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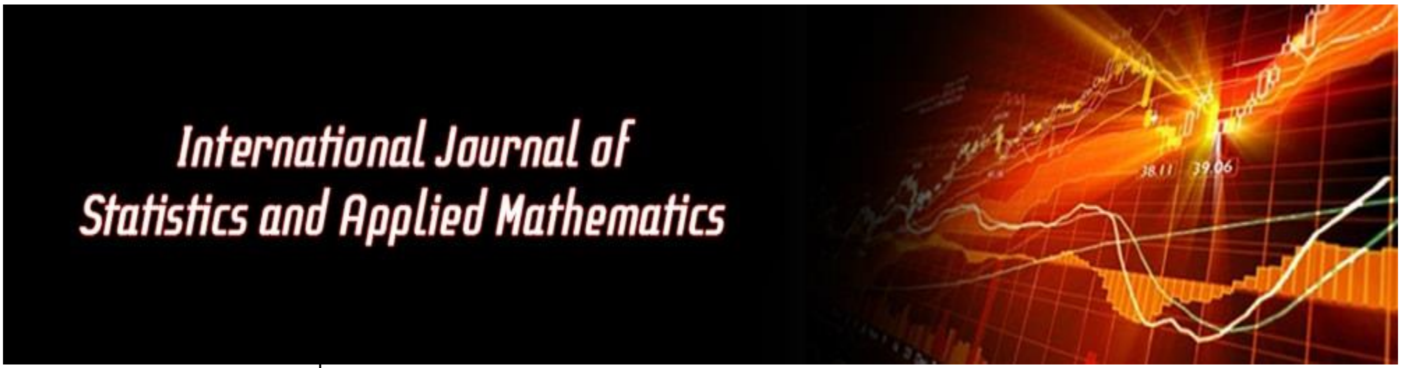


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# On contra βwg-continuous functions in topological spaces

**Govindappa Navalagi and Kantappa M Bhavikatti**

**Abstract**
The main aim of this paper is to define and study the notions of contra βwg-continuous, almost contra βwg-continuous functions and discussed the relationship with other contra continuous functions and obtained their characteristics. Further we introduce the concepts of contra βwg-irresolute, contra βwg-closed functions and obtain some of their properties.

**Keywords:** βwg-continuous, contra βwg-continuous, almost contra βwg-continuous, contra βwg-irresolute, contra βwg-closed functions, βwg-locally indiscrete space.

## 1. Introduction
In 1996, Dontchev. [4], introduced the notion of contra continuous functions. Jafari and Noiri [9] introduced contra precontinuous functions. Ekici. E [6] introduced almost contra precontinuous functions in 2004. Dontchev and Noiri [5], introduced and investigated contra semi-continuous functions and RC-continuous functions between topological spaces. Veerakumar [26] also introduced contra pre-semi-continuous functions. S.Sekar and P. Jayakumar [21] introduced contra gp*-continuous functions. Recently, Govindappa. Navalagi and Kantappa. M. Bhavikatti [15] introduced βwg-continuous functions. In this paper we introduce and study the new class of functions called contra βwg-continuous and almost contra βwg-continuous functions in topological spaces. Also we define the notions of contra βwg-irresolute, contra βwg-closed functions, βwg-locally indiscrete space and study some of their properties.
Throughout this paper (X, τ), (Y, σ) and (Z, η) (or simply X, Y, and Z) represent the non-empty topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of X, the closure of A and interior of A will be denoted by Cl (A) and Int (A) respectively. The union of all βwg-open sets of X contained in A is called βwg-interior of A and it is denoted by βwgInt (A). The intersection of all βwg-closed sets of X containing A is called βwg-closure of A and it is denoted by βwg Cl (A). Also the collection of all βwg-open subsets of X containing a fixed point x is denoted by βwg O (X, x).

