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S Saalini
Department of Statistics,
Loyola College, Chennai,
Tamil Nadu, India

C Santharam
Associate Professor (Retired),
Department of Statistics,
Loyola College, Chennai,
Tamil Nadu, India

Correspondence
S Saalini
Department of Statistics,
Loyola College, Chennai,
Tamil Nadu, India

Efficiency of NNBD and NNBIBD using autoregressive model

S Saalini and C Santharam

Abstract
Neighbour Balanced Block Designs, permitting the estimation of direct and neighbour effects, are used when the treatment applied to one experimental plot may affect the response on neighbouring plots as well as the response on the plot to which it is applied. The allocation of treatments in these designs is such that every treatment occurs equally often with every other treatment as neighbours. Neighbour Balanced Block Designs for observations correlated within a block have been investigated for the estimation of direct as well as left and right neighbour effects of treatments. It is observed that efficiency for direct as well as neighbour effects is high, in case of Complete block designs i.e. m = 0 for Nearest Neighbour correlation structure with ρ in the interval 0.1 to 0.7. In case of incomplete block designs m = 1,2,…, v-4 for Nearest Neighbour correlation structure turns out to be more efficient with ρ in the interval 0.1 to 0.7 using AR (1) model.

Keywords: Neighbour balanced block design, generalized least squares, autoregressive model, efficiency

1. Introduction
A Neighbour balanced block design (NBBD) with one-sided neighbour effects (say, left side) is said to be circular if the treatment in the left border is the same as the treatment in the right-end inner plot and a NBB design with two-sided neighbour effects is said to be circular if the treatment in the left border is the same as the treatment in the right-end inner plot and the treatment in the right border is the same as the treatment in the left-end inner plot. The assumptions in the classical (Fisherian) block model are that the response on a plot to a particular treatment does not affect the response on the neighbouring plots and the fertility associated with plots in a block is constant. However, in many fields of agricultural research, like horticultural and agro-forestry experiments, the treatment applied to one experimental plot in a block may affect the response on the neighbouring plots if the blocks are linear with no guard areas between the plots. The estimates of treatment differences may therefore deviate because of interference from neighbouring units. Neighbour balanced block designs, where the allocation of treatments is such that every treatment occurs equally often with every other treatment as neighbours, are used for modeling and controlling interference effects between neighbouring plots. Azais *et al*. (1993) [16] obtained a series of efficient neighbour designs with border plots that are balanced in v-1 blocks of size v and v blocks of size v-1, where v is the number of treatments. Druilhet (1999) [15] studied optimality of circular neighbour balanced block designs obtained by Azais *et al*. (1993) [16]. Has given some designs for studying one-sided neighbour effects. These neighbour balanced block designs have been developed under the assumption that the observations within a block are uncorrelated. In situations where the correlation structure among the observations within a block is known, may be from the data of past similar experiments, it may be advantageous to use this information in designing an experiment and analyzing the data so as to make more precise inference about treatment effects (Gill and Shukla, 1985) [1]. Considered two related models for interference and have shown that optimal designs for one model can be obtained from optimal designs for the other model. Have given variance balanced designs under interference and dependent observations. Tomar and Seema Jaggi (2007) [3] observed that efficiency is quite high, in case of complete block designs for both AR (1) and NN correlation structures.

Santharam. C & K. N. Ponnuswamy (1997) [2] observed that the performance of NNBD is quite satisfactory for the remaining models. Neighbour balanced block designs for observations correlated within a block have been investigated for the estimation of direct as well as left and right neighbour effects of treatments. The performance of these designs for AR (1), MA (1) and ARMA (1, 1) and NN error correlation structure is studied when generalized least squares estimation is used R. Senthil Kumar and C. Santharam (2013) [5]. The performance of these designs for Auto Regressive with first order when generalized least squares estimation is used. Also we have investigated the efficiency of Nearest Neighbour Balanced Block Design (NNBD) in comparison to regular block design when the error follows first order correlated models. And we have also investigated the efficiency of Nearest Neighbour Balanced Incomplete Block Design (NNBIBD) in comparison to regular block design when the error follows first order correlated model. Finally we have presented the results and conclusion.

