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Control charts and cusum charts under linear trend with log-logistic distribution

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Abstract
This paper intends to assess the performance of Control Charts and CUSUM charts under linear trend with Log-Logistic distribution. A distinct approach, in which the operation of the scheme is regarded as forming a Markovian Chain, is set out. The Run Length properties of Control Charts and CUSUM schemes under conditions of slippage in the mean level by step-change to a new sustained level are well documented. Such control procedures are often used where a genuine out of control signal may result from gradual, rather than step changes. This paper presents, the results of evaluation of run length under linear trend with Log-Logistic distribution.

Keywords: Control chart, cumulative sum technique, average run length, run length distribution, linear trend, non-homogeneous morkovchain, transition matrix

1. Introduction

Cumulative Sum Schemes Control Charts were introduced in 1954 by Page (1954) [11]. These charts may be used in several situations where a production process is expected to change at an unknown time from an” in control state” to an “out of control state”. As soon as one has evidence that the out of control state has observed one would wishes to stop the production process to take remedial measures. CUSUM schemes have proven to be optimal stopping rules in the sense that to minimize expected run length under the out of control state given that the stopping rules has a fixed expected run length under the in-control state Moustakides (1986) [10].

CUSUM control charts have found an interesting variety of applications since their introduction. Several researchers namely Johnson (1966) [5], Hinkley (1970) [3], Brook and Evans (1972) [1], Ashish and Srivastava (1975) [14], Hawkins (1977) [4], Khan (1978) [8], Koning and Does (1988) [9], Rendtel (1990) [12], Rogerson (2006) [13], Cox (2009) [2] have attempted performance of CUSUM charts under various conditions. In most of the research problems the CUSUM chart performance is mainly assessed based on the Average Run Length or its distribution. In other words, the effectiveness of monitoring procedures like Shewart charts with Action limit only, control charts with Warning lines and CUSUM procedures can be demonstrated when there is a slippage in mean level from a target value. This can be done with the help of ARL or other features of run length distributions. The ARL is usually measured on the assumption of step change i.e. abrupt change from the process average. The main purpose of this paper is to assess the performance of control charts and CUSUM charts under linear trend with non-normal distribution namely, Log-Logistic. This distribution is considered because of its applications in the real world. This distribution and its importance is discussed as and when the CUSUM schemes and other control charts performance are assessed. In the subsequent sections we discuss Shewart chart with action line, control chart with warning line and CUSUM procedures.

2. Shewart Control Chart with Action Lines

In the construction of control charts we are using two sets of limits such as action limits or outer limits and warning limits or inner limits. When action lines point plots outside of this limit, a search for an assignable cause is made and corrective action is taken if necessary.

Shewart control chart with only action lines, it is denoted by ‘Scheme A’ and specified distributional assumptions, the evaluation of run length properties follows Geometric distribution with parameter P_A that is

$$\text{ARL} = \frac{1}{P_A} \tag{2.1}$$

Where P_A is the Probability of action limit for a specified process mean.

3. Shewart Control Chart with Warning Lines

In a Shewart control chart with action and warning lines we take decisions with monitoring procedures depend on preceding observations as well as the most recent value. It is denoted by ‘Scheme W’. If one or more points fall between the warning line and the central line or very close to the warning line, we should be suspicious that the process may not be operating properly. One possible action to take when this occurs is to increase the sampling frequency. The use of warning limit can increase the sensitivity of the control chart. The complete run length distribution is obtained by using successive powers of the transition matrix. In particular, the ARL is found to be

$$\frac{1 + P_W - P_A}{P_A + P_W\,(P_W - P_A)} \tag{3.1}$$

Where P_W is the Probability of a violation of warning line which includes more extreme action line violation P_A. The action line scheme is having only two states one Transient and another one is absorbing.

4. Transition Matrices for Control Chart and Cusums with Log-Logistic Distribution

4.1 The Importance of Log Logistic Distribution

In Probability and Statistics, the log-logistic distribution is a continuous probability distribution for a non-negative random variable. It is used in survival analysis as a parametric model for events whose rate increases initially and decreases later, for example mortality from cancer following diagnosis or treatment. It has also been used in hydrology to model stream flow and precipitation, and in economics as a simple model of the distribution of wealth or income. The log-logistic distribution is the probability distribution of a random variable whose logarithm has a logistic distribution. It is similar in shape to the log-normal distribution but has long tails.