## 2. Preliminaries
We recall the following definitions which are useful in the sequel.

**Definition 2.1** A subset A of a topological space (X, τ) is called
(i) Semi-open [11] if $A \subseteq cl(int(A))$ and semi-closed if $int(cl(A)) \subseteq A$.
(ii) Preopen [13] if $A \subseteq int(cl(A))$ and preclosed if $cl(int(A)) \subseteq A$.
(iii) α-open [16] if $A \subseteq int(cl(int(A)))$ and α-closed if $cl(int(cl(A))) \subseteq A$.
(iv) semi-preopen [1] (β-open) if $A \subseteq cl(int(cl(A)))$ and
(v) Semi-preclosed (β-closed) if $int(cl(int(A))) \subseteq A$.
(vi) Regular open [23] if $A = int(cl(A))$ and regular closed if $A = cl(int(A))$.

**Definition 2.2:** A subset A of a topological space (X, τ) is called
(i) Generalized preclosed (briefly, gp-closed) [12] if pcl (A) ⊆ U whenever A ⊆ U and U is open in X.
(ii) Generalized semi-preclosed (briefly, gsp-closed) [3] if spcl (A) ⊆ U whenever A ⊆ U and U is open in X.
(iii) Generalized pre regularclosed (briefly, gpr-closed) [7] if pcl (A) ⊆ U whenever A ⊆ U and U is regular open in X.
(iv) Generalized star preclosed (briefly, g*p-closed set) [23] if pcl (A) ⊆ U whenever A ⊆ U and U is g-open in X.
(v) Generalized pre star closed (briefly, gp*-closed set) [10] if cl (A) ⊆ U whenever A ⊆ U and U is gp-open in X.
(vi) Bwg-closed [14] if βcl (A) ⊆ U whenever A ⊆ U and U is αg-open in X.
(vii) Pre semi-closed [25] if spcl (A) ⊆ U whenever A ⊆ U and U is g-open in X.

**Definition 2.3:** [15] A function f: (X, τ) → (Y, σ) is called βwg-continuous if $f^{-1}(V)$ is βwg-closed set in (X, τ) for every closed set V in (Y, σ).

**Definition 2.4:** [15] A function f: (X, τ) → (Y, σ) is called βwg-irresolute if $f^{-1}(V)$ is ßwg-closed set in (X, τ) for every βwg-closed set V in (Y, σ).

**Definition 2.5:** A function f: (X, τ) → (Y, σ) is called
1. Contra continuous [4] if $f^{-1}(V)$ is closed set in X for each open set V of Y.
2. Contra pre-continuous [9] if $f^{-1}(V)$ is preclosed set in X for each open set V of Y.
3. Contra semi-continuous [5] if $f^{-1}(V)$ is semi-closed set in X for each open set V of Y.
4. Contra α-continuous [8] if $f^{-1}(V)$ is α-closed set in X for each open set V of Y.
5. Contra pre semi-continuous [26] if $f^{-1}(V)$ is pre semi-closed set in X for each open set V of Y.
6. Contra gp-continuous [20] if $f^{-1}(V)$ is gp-closed set in X for each open set V of Y.
7. Contra gpr-continuous [20] if $f^{-1}(V)$ is gpr-closed set in X for each open set V of Y.
8. Contra rg-continuous if $f^{-1}(V)$ is rg-closed set in X for each open set V of Y.
9. Contra Ag-continuous if $f^{-1}(V)$ is αg-closed set in X for each open set V of Y.
10. Contra Ag*-continuous if $f^{-1}(V)$ is Ag*-closed set in X for each open set V of Y.
11. Contra gsp-continuous [20] if $f^{-1}(V)$ is gsp-closed set in X for each open set V of Y.
12. Contra gp*-continuous [21] if $f^{-1}(V)$ is gp*-closed set in X for each open set V of Y.
13. Contra (gsp)*-continuous [19] if $f^{-1}(V)$ is (gsp)*-closed in X for each open set V of Y.
14. Contra g*p-continuous [18] if $f^{-1}(V)$ is g*p-closed set in X for each open set V of Y.

**Definition 2.6:** A function f: (X, τ) → (Y, σ) is called
1. Perfectly continuous [17] if $f^{-1}(V)$ is clopen in X for every open set V of Y.
2. Almost continuous [22] if $f^{-1}(V)$ is open in X for each regular open set V of Y.
3. Almost βwg-continuous [15] if $f^{-1}(V)$ is βwg -open in X for each regular open set V of Y.
4. Almost contra g*p-continuous [18] if $f^{-1}(V)$ is g*p-closed in X for each regular open set V of Y.
5. Pre-closed [13] if f (U) is pre-closed in Y for each closed set U of X.
6. Contra pre-closed [2] if f (U) is pre-closed in Y for each open set U of X.