2. Model and Definition

Let Δ be a class of binary neighbour balanced block designs with n = bk units that form b blocks each containing k units. Let Y_{ij} be the response from the i^{th} plot in the j^{th} block (i = 1, 2... k; j = 1, 2... b). It is assumed that the experiment is conducted in small plots in well separated linear blocks with no guard areas between the plots in a block. Further, the layout includes border plots at both ends of every block, i.e. at 0^{th} and $(k + 1)^{th}$ position and observations for these units are not modeled. It is further assumed that the design is circular, that is the treatment on border plots is same as the treatment on the inner plot at the other end of the block. Following fixed effects additive model is considered for analyzing a neighbour balanced block design under correlated observations:

$$Y_{ij} = \mu + \tau_{(i,j)} + l_{(i-1,j)} + \gamma_{(i+1,j)} + \beta_j + e_{ij}\,, \qquad (2.1)$$

Where μ is the general mean, τ (i, j) is the direct effect of the treatment in the i^{th} plot of j^{th} block, β_j is the effect of the j^{th} block. $l_{(i-1,j)}$ is the left neighbour effect due to the treatment in the $(i-1)^{th}$ plot of j^{th} block. $\gamma_{(i+1,j)}$ is the right neighbour effect due to the treatment in the $(i+1)^{th}$ plot in j^{th} block. e_{ij} are error terms distributed with mean zero and a variance-covariance structure $\Omega = I_b \otimes \Lambda$ (I_b is an identity matrix of order b and ⊗ denotes the kronecker product). Assuming no correlation among the observations between the blocks and correlation structure between plots within a block to be the same in each block, Λ is the correlation matrix of k observations within a block. If the errors within a block follow a first order Autoregressive AR(1) structure, then Λ is a k × k matrix with (i, i')th entry (i, i' = 1, 2,..., k) as $\rho^{|i-i'|}$, |ρ| < 1. The NN correlation structure, the Λ is a matrix with diagonal entries as 1 and off-diagonal entries as ρ. Model (2.1) can be rewritten in the matrix notation as follows:

$$Y = \mu + \Delta'\tau + \Delta'_1\, l + \Delta'_2\, \gamma + D'\beta + e \qquad (2.2)$$

Where Y is $n\times1$ vector of observations, 1 is $n\times1$ vector of ones, Δ϶ is an $n\times v$ incidence matrix of observations versus direct treatments, τ is $v\times1$ vector of direct treatment effects, Δ'_1 is a $n\times v$ matrix of observations versus left neighbour treatment, Δ'_2 is a $n\times v$ matrix of observations versus right neighbour treatment, l is $v\times1$ vector of left neighbour effects, γ is $v\times1$ vector of right neighbour effects, D' is an $n\times b$ incidence matrix of observations versus blocks, β is $b\times1$vector of block effects and e is $n\times1$ vector of errors.

2.1 Information Matrix

The joint information matrix for estimating the direct and neighbour (left and right) effects under correlated observations estimated by generalized least squares is obtained as follows:

$$c = \begin{bmatrix} \Delta(I_b \otimes \Lambda^*)\Delta' & \Delta(I_b \otimes \Lambda^*)\Delta'_1 & \Delta(I_b \otimes \Lambda^*)\Delta'_2 \\ \Delta_1(I_b \otimes \Lambda^*)\Delta' & \Delta_1(I_b \otimes \Lambda^*)\Delta'_1 & \Delta_1(I_b \otimes \Lambda^*)\Delta'_2 \\ \Delta_2(I_b \otimes \Lambda^*)\Delta' & \Delta_2(I_b \otimes \Lambda^*)\Delta'_1 & \Delta_2(I_b \otimes \Lambda^*)\Delta'_2 \end{bmatrix} \qquad (2.3)$$

$$\wedge^* = \wedge^{-1} - (1'_k \wedge^{-1} 1_k)^{-1} \wedge^{-1} 1_k\, 1'_k \wedge^{-1}$$

