International Journal of Statistics and Applied Mathematics 2018; 3(3): 48-50

ISSN: 2456-1452
Maths 2018; 3(3): 48-50

www.mathsjournal.com
Received: 07-03-2018
Accepted: 08-04-2018

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Study of some important theorem on a fuzzy metric space

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Abstract
Fuzzy metric spaces have derived several interesting and appealing result on it. The notion of fuzzy metric space by generalizing the concept of the probabilistic metric space to fuzzy situation.

Keywords: Fuzzy metric spaces, fuzzy point, fuzzy set

1. Introduction
Kramosil and Michalek introduced the notion of fuzzy metric space by generalizing the concept of probabilistic metric space to fuzzy situation our aim is to study the method by which one can generalize the notion of the metric space by setting the distance between two points to be non-negative fuzzy number.
In fuzzy mathematics, the concept of fuzzy point was introduced by CK. Wong redefined by Pupao-Ming and Liu-Ying-Ming. We will follow the definition of fuzzy point given by Pu and Liu. We have defined fuzzy metric space which is regarded as the generalization of ordinary

1.1 Fuzzy Point: A fuzzy set A in x is called fuzzy point iff it taken the value 0 for all y$\in$X except one, say x$\in$X. If its value at X is K where 0<K≤1, *we denote the* fuzzy point byX_k, where the point X is called as its support. That is a fuzzy set A is fuzzy point iff A(X) = 0 for all x$\in$X except at one X and the value of A at X is denoted by A(X) = K, where 0<K≤1.

1.2 Contained: The fuzzy point X_k is said to be contained in a fuzzy set A or it belongs to A, denoted by $X_k \in$A iff K≤A(X).
Obviously, every fuzzy set A in X can be expressed as the union of all fuzzy points which belongs to A. that is, ifX_λ X_μ………..$\in$A.
Then
$A(X) = U\{X_{\lambda,} X_\mu \ldots\ldots\ldots\ldots\}$

1.3 Distance: Let A be a fuzzy set in X. let $X_{\lambda,}$ $X_{\mu,}$ $X_\nu \in$A, Where$[0 < \lambda \leq 1, 0 < \mu \leq 1, 0 < \nu \leq 1]$. We define a fuzzy distance function or metric d on A such that d (X_λ, Y_μ) = {|A(x)-A(Y)|}

Where,
$\boldsymbol{\lambda}$≤A(X) and μ≤A(Y)
Now, we will verify the four properties of metric space which are also the properties of fuzzy metric space.
(a) D $(X_{\lambda,} Y_\mu)$ >0

As |A(X)-A(Y)| > 0, iff X$\neq Y$

(b) D (X_λ, Y_μ) = 0

As A(X)-A(Y) = 0 iff X=Y

(C) D (X_λ, Y_μ) = d (Y_μ,X_λ)

Since,| A(X)-A(Y)| = |A(Y)-A(X)|

Now we consider the fourth properties and will prove that

D(X_λ ,Z_ν) ≤d (X_λ,Y_μ) +d ($Y_{\mu,}Z_\nu$)

Since |A(X)-A (Z)| = |A(X)-A(Y) + A(Y)-A (Z)|

Hence | A(X)-A (Z)|≤|A(X)-A(Y)|+ |A(Y)-A (Z)|

As |**λ**-ν|≤|**λ**-μ|+|μ-ν|

Thus d(X_λ ,Z_ν) ≤d (X_λ, Y_μ) + d ($Y_{\mu,}Z_\nu$)

1.4 Fuzzy metric space: Let A be a fuzzy set in X, and d be a real valued fuzzy metric. Let, $Y_{\mu,}Z_\nu X_\lambda$ are fuzzy points and they belong to A and d (X_λ,Y_μ) = |A(X)-A(Y)| where 0<**λ**≤1, 0<μ≤1, d is called fuzzy metric define on AXA if it satisfies the following conditions:-

(a) D ($X_{\lambda,}Y_\mu$) >0

(b) D (X_λ, Y_μ) = 0, if x=y

(c) D (X_λ, Y_μ) = d (Y_μ,X_λ)

(d) D(X_λ ,Z_ν) ≤ d (X_λ,Y_μ) + d ($Y_{\mu,}Z_\nu$)

Where 0<**λ**≤1, 0<μ≤1, 0<ν≤1.