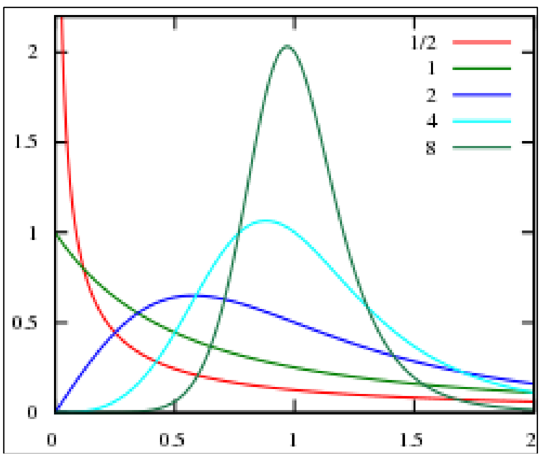
The parameter $\alpha > 0$ is a scale parameter and is also the median of the distribution. The parameter $\beta > 0$ is a shape parameter.

The cumulative distribution function is

$$\begin{aligned} F(x;\alpha,\beta) &= \frac{1}{1+(x/\alpha)^{-3}} \\ &= \frac{(x/\alpha)^3}{1+(x/\alpha)^3} \\ &= \frac{x^3}{\alpha^3 + x^3} \end{aligned} \tag{4.1}$$

Where $x > 0$, $\alpha > 0$, $\beta > 0$.

Probability density function



$\alpha = 1$, values of β as shown in legend

The probability density function is

$$f(x;\alpha,\beta) = \frac{(\beta/\alpha)(x/\alpha)^{\beta-1}}{[1+(x/\alpha)^{\beta}]^2} \qquad (4.2)$$

Where, $x > 0$, $\alpha > 0$, $\beta > 0$.

The kth raw moment exists only when $k < \beta$, when it is given by

$$\begin{aligned} E(X^k) &= \alpha^k B(1-k/\beta,\ 1+k/\beta) \\ &= \alpha^k \frac{k\pi/\beta}{\sin(k\pi/\beta)} \end{aligned} \qquad (4.3)$$

Where B () is the beta function.

$$E(X) = \alpha b/\sin b, \quad \beta > 1,$$

And the variance is

$$\operatorname{Var}(X) = \alpha^2(2b/\sin 2b - b^2/\sin^2 b), \ \beta > 2.$$

If X has a log-logistic distribution with scale parameter α and shape parameter β then $Y = \log(X)$ has a log-logistic distribution with location parameter log (α) and scale parameter $1/\beta$. This situation is of special interest because a CUSUM appropriate to a Log Logistic distribution may be used.

4.2Transition Matrix for Control Chart

The Transition matrices representation for control charts are give below. In case of row labels refer to states at sample (i-1) and column heading to states at sample i. The upper left column partition is the reduced transition matrix after deleting row and column for the absorbing states.

Table 4.1

A. Transition matrix for "Action only" (Shewart) chart		
	clear	**signal**
Clear	$1-P_A$	P_A
Signal	0	1

Table 4.2

W. Transition matrix for "Action and Warning" control chart			
	clear	**Warning**	**signal**
clear	$1-P_A$	P_W-P_A	P_A
Warning	$1-P_W$	0	P_W
Signal	0	0	1

4.3 Transition Matrix for Cusum Chart with Log-Logistic Distribution

Brooks and Evans (1972) show that CUSUM procedures may be viewed as Markov chains. However for, continuous distributions, it is necessary to consider the discretization for the markov chain representation and the various states then corresponds to values of the CUSUM at any step. For an instance consider a scheme with decision interval H and reference value K are designed to detect upward shift from a target value. A set of (m + 1) states can be interpreted as the CUSUM values of

$$\leq 0, 0 \text{ to } \frac{H}{m}, \frac{H}{m} \text{ to } \frac{2H}{m}, \text{etc} \ ------- (m-1)\frac{H}{m} \text{ to } < H, ------- \geq H. \qquad (4.3.1)$$

The last of these states that is violation of the decision interval can be thought of as an absorbing barrier.

