668	14.54672	304.6018	12.18407
0.05	458.5292	22.92646	366.6975	18.33487	386.6888	19.33444	338.5666	16.92833	436.9813	21.84906	355.0345	17.75173	297.3097	14.86548
0.06	449.8754	26.99253	359.4692	21.56815	378.2328	22.69397	331.4218	19.88531	428.3275	25.69965	346.5785	20.79471	290.1649	17.40989
0.08	433.0786	34.64629	345.4393	27.63515	361.8396	28.94716	317.5626	25.40501	411.5307	32.92246	330.1853	26.41482	276.3057	22.10446
0.1	416.9404	41.69404	331.9596	33.19596	346.1159	34.61159	304.2588	30.42588	395.3925	39.53925	314.4616	31.44616	263.0019	26.30019
0.15	379.3092	56.89639	300.5274	45.07911	309.5695	46.43542	273.2903	40.99354	357.7613	53.6642	277.9152	41.68728	232.0334	34.80501
0.2	345.2592	69.05184	272.0864	54.41728	276.6742	55.33484	245.3479	49.06957	323.7113	64.74226	245.02	49.00399	204.091	40.8182
0.25	314.4494	78.61236	246.3519	61.58798	247.09	61.77249	220.1483	55.03707	292.9015	73.22538	215.4357	53.85893	178.8914	44.72285
0.3	286.5716	85.97148	223.0664	66.91992	220.5084	66.15252	197.435	59.23051	265.0237	79.50711	188.8542	56.65625	156.1782	46.85345
0.4	238.5223	95.40891	182.9322	73.1729	175.2601	70.10405	158.5611	63.42442	216.9744	86.78975	143.6059	57.44235	117.3042	46.92167
0.5	199.1828	99.59141	150.0732	75.03658	138.9849	69.49247	127.1187	63.55935	177.6349	88.81746	107.3307	53.66535	85.86181	42.93091
0.6	166.9744	100.1846	123.1704	73.90225	110.0736	66.04414	101.7867	61.07203	145.4265	87.25589	78.41932	47.05159	60.52984	36.3179
0.8	119.0144	95.21152	83.11087	66.48869	69.21248	55.36999	65.28345	52.22676	97.4665	77.9732	37.55824	30.04659	24.02657	19.22126
1	86.86585	86.86585	56.25815	56.25815	44.40481	44.40481	42.38135	42.38135	65.31796	65.31796	12.75057	12.75057	12.38135	12.38135
1.25	41.84639	52.30799	34.77545	43.46931	27.34616	34.18269	25.96111	32.45139	20.2985	25.37312	10.96111	13.70139	10.96111	13.70139
1.5	24.04675	36.07013	21.74554	32.61831	9.591441	14.38716	17.57647	26.36471	22.68128	34.02191	7.591441	11.38716	7.576474	11.36471
2	8.846306	17.69261	8.609056	17.21811	7.325413	14.65083	9.539973	7.325413	7.480828	14.96166	5.325413	10.65083	6.997273	13.99455
2.5	3.254374	8.135936	3.938282	9.845705	5.369215	13.42304	7.539973	5.369215	1.888896	4.722241	3.369215	8.423037	4.997273	12.49318
3	1.197217	3.591652	1.253455	3.760366	2.361453	7.084358	5.110853	2.361453	0.197217	0.591652	0.361453	1.084358	2.568153	7.704458

7. Conclusions

The various control schemes considered here are, in effect, continuous hypothesis tests. These hypotheses can be stated as

$$H_0 : \mu = T$$

Against the alternatives

$$\left.\begin{array}{l} H_1 : \mu < T \\ H_1 : \mu < T \end{array}\right\} \text{"Single-sided" schemes}$$

$H_0 : \mu \neq T$ "Two-sided" schemes

In real world, it is frequently unknown whether the process averages μ will change suddenly or gradually. Most of the ARL calculations are based on the one standard deviation schemes under a trend alternative.

If trend is expected in the process average, this prior knowledge will be incorporated in to the design of control procedures while considering sampling intervals.

In the case of Rayleigh distribution suggest that there exists less difference in performance among schemes A, W and C under the linear trend than under step change conditions. Where as in scheme C with Rayleigh distribution the lower ARL for slippage of 0.2 to 0.1 standard errors is noted. It can be observed that the standard C scheme with Rayleigh distribution gives somewhat quicker response over the range $0.015 \leq \Delta \leq 0.6$ as compared with schemes of A and W schemes over the range $0.03 \leq \Delta \leq 0.3$.

These results are broadly compatible for those relating to step changes, in that, for example with ARL ≅ 6, at $\Delta = 0.3$ for W and C schemes, the process mean must have shifted above two standard errors by the time the trend is detected. Similarly, for A and C shift is about 3 standard error with ARL ≅ 4.9 at $\Delta = 0.6$. For step changes greater than $2.5\sigma_e$, it is observed that lower ARL for W when compared with C scheme. The same situation is prevailed in the case of slippage greater than $2.5\sigma_e$. In table 6.2, the values of Δ ARL gives for further clarification on the point of selection A, W and C alternatives.

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