with the above $3v\times3v$ information matrix (C) for estimating the direct effects and neighbour effects of treatments in a block design setting is symmetric, non-negative definite with row and column sums equal to zero. The information matrix for estimating the direct effects of treatments from (2.3) is as follows:

$$C\tau = C_{11} - C_{12}\, C^{-1}_{22}\, C_{21} \qquad (2.4)$$

Where

$$C_{11} = \Delta(I_b \otimes \wedge^*)\Delta'$$

$$C_{12} = [\Delta(I_b \otimes \wedge^*)\Delta'_1 \;\; \Delta(I_b \otimes \wedge^*)\Delta'_2] \text{ and}$$

$$C_{22} = \begin{bmatrix} \Delta_1(I_b \otimes \Lambda^*)\Delta'_1 & \Delta_1(I_b \otimes \Lambda^*)\Delta'_2 \\ \Delta_2(I_b \otimes \Lambda^*)\Delta'_1 & \Delta_2(I_b \otimes \Lambda^*)\Delta'_2 \end{bmatrix}$$

Similarly, the information matrix for estimating the left neighbour effect of treatments (C_l) and right neighbour effect of treatments ($C\gamma$) can be obtained.

2.2 Construction of Design

Tomer *et al*. (2005) has constructed neighbour balanced block design with parameters *v* (prime or prime power), *b* = *v* (*v*-1), *r* = (*v*-1) (v-m), k = (v-m), *m* = 1, 2…., *v*-4 and λ = (v-m) using Mutually Orthogonal Latin Squares (MOLS) of order *v*. This series of design has been investigated under the correlated error structure. It is seen that the design turns out to be pair-wise uniform with α = 1 and also variance balanced for estimating direct (V_1) and neighbour effects ($V_2 = V_3$).

Definition 1. A block design is neighbour balanced if every treatment has every other treatment appearing as a neighbour (left and right) constant number of times (say, λ).

Definition 2. A neighbour balanced block design is called pair-wise uniform on the plots if each treatment s (= 1, 2,..., v) occurs equally often in each plot position i (= 1, 2,..., k) and each pair of treatments s and s', s ≠ s' (=1,...,v) occurs equally often (α times) within the same block in each unordered pair of plot positions i and i', i ≠ i'(=1, 2..k)

Definition 3. A neighbour balanced block design with correlated observations permitting the estimation of direct and neighbour (left and right) effects, is called variance balanced if the variance of any estimated elementary contrast among the direct effects is constant, say V_1, the variance of any estimated elementary contrast among the left neighbour effects is constant, say V_2, and the variance of any estimated elementary contrast among the right neighbor effects is constant, say V_3. The constants V_1, V_2 and V_3 may not be equal. A block design is totally balanced if $V_1 = V_2 = V_3$.

3. Comparison of Efficiency

In this section, a quantitative measure of efficiency of the designs in compared to the universally Optimal neighbour balanced design for *v* treatments in (*v*-1) complete blocks of Azais *et al*. (1993) [16] considering observations to be correlated within the blocks. We compare the average variance of an elementary treatment contrast $\hat{\tau}_s - \hat{\tau}_{s'}$ in both cases. The average variance of an elementary treatment contrast for direct effects of the neighbour balanced design of Azais *et al*. (1993) [16] estimated by generalized least squares methods, is given by

$$V_A = \frac{2\sigma^2}{v-1}\sum_{s=1}^{v-1}\theta_s^{-1}$$

Where Θ_s's are the (v-1) non-zero Eigen values of C_τ for Azais *et al.* (1993) [16], σ^2 is the variance of an observation. The efficiency factor (E_τ) for direct effects of the neighbour balanced pair-wise uniform block design is thus given as:

$$E_\tau = \frac{(v-1)\ \sum_{s=1}^{v-1}\theta_s^{-1}}{(v-m)\sum_{s=1}^{v-1}\delta_s^{-1}}$$

δ_s's are the (v-1) non-zero Eigen values of C_τ. Similarly the efficiency (E_l) and (E_τ) for neighbour effects (left and right) of treatments is obtained. The ranges of correlation coefficient (ρ) for different correlation structures investigated are $|\rho| \leq 0.7$ for AR (1).