In the usual Markov chain notation with Transition matrix P and reduced matrix are one which is obtained from the deletion of the row and column representing the absorbing barrier. The well-known result for obtaining ARL from an initial zero CUSUM is the sum of the elements in the first row of $(I-R)^{-1}$. While considering the states the degree of discretization has some effect on the accuracy of ARL determination. In the present study 20 states transition matrices were used. Thus for H = 5 and K = 0.5 with μ at the target value. The transition matrix in general and for the particular study is shown in tables 4.3 and 5.1 respectively.

Table 4.3: C. Transition matrix for CUSUM scheme H, K (m + 1 states)

		0	$\frac{H}{m}$	$\frac{2H}{m}$	--------------	$\frac{(m-1)H}{m}$	≥H
	≤0	$P_{0,0}$	$P_{0,1}$	$P_{0,2}$	--------------	$P_{0,m-1}$	$P_{0,m}$
CUSUM at	$\frac{H}{m}$	$P_{1,0}$	$P_{1,1}$	$P_{1,2}$	--------------	$P_{1,m-1}$	$P_{1,m}$
(i - 1)th sample	$\frac{2H}{m}$	$P_{2,0}$	$P_{2,1}$	$P_{2,2}$	---------------	$P_{2,m-1}$	$P_{2,m}$
	- - - -	- - - -	- - - -	- - - -	- - - -	- - - -	- - - -
	$\frac{(m-1)H}{m}$	$P_{m-1,0}$	$P_{m-1,1}$	$P_{m-1,2}$	---------------	$P_{m-1,m-1}$	$P_{m-1,m}$
	≥H	0	0	0	--------------	0	1

The entries in the above matrix need some explanation. In the first row, all entries corresponding moves from an initial zero CUSUM, and in the first entry, it indicates that a sample i, the CUSUM remains at or below zero. This means the sample value should not exceed the reference value k. Thus

$$P_{0,0} = P(x \le K) \tag{4.3.2}$$

For a move from state zero to H/m, the ith sample must have a value between K and (K +H/m). So that the subtraction of reference value gives a CUSUM contribution of H/m. After the discretization,

$$P_{0,1} = P(x = K + 2H/m) \tag{4.3.3}$$

5. Run Length Calculation under Linear Trend with Log Logistic Distribution

For calculating Run length distribution under linear trend with Log Logistic distribution, we make use of the discussion and procedure given in chapter-II. From this discussion we can obtain Transition Probability Matrix for the standard control charts viz., Action scheme, Warning scheme and CUSUM scheme.

For an illustration, consider W chart with action line at $3.09\sigma_e$ from a target value and warning limit at $1.96\sigma_e$. Let the process slippage to a value $1.00\sigma_e$ from the target value, then for Log-logistic variable

$$P_A = 1 - \Phi(3.09 - 1) = 0.015299$$

$$P_W = 1 - \Phi(1.96 - 1) = 0.0.146607$$

The transition matrix in this case is given by

$$P = \begin{bmatrix} 0.853393 & 0.131308 & 0.015299 \\ 0.853393 & 0 & 0.146607 \\ 0 & 0 & 1 \end{bmatrix}$$

The ARL is

$$\frac{1 + P_W - P_A}{P_A + P_W(P_W - P_A)} = 32.74439194$$

The probability of a signal at the first sample $P_{0,3}$ is 0.015299. Squaring P, we get

$$\begin{bmatrix} 0.844923 & 0.110376 & 0.04470 \\ 0.724147 & 0.110376 & 0.155077 \\ 0 & 0 & 1 \end{bmatrix}$$

The element $P^2_{0.3}$ is 0.04470. Obviously the probability of a signal a sample 2 is 0.030542. Similarly $P^3_{0.3}$ is 0.072441, gives 0.030542 is the probability run length for three samples.

In the case of non-homogenous transition matrix, it is necessary to multiply original P by new transition matrix obtained after allowing a step change in the mean level. We denote the rate of change by Δ and we use ^1P, ^2P etc, for the first, second etc samples transition matrices are deduced. In general the Cumulative probability of signal before the ith sample is (0, m)th element of the product, i.e. $^1P\ ^2P\ ^3P$-------^iP.