**Definition 2.7:** Let A be a subset of a space (X, τ).
1. The set ∩ {U ∈τ/A ⊆ U} is called the kernel of A and is denoted by ker (A).
2. The set ∩ {F ∈ X/A ⊆ F, F is β-closed} is called the β-closure of A and is denoted by βc1 (A).

**Lemma 2.8:** The following properties hold for subsets A, B of a space X:
1. X ∈ ker (A) if and only if U ∩ A ≠ φ for any F∈C(X, x).
2. A ⊆ ker (A) and A = ker (A) if A is open in X.
3. If A ⊆ B, then ker (A) ⊆ ker (B).

**Lemma 2.9:** [15] For x∈X, x∈ βwg Cl (A) if and only if U ∩ A ≠ φ for every βwg-open set U containing x.
Proof: Necessary part: Suppose there exists βwg -open set U containing x such that U ∩ A ≠ φ. Since A ⊆ X - U, βwg Cl (A) ⊆ X-U. This implies x∉βwg Cl (A). This is a contradiction.
Sufficiency part: Suppose that x ∉ βwg-cl (A). Then there exists a βwg-closed subset F containing A such that x ∉ F. Then x ∈ X-F is ßwg-open, (X - F) ∩A = φ. This is contradiction.

**3. Contra βwg-Continuous Functions**
In this section, we introduce and study new class of continuous functions called contra βwg-continuous functions and investigate some of their properties in the following.

**Definition 3.1:** A function f: X→ Y is called contra beta weakly generalised (briefly, βwg-continuous) continuous if $f^{-1}(V)$ is βwg-closed set in X for every open set V in Y.

**Example 3.2:** Let X = Y = {a, b, c, d} with topologies, τ = {X, φ, {a}, {b}, {a, b}} and σ = {Y, φ, {b,c}, {a,b,c}, {b,c,d}}. Now βwgC(X) ={X, φ,{a},{b},{c},{d},{a, c}, {a,d}, {b,c}, {b,d}, {c,d}, {a, c, d},{b, c, d}, X}. Define a function f: X → Y by f (a) = d, f (b) = a, f(c) = b and f (d) = c. Then f is contra βwg-continuous function, since every open set in Y is βwg-closed in X**.**

**Theorem 3.3:** Every contra continuous (resp. contra pre-continuous) function is contra βwg -continuous but not conversely.
**Proof:** Let U be an open set in Y then $f^{-1}(U)$ is closed (resp. pre-closed) in X. Since every closed (resp. pre-closed) set is a βwg-closed set. Therefore f is contra βwg -continuous.

**Example 3.4:** Let X = {a, b, c, d} = Y with topologies τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}} and σ ={Y, φ, {a}, {b, c}, {a, b, c}}. Define f: X → Y by f (a) = c, f (b) = d, f (c) = b and F (d) = a. Then f is contra βwg-continuous but not contra continuous and contra pre-continuous. Since {b,c} is an open set in Y but $f^{-1}(\{b,c\})$ = {a,c} is βwg-closed but not closed and pre-closed in X.
We define the following

**Definition 3.5:** A function f: X → Y is called contra αg*-continuous if $f^{-1}(V)$ is α-closed set in X for each open set V of Y

**Theorem 3.6:** Every contra Ag*-continuous (resp. contra gp*-continuous) function is contra βwg -continuous but not conversely.
**Proof:** Let V be an open set in Y then $f^{-1}(V)$ is Ag*-closed (resp. gp*-closed) in X. Since every Ag*-closed (resp. gp*-closed) set is an ßwg -closed set. Therefore f is contra βwg-continuous.