3.1 Efficiency of NNBD in comparison to regular block design

For a regular block design, the variance of an elementary treatment contrast, estimated by ordinary least squares methods, is given by

$$v_1 = 2r^{-1}\sigma^2(1-\bar{\rho})$$

Where σ^2 is the variance of an observation, ρ is the average correlation between observations from any two plots within a block, the average being taken over all possible randomizations; r is the number of replications. If observations within a block follow an errors-in-variable autoregressive model, then according to

$$\sigma^2 = \sigma_\varepsilon^2(1+\alpha(1-\rho^2)(1-\rho^2)^{-1})$$

And

$$\bar{\rho} = \frac{2\rho}{\{1+\alpha(1-\rho^2)\}(t-1)(1-\rho)}\left\{1-\frac{1-\rho^t}{t(1-\rho)}\right\}$$

So that

$$V_1 = \frac{2\sigma_\varepsilon^2}{r(1-\rho^2)}\left[1+\alpha(1-\rho^2)-\frac{2}{(t-1)(1-\rho)}\left\{1-\frac{(1-\rho^t)}{t(1-\rho)}\right\}\right]$$

For generalized least squares estimation, the average variance of an elementary treatment contrast, estimated from a design d, is

$$V_2 = \frac{2\sigma^2}{v-1}\sum_{s=1}^{v-1}\theta_s^{-1}$$

Where Θ_s's are non-zero values of C_d. We define the efficiency of a design d relative to a regular block design as V_1 / V_2.
The Table 1 shows the parameters of neighbour balanced pair-wise uniform complete block designs for v = 6 and 7, m = 0 along with the efficiency for direct and neighbour effects (left and right) has been shown. The efficiency values have been reported under the AR (1) with ρ in the interval-0.7 to 0.7. The values in the tables shows that as ρ increases from 0.1 to 0.7, the gain in efficiency also increases under AR (1) models. Tables 2 and Table 3 shows the efficiencies of NNBD with t = 6, r = 30 and t = 6, r = 42, ρ = -0.7 to 0.7 and α = 1. The values in the tables show that as ρ increases from 0.1 to 0.7 the gain in efficiency also increases under AR (1) model.

Table 1. Efficiency of neighbour balanced pair-wise uniform complete block designs (AR (1))

Parameters					Correlation structure			
					AR(1)			
v	b	m	r	K = λ	p	E_τ	E_l	E_g
6	30	0	30	6	-0.7	0.5193	0.5538	0.5918
					-0.5	0.6493	0.6912	0.6478
					-0.3	0.7377	0.7267	0.7441
					-0.1	0.8026	0.7955	0.7967
					0	0.8333	0.8333	0.8333
					0.1	0.8616	0.6899	0.8592
					0.3	0.9882	0.7924	1.0091
					0.5	1.1803	0.9000	1.1436
					0.7	1.2317	0.9318	1.2472
6	30	1	25	5	-0.7	0.8175	0.7127	0.7192
					-0.5	0.8569	0.7915	0.8433
					-0.3	0.8911	0.8912	0.8913
					-0.1	0.9412	0.9391	0.9411
					0	1.0000	1.0000	1.0000
					0.1	1.0591	1.0912	1.0642
					0.3	1.1312	1.1761	1.1361
					0.5	1.1866	1.3912	1.2967
					0.7	1.2413	1.5167	1.4715
7	42	0	42	7	-0.7	0.5341	0.5712	0.5697
					-0.5	0.6678	0.6851	0.5635
					-0.3	0.7588	0.7475	0.7654
					-0.1	0.8255	0.8182	0.8194
					0	0.8571	0.8571	0.8571
					0.1	0.8863	0.8870	0.8837
					0.3	1.0164	1.0188	1.0379
					0.5	1.2141	1.1571	1.1763
					0.7	1.2669	1.2943	1.2829
7	42	1	36	6	-0.7	0.7912	0.7018	0.6913
					-0.5	0.8134	0.7211	0.7042
					-0.3	0.8712	0.8167	0.8512
					-0.1	0.9238	0.9212	0.9314
					0	1.0000	1.0000	1.0000
					0.1	1.0391	1.0451	1.0911
					0.3	1.1864	1.0917	1.1121
					0.5	1.2681	1.2912	1.2912
					0.7	1.3785	1.4102	1.3961