The function d with properties (a) to (d) is called fuzzy distance or fuzzy metric space on A. the element of A are called fuzzy points of the fuzzy metric space (A, d)
Now we will prove that the following properties.

Theorem 1.1 Given any three fuzzy points, $Y_{\mu,}Z_\nu X_\lambda$ in fuzzy metric space (A, d) then,

|d ($X_{\lambda,}Z_\nu$)-d (Y_μ,Z_ν)| ≤d (X_λ, Y_μ).

Proof: By condition (d) of the definition of fuzzy metric space, we have
D (X_λ ,Z_ν) ≤d (X_λ, Y_μ) + d ($Y_{\mu,}Z_\nu$)

Hence,

D (X_λ ,Z_ν) -d ($Y_{\mu,}Z_\nu$) ≤ d (X_λ,Y_μ … (1)

by (1), we have

|A(X)-A(Y)|-|A(Y)-A (Z) |≤|A(X)-A(Y)|

Interchanging x and y, we obtain

|A(Y)-A (Z)|-|A(X)-A (Z) |≤|A(Y)-A(X)| = |A(X)-A(Y)|

d(Y_μ,Z_ν)-d ($X_{\lambda,}Z_\nu$) ≤ d(Y_μ,Z_ν) …(2)

From (1) and (2), we have

|d($X_{\lambda,}Z_\nu$) -d(Y_μ,Z_ν)| ≤ d (X_λ,Y_μ).

Hence the result.

Theorem 1.2: Given any for points $Y_{\mu,} Z_{\nu} X_{\lambda}, t_{\eta}$ in a fuzzy metric space (A, d),

$d(X_{\lambda,} Z_{\nu}) + d(Y_{\mu,} t_{\eta}) \geq |d(X_{\lambda}, Y_{\mu}) - d(Z_{\nu}, t_{\eta})|$.

Proof: By condition (d) of the definition of fuzzy metric space, we have

$d(X_{\lambda,} Z_{\nu}) \leq d(X_{\lambda}, Y_{\mu}) + d(Y_{\mu}, Z_{\nu})$

Where $0<\lambda\leq1$, $0<\mu\leq1$, $0<\nu\leq1$.

Hence $d(X_{\lambda,} Z_{\nu}) - d(Y_{\mu}, Z_{\nu}) \leq d(X_{\lambda}, Y_{\mu})$.

That is $|A(X)-A(Z)|-|A(Y)-A(Z)| \leq |A(X)-A(Y)|$

Interchanging y and z, we have

$|A(X)-A(Y)|-|A(Z)-A(Y)| \leq |A(X)-A(Z)|$

$|A(X)-A(Y)|-|A(Z)-A(Y)| \leq d(X_{\lambda,} Z_{\nu})$ …. (1)

Again, when we take three fuzzy points $Y_{\mu,} Z_{\nu}, t_{\eta}$ then by properties (d) of the definition of fuzzy metric space, we have

$D(Y_{\mu,} t_{\eta}) \leq d(Y_{\mu}, Z_{\nu}) + d(Z_{\nu}, t_{\eta})$

Or, $d(Y_{\mu,} t_{\eta}) - d(Z_{\nu}, t_{\eta}) \leq d(Y_{\mu}, Z_{\nu})$

That is, $|A(y)-A(t)|-|A(z)-A(t)| \leq |A(y)-A(z)|$

Interchanging z and t, we get

$|A(y)-A(z)|-|A(t)-A(z)| \leq |A(y)-A(t)|$

$|A(y)-A(z)|-|A(t)-A(z)| \leq d(Y_{\mu,} t_{\eta})$ ….(2)

From (1) and (2) we obtain

$d(X_{\lambda,} Z_{\nu}) + d(Y_{\mu,} t_{\eta}) \geq |A(x)-A(y)|-|A(z)-A(y)|+|A(y)-A(z)|-|A(t)-A(z)|$

$=|A(x)-A(y)|-|A(y)-A(z)|+|A(y)-A(z)|-|A(z)-A(t)|$

$=|A(x)-A(y)|-|A(z)-A(t)|$, by virtue of properties (c).

Hence, $d(X_{\lambda,} Z_{\nu}) + d(Y_{\mu,} t_{\eta}) \geq |d(X_{\lambda}, Y_{\mu}) - d(Z_{\nu}, t_{\eta})|$.

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