**Example 3.7:** Let X = Y = {a, b, c, d} with topologies τ = {X, {a, b}, φ} and σ = {Y, φ, {c}, {d}, {c, d}}. Define f: X → Y by f(a) = c, f(b) = a, f(c) = d, f(d) = b .Then f is contra βwg-continuous but not contra αg*-continuous and contra gp*-continuous, since {c,d} is an open set in Y but $f^{-1}(\{c,d\})$ = {a,c} is not αg*-closed and gp*-closed in X.

**Theorem 3.8:** Every contra βwg-continuous function is contra pre-semi-continuous but not conversely.
**Proof:** Let U be an open set in Y then $f^{-1}$ (U) is βwg - closed set in X. Since every βwg-closed set is pre semi-closed set then $f^{-1}(U)$ is pre-semi-closed in X. Therefore f is contra pre-semi-continuous.

**Example 3.9:** Let X = {a, b, c, d} = Y with topologies τ = {X, φ, {a}, {b}, {a, b}, {a, b, c}} and σ = {Y, φ, {a}, {b, d}, {a, b, d}}. Define a function f: V → Y by f (a) = a, f (b) = d, F (c) = c and f (d) = b. Then f is contra pre semi-continuous but not contra βwg-continuous, since {a,b,d} is an open set in Y but $f^{-1}(\{a,b,d\})$ = {a,b,d} is not βwg-closed in X.

**Theorem 3.10: (i)** Every contra βwg-continuous function is contra gp-continuous.
1. Every contra βwg-continuous function is contra gpr-continuous.
2. Every contra βwg -continuous function is contra g-continuous.
3. Every contra βwg-continuous function is contra g*p-continuous.
4. Every contra βwg-continuous function is contra αg-continuous
   (resp.gs-continuous, gsp-continuous).

**Proof:** The proof is straight forward from the definition 3.1 and Theorem 3.3.

**Remark 3.11:** The converses of theorem 3.10, is not true as shown in the following examples.

**Example 3.12:** Let X = Y = {a, b, c}, τ = {X, φ, {a}, {b}, {a, b}} and σ = {Y,φ, {a, b}}. Define f: X → Y by f (a) = b, f (b) = and f(c) = c. Then f is contra gpr-continuous but not contra βwg-continuous, since {a,b} is an open set in Y, $f^{-1}(\{a,b\})$ = {a,b} is gpr-closed but not βwg-closed in X.

**Example 3.13:** Let X = {a,b,c,d}= Y with topologies τ = { X, φ, {a}, {b}, {a,b}, {a,b,c}} and σ = {Y, φ, {c}, {a,b,d}}. Define f: X → Y by f (a) = b, f (b) = a,f(c) = c and f (d) = d. Then f is contra g*p-continuous but not contra βwg-continuous, since {a,b,d} is open set in Y, $f^{-1}$ ({a,b,d}) = {a,b,d} is g*p-closed but not βwg-closed in X.

**Example 3.14:** Let X = Y = {a, b, c} with topologies, τ = {φ, {a}, X} and σ = {Y, φ, {a}, {a,b}, {b}}. Define f: X → Y by f (a) = a, f (b) = c and f(c) = b. Then f is contra gp-continuous (resp. contra gs-continuous, contra αg-continuous, contra gsp-continuous, contra g-continuous) but not contra βwg - continuous, since {a,b} is an open set in Y, $f^{-1}$ ({a, b}) = {a, c} is gp-closed (resp. gs-closed, αg-closed, gsp-closed, g-closed) set but not βwg-closed in X.
Also, we define and obtain the following

**Definition 3.15:** A function f: X→Y is called a
1. Contra rg-continuous if $f^{-1}(V)$ is rg-closed set in X for each open set V of Y
2. Contra Ag-continuous if $f^{-1}(V)$ is αg-closed set in X for each open set V of Y.
3. Contra Ag*-continuous if $f^{-1}(V)$ is Ag*-closed set in X for each open set V of Y.

**Theorem 3.16:** Every contra βwg-continuous function is contra rg-continuous but not conversely.

**Proof:** Let U be an open set in Y then $f^{-1}(U)$ is βwg-closed set in X. Since every βwg-closed set is rg-closed set then $f^{-1}(U)$ is rg-closed in X. Therefore f is contra rg-continuous.