Table 2. Efficiency of NNBD using AR (1) model (t = 6, r = 30 and α = 1)

ρ	E_τ	E_l	E_γ
-0.7	1.87895	1.67649	1.73107
-0.5	1.71941	1.25539	1.51658
-0.3	0.68845	0.67821	0.69446
-0.1	0.70029	0.69405	0.69511
0	0.70937	0.70937	0.70937
0.1	0.71791	0.71852	0.71586
0.3	0.78973	0.79100	0.80581
0.5	0.88111	0.83979	0.83190
0.7	0.91122	0.85548	0.85729

Table 3: Efficiency of NNBD using AR (1) model (t = 7, r = 42 and α = 1)

ρ	E_τ	E_l	E_γ
-0.7	1.95857	1.22579	1.02243
-0.5	1.75410	1.76294	1.42976
-0.3	1.21813	0.95373	1.17658
-0.1	0.98645	0.97767	0.97916
0	1.00065	1.00065	1.00065
0.1	1.01477	1.01564	1.01188
0.3	1.12471	1.12738	1.14849
0.5	1.28684	1.22649	1.28383
0.7	1.29925	1.25155	1.32472

4. Efficiency of NNBIBD in comparison to regular block design

In this section, a quantitative measure of efficiency of NNBIBD is also derived when the errors follow AR (1) Model. We have derived the errors within a block that follow AR (1) models. The Tables 4 and 5 show the efficiencies of NNBIBD with t = 6, r = 25 and t = 7, r = 36, ρ = -0.7 to 0.7 and α =1. The values in the tables show that as ρ increases from 0.1 to 0.7, the gain in efficiency also increases under AR (1) models.

Table 4: Efficiency of NNBIBD using AR (1) model (t = 6, r = 25 and α = 1)

ρ	E_τ	E_l	E_γ
-0.7	2.24291	1.47095	1.22691
-0.5	2.01493	1.21556	1.15571
-0.3	1.96176	1.14448	1.17190
-0.1	1.68374	1.11320	1.17499
0	1.20078	1.20078	1.20078
0.1	1.21773	1.21877	1.21426
0.3	1.34966	1.35285	1.37819
0.5	1.54420	1.44179	1.54059
0.7	1.69670	1.47386	1.66967

Table 5: Efficiency of NNBIBD using AR (1) model (t = 7, r = 36 and α = 1)

ρ	E_τ	E_l	E_γ
-0.7	0.84211	1.52257	1.85292
-0.5	0.79264	1.46463	1.76934
-0.3	0.80319	1.43972	0.81020
-0.1	0.81700	0.80973	0.81096
0	0.82760	0.82760	0.82760
0.1	0.83756	0.83828	0.83517
0.3	0.92065	0.92283	0.94011
0.5	1.02796	0.97976	1.02888
0.7	1.89975	1.22972	1.28350

5. Conclusion

We have investigated the efficiency of neighbour balanced pair-wise uniform block designs when observations within a block are assumed to be correlated and generalized least square estimation is used when the errors follow AR (1) model. The efficiency values have been reported under the AR (1) with ρ in the interval -0.7 to 0.7. It is seen that efficiency for direct as well as neighbour effects is high. We have investigated the efficiency of both NNBD and NNBIBD in comparison to regular block design when the errors follow first order AR model. The values in the tables show that as ρ increases from 0.1 to 0.7, the gain in efficiency also increases under AR (1) model. The efficiency in NNBIBD over RBD is high for direct as well as neighbour effects of treatments when compare to NNBD over RBD using AR (1) model.

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