Individual terms of the run length probability distribution are obtained by successive differences of cumulative probabilities.
Reconsider W scheme with A = 3.09, W = 1.96, with 0 shift we get
P_A =0.0002
P_W =0.020433

We get

$$^{1}P = \begin{bmatrix} 0.979567 & 0.020232 & 0.0002 \\ 0.979567 & 0 & 0.020433 \\ 0 & 0 & 1 \end{bmatrix}$$

At the second sample, the $0.5\sigma_e$ shift i.e. $\Delta = 0.5$ gives
P_A = 0.002427
P_W = 0.063859

We get

$$^{2}P = \begin{bmatrix} 0.936141 & 0.061431 & 0.002427 \\ 0.936141 & 0 & 0.063859 \\ 0 & 0 & 1 \end{bmatrix}$$

Also

$$^{1}P\ ^{2}P = \begin{bmatrix} 0.935954 & 0.060176 & 0.00387 \\ 0.917014 & 0.060176 & 0.022810 \\ 0 & 0 & 1 \end{bmatrix}$$

From the above calculations we get the cumulative probabilities of a signal at sample 2 is 0.008545. Subtracting the $(0, m)^{th}$ element of ^{1}P gives 0.00367. The above computations are presented for the sake of illustration allowing slippages at different levels similar type of calculated and re-designated as 1P 2P 3P-------
For different slippages in mean level, say Δ = 0.01, 0.02, 0.05, 0.1, 0.2 are computed and summarized below.
In case of CUSUM scheme the transition matrix can be obtained by making use of the formulae given in section 4.3.3 These are summarized in the following matrix.

Table 5.1: Signaling Table for Action & Warning Charts

Shift	**P_A**	**P_W**		**0.9801589**	**0.0177707**	**0.0020704**					
0	0.0020704	0.0198411	^{1}P	0.9801589	0	0.0198411					
				0	0	1		0.9777296	0.0177699	0.0045005	
							^{1}P *^{2}P	0.9603186	0.0177699	0.0219114	0.00243
0.01				0.9797581	0.0181297	0.0021123		0	0	1	
	0.0021123	0.0202419	^{2}P	0.9797581	0	0.0202419					
				0	0	1		0.9772805	0.0181215	0.0045979	
							^{2}P*^{3}P	0.9595253	0.0181215	0.0223532	0.002486
0.02				0.9793492	0.0184959	0.0021549		0	0	1	
	0.0021549	0.0206508	^{3}P	0.9793492	0	0.0206508					
				0	0	1		0.9759645	0.019234	0.0048014	
							^{3}P*^{4}P	0.9578742	0.019234	0.0228917	0.002646
0.05				0.9780722	0.0196396	0.0022882		0	0	1	
	0.0022882	0.0219278	^{4}P	0.9780722	0	0.0219278					
				0	0	1		0.9735333	0.0212292	0.0052375	
							^{4}P*^{5}P	0.9543696	0.0212292	0.0244012	0.002949
0.1				0.975766	0.0217051	0.0025288		0	0	1	
	0.0025288	0.024234	^{5}P	0.975766	0	0.024234					
				0	0	1		0.9679466	0.0258683	0.0061851	
							^{5}P*^{6}P	0.9468839	0.0258683	0.0272478	0.003656
0.2				0.9704006	0.0265107	0.0030887		0	0	1	
	0.0030887	0.0295994	^{6}P	0.9704006	0	0.0295994					
				0	0	1		0.9431442	0.0468759	0.00998	
							^{6}P*^{7}P	0.9180633	0.0468759	0.0350609	0.006891
0.5				0.9460663	0.0483057	0.005628		0	0	1	
	0.005628	0.0539337	^{7}P	0.9460663	0	0.0539337					
				0	0	1		0.8485901	0.1242265	0.0271834	
								0.8073664	0.1242265	0.0684071	0.021555
1				0.853393	0.1313085	0.0152985	^{7}P*^{8}P	0	0	1	
	0.0152985	0.146607	^{8}P	0.853393	0	0.146607					
				0	0	1					

In the above tables, if we multiply 3.2with 3.3 it gives cumulative probability of signal by sample 2 is 0.002064. Similarly one can compute the moments with expected values of appropriate functions of run length.