**Example 3.17:** Let X = Y = {a, b, c}, τ = {φ, {a}, X} and σ = {φ, {c}, {a, c}, Y}. Now RGC(X) =P(X) and βwgC (X) = {X, φ, {b}, {c}, {b, c}}. Define f: X → Y by f (a) = c, f (b) = b and f(c) = a. Then f is contra rg-continuous but not contra βwg-continuous, since {a,c} is an open set in Y, $f^{-1}$({a, c}) = {a, c} is rg-closed but not βwg-closed in X.

**Theorem 3.17:** If f: X → Y contra (gsp)*-continuous then f is contra βwg-continuous function but not conversely.
**Proof:** Let G be an open set in Y. Since f is contra (gsp)*-continuous, then $f^{-1}$(G) is (gsp)*-closed set in X. Since every contra (gsp)*-closed set is βwg -closed set then $f^{-1}$(U) is βwg-closed in X. Therefore f is contra βwg-continuous.

**Example 3.18:** Let X = Y = {a, b, c}, τ = {φ, {c}, {a, c}, X} and σ = {Y,φ, {c}}. Define the function f: X → Y by f (a) = b, f (b) = and f(c) = c. Then f is contra βwg-continuous but not contra (gsp)*-continuous, since {c} is an open set in Y, $f^{-1}$({c}) = c} is βwg-closed but not (gsp)*-closed in X.

**Definition 3.19:** A space (X, τ) is called βwg-locally indiscrete if every βwg-open set is closed.

**Theorem 3.20:** (i) If a function f: X → Y is βwg-continuous and (X, τ) is βwg-locally indiscrete then f is contra continuous.
(ii) If a function f: X → Y is contra βwg-continuous and (X, τ) is $_{\beta wg}T_b$ space then f is contra continuous.
(iii) If a function f: X → Y is contra βwg -continuous and (X, τ) is $_{\beta wg}T_{1/2}$ space then f is contra precontinuous.

**Proof:** (i) Let G be open in (Y, σ). By assumption, $f^{-1}$(G) is βwg-open in X. Since X is locally indiscrete, $f^{-1}$(G) is closed in X. Hence f is contra continuous.
(ii) Let G be open in (Y, σ). By assumption, $f^{-1}$(G) is βwg-closed in X. Since by definition, X is $_{\beta wg}T_b$ space, $f^{-1}$(G) is closed in X. Hence f is contra continuous.
(iii) Let G be open in (Y, σ). By assumption, $f^{-1}$(G) is βwg-closed in X. Since by definition, X is $_{\beta wg}T_{1/2}$ space, $f^{-1}$(V) is pre-closed in X. Hence f is contra pre-continuous.

**Theorem 3.21:** If a function f: (X, τ) → (Y, σ) is contra βwg-continuous and (X, τ) is $_{\beta wg}T_d$ -space then f is contra g-continuous.

**Proof:** Let V be open in (Y, σ). By assumption, $f^{-1}$(V) is βwg-closed in X. Since X is $_{\beta wg}T_d$ -space, $f^{-1}$(V) is g-closed in X. Hence f is contra g-continuous.

**Theorem 3.22:** If a function f: (X, τ) → (Y, σ) is contra βwg-continuous and (X, τ) is $_{\beta wg}T_\alpha$- space then f is contra α-continuous.

**Proof:** Let V be open in (Y, σ). By assumption, $f^{-1}$(V) is βwg-closed in X. Since X is $_{\beta wg}T_\alpha$ -space, $f^{-1}$(V) is α-closed in X. Hence f is contra α-continuous.

**Theorem 3.23:** The following are equivalent for a function f: (X, τ) → (Y, σ):
F is contra βwg-continuous;

1. For every closed subset F of Y, $f^{-1}$ (F) $\in$ βwgO (X);
2. F (βwgC1 (A)) $\subseteq$ ker (f (A)) for every subset A of X ;
3. βwgC1 ($f^{-1}$ (B)) $\subseteq$ $f^{-1}$ (ker (B)) for every subset of B of Y.