6. Run Length Properties of Standard Control Charts under Linear Trend with Log Logistic Distribution

The following table 6.1 gives the average run length and other properties of the run length distributions for three basic control charts namely a scheme, W scheme and CUSUM scheme. In these tables Δ values ranges from 0 to 1. Here we note that the shift will be detected rapidly when the trend is greater than 1 standard error per sample in all charts methods.

Table 6.1: Run Length Properties of Control Procedures under Linear Trend

			Scheme	
Δ		**A**	**W**	**C**
0	ARL	471.9074	328.6031	370.7045
	DEL*ARL	0	0	0
0.01	ARL	447.7653	313.0634	192.5529
	DEL*ARL	4.477653	3.130634	1.925529
0.02	ARL	425.1167	298.4103	172.5522
	DEL*ARL	8.502335	5.968205	3.451045
0.05	ARL	365.1412	259.2238	139.6568
	DEL*ARL	18.25706	12.96119	6.982838
0.1	ARL	286.8255	207.0704	129.5647
	DEL*ARL	28.68255	20.70704	12.95647
0.2	ARL	185.0777	137.0706	125.9978
	DEL*ARL	37.01555	27.41412	25.19957
0.5	ARL	69.9616	52.43937	118.6587
	DEL*ARL	34.9808	26.21969	59.32937
1	ARL	33.06159	22.27797	116.6856
	DEL*ARL	33.06159	22.27797	116.6856

The entries in the third and fourth column are obtained by using equation (2.1) and (3.1) the first column ARL are obtained by using from initial zero CUSUM which is the sum of the elements in the first row of $(I - R)^{-1}$ of respective slippage values. Here CUSUM scheme is operated with H = 5, K = 0.5 under log logistic distribution. The following table gives values of ARL and Δ*ARL (displacement ARL) for these basic control charts and the corresponding data for further two CUSUM scheme with H =8, K=0.25 and H = 2.5, K =1.

Control Charts							Cusum schemes							
		A=3.09		A=3		A=3.09,W=1.96	A=3,W=2		H=8,K=0.25		H=5,K=0.5		H=2.5,K=1	
Δ	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**	**ARL**	**Δ*ARL**
0	471.9074	0	328.6031	0	131.1323	0	134.1699	0	349.8561	0	370.7045	0	360.8578	0
0.005	459.6429	2.298215	320.7187	1.603594	129.5022	0.647511	132.2731	0.661365	250.2714	1.251357	242.5532	1.212766	231.3672	1.156836
0.01	447.7653	4.477653	313.0634	3.130634	127.8953	1.278953	130.4084	1.304084	188.856	1.88856	192.5529	1.925529	192.3664	1.923664
0.015	436.261	6.543915	305.6296	4.584445	126.3115	1.894673	128.5753	1.928629	149.8559	2.247839	182.5526	2.738288	181.3656	2.720484
0.02	425.1167	8.502335	298.4103	5.968205	124.7504	2.495008	126.7733	2.535467	143.868	2.877359	172.5522	3.451045	171.3648	3.427295
0.025	414.3201	10.358	291.3983	7.284958	123.2117	3.080291	125.002	3.125049	139.8557	3.496393	154.5519	3.863799	161.364	4.0341
0.03	403.8588	12.11576	284.5871	8.537613	121.695	3.650851	123.2606	3.697819	133.9155	4.017464	148.6097	4.458291	154.3022	4.629067
0.04	383.8963	15.35585	271.5413	10.86165	118.7268	4.749073	119.8661	4.794645	131.9423	5.277693	142.6338	5.705352	151.2722	6.050887
0.05	365.1412	18.25706	259.2238	12.96119	115.8435	5.792176	116.5858	5.829292	149.9662	7.498308	139.6568	6.982838	147.2451	7.362256
0.06	347.5121	20.85072	247.5891	14.85534	113.0428	6.78257	113.4159	6.804955	129.9897	7.799379	135.6797	8.140782	143.2181	8.593085
0.08	315.3352	25.22681	226.2008	18.09607	107.6805	8.614443	107.3923	8.591388	127.0356	10.16285	132.7255	10.61804	141.1641	11.29313
0.1	286.8255	28.68255	207.0704	20.70704	102.6229	10.26229	101.767	10.1767	123.8687	12.38687	129.5647	12.95647	138.3536	13.83536
0.15	228.7016	34.30524	167.4577	25.11866	91.20719	13.68108	89.27462	13.39119	122.186	18.3279	127.8849	19.18273	136.9764	20.54646
0.2	185.0777	37.01555	137.0706	27.41412	81.36279	16.27256	78.73662	15.74732	120.2839	24.05677	125.9978	25.19957	133.8433	26.76866
0.25	151.9843	37.99607	113.5382	28.38455	72.88245	18.22061	69.83811	17.45953	118.3742	29.59355	123.11	30.7775	130.711	32.67775
0.3	126.6212	37.98635	95.14802	28.54441	65.58263	19.67479	62.31365	18.6941	117.4573	35.2372	122.2214	36.66643	126.5795	37.97385
0.4	91.6571	36.66284	69.1273	27.65092	53.89679	21.55872	50.53079	20.21232	116.6038	46.64151	120.4418	48.17672	124.3188	49.72753
0.5	69.9616	34.9808	52.43937	26.21969	45.23702	22.61851	42.00476	21.00238	115.7263	57.86315	118.6587	59.32937	122.0611	61.03056
0.6	56.04004	33.62402	41.4113	24.84678	38.79635	23.27781	35.771	21.4626	115.3414	69.20482	117.8721	70.72324	119.8063	71.88376
0.8	40.56173	32.44938	28.70338	22.9627	30.3401	24.27208	27.70622	22.16498	114.981	91.98484	117.2871	93.82971	119.3048	95.44385
1	33.06159	33.06159	22.27797	22.27797	25.43894	25.43894	23.07402	23.07402	114.8988	114.8988	116.6856	116.6856	116.8139	116.8139
1.25	28.39658	35.49572	18.1321	22.66513	21.9301	27.41263	6.503641	8.129552	114.7245	143.4056	116.1587	145.1984	116.2148	145.2686
1.5	26.033	39.0495	15.96246	23.94369	19.93487	29.90231	4.608738	6.913108	114.2181	171.3272	115.6026	173.404	115.6317	173.4475
2	17.25648	34.51296	10.24578	20.49156	5.264789	10.52958	2.031458	4.062916	113.8119	227.6238	114.3994	228.7987	115.1991	230.3982
2.5	6.321457	15.80364	5.21478	13.03695	3.215478	8.038695			113.7027	284.2569	114.7966	286.9915	114.4628	286.1569
3	2.124578	6.373734	2.312457	6.937371	3.215478	9.646434			113.3756	340.1267	114.63	343.89	114.3805	343.1414

7. Conclusions

The various control schemes considered here are, in effect, continuous hypothesis tests. These hypotheses can be stated as

$$H_0 : \mu = T$$

Against the alternatives

$$\left.\begin{matrix} H_1 : \mu < T \\ H_1 : \mu < T \end{matrix}\right\} \text{"Single-sided" schemes}$$

$H_0 : \mu \neq T$ "Two-sided" schemes

In real world, it is frequently unknown whether the process averages μ will change suddenly or gradually. Most of the ARL calculations are based on the one standard deviation schemes under a trend alternative.

If trend is expected in the process average, this prior knowledge will be incorporated in to the design of control procedures while considering sampling intervals.

In the case of Log Logistic distribution suggest that there exists less difference in performance among schemes A, W and C under the linear trend than under step change conditions. Where as in scheme C with Log Logistic distribution the lower ARL for slippage of 0.2 to 0.1 standard errors is noted. It can be observed that the standard C scheme with Log Logistic distribution gives somewhat quicker response over the range $0.015 \leq \Delta \leq 0.6$ as compared with schemes of A and W schemes over the range $0.03 \leq \Delta \leq 0.3$.

These results are broadly compatible for those relating to step changes, in that, for example with ARL ≅ 6, at Δ = 0.3 for W and C schemes, the process mean must have shifted above two standard errors by the time the trend is detected. Similarly, for A and C shift is about 3 standard error with ARL ≅ 4.9 at Δ= 0.6. For step changes greater than 2. $5\sigma_\varepsilon$, it is observed that lower ARL for W when compared with C scheme. The same situation is prevailed in the case of slippage greater than $2.5\sigma_\varepsilon$. In table 6.2, the values of Δ ARL gives for further clarification on the point of selection A, W and C alternatives.

8. References

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