**Proof:** The implications **(i)** ⇒ **(ii)** is obvious.
**(ii) ⇒ (III):** Let A be any subset of X. Suppose that y$\notin$ ker (f (A)). Then by Lemma 2.10, there exists F∈C(X,y) such that f (A) ∩ F ≠ φ. Thus A ∩ $f^{-1}$ (F) = φ and
βwg-cl (A) ∩ $f^{-1}$ (F) = φ .Therefore, we obtain f (βwg-cl(A)) ∩ $f^{-1}$ (F) = φ and y$\notin$ f(βwgC1(A)). This implies that f(βwgC1(A)) $\subseteq$ ker ( f (A)).
**(iii) ⇒ (IV):** Let B be any subset of Y. By (IV) and Lemma 2.10, we have
f(βwgCl($f^{-1}$(B))) $\subseteq$ ker(f($f^{-1}$(B))) $\subseteq$ ker(B) and βwgCl($f^{-1}$(B)) $\subseteq$ $f^{-1}$(ker(B)).
**(iv) ⇒ (i):** Let V be any open set of Y. Then by Lemma 2.10, we have
βwgC1 ($f^{-1}$ (V)) $\subseteq$ $f^{-1}$ (ker (V)) = $f^{-1}$ (V) and βwg-c1 ($f^{-1}$ (V)) = $f^{-1}$(V).
This shows that $f^{-1}$(V) is βwg-closed in X.

**Remark 3.24:** The Composition of two contra βwg-continuous maps need not be contra Bwg-continuous map and this can be shown by the following example.

**Example 3.25:** Let X = Y ={a,b,c,d} = Z with topologies, τ ={X, φ, {a}, {b}, {a,b}, {b,c}, {a,b,c}, X}, σ = {Y, φ,{a},{b,c},{a,b,c}} and η = {Z, φ,{a,b}}.
Define a functions f: X → Y by f (a) = b, f (b) = a, f(c) = d, f (d) = c and g: X → Z by g (a) = d, g (b) = c, g (c) = b and g (d) = a. Then both f and g are contra βwg-continuous functions. But gof is not contra βwg-continuous map, since {b,c,d}is an open set in Z, then $(gof)^{-1}$({b,c,d}) = $f^{-1}$ ($g^{-1}$({b,c,d})) = $f^{-1}$({a,b,c}) ={a,b,d} is not a βwg-closed set in X.

**Remark 3.26:** The following two examples will show that the concept of βwg-continuity and contra βwg-continuity are independent from each other.

**Example 3.27:** Let X = Y = {a,b,c,d}, τ = {X, φ,{a},{b,c},{a,b,c}} and σ = {Y, φ, {a}, {b}, {a,b}, {b,c}, {a,b,c}}. Define f: (X, τ) → (Y, σ) by f (a) = d f (b) = a, f(c) = b and f (d) = c. Then f is contra βwg-continuous but f is not βwg-continuous, since {a,d} is a closed set in Y, $f^{-1}$({a,d}) = {a, b}is not a βwg-closed in X.

**Example 3.28:** Let X = Y = {a, b, c, d}, τ = {X, φ, {a}, {a, b}} and σ = {Y, φ, {c}, {b, c}, {a, c, d}}. Define a function f: X → Y by f (a) = c, f (b) = a, f(c) = b and f (d) = d. Then f is βwg-continuous but f is not contra βwg-continuous, since {a,c, d} is an open set in Y, $f^{-1}$({a, c, d}) = {a, b, d}is not a βwg-closed in X.

**Theorem 3.29:** If f: X → Y is a contra βwg -continuous function and g: Y → Z is a continuous function then g o f: X → Z is contra βwg-continuous.

**Proof:** Let U be an open set in Z. Then $g^{-1}$ (U) is open in Y. Since f is contra βwg-continuous, $f^{-1}$ ($g^{-1}$ (U)) = (g o f)$^{-1}$ (U) is βwg -closed in X. Therefore g o f: X → Z is contra βwg-continuous.

**Theorem 3.30:** If f: X → Y is a βwg-irresolute function and g: Y → Z is a contra Bwg-continuous function then g o f: X → Z is contra βwg-continuous function.

**Proof:** Let G be an open set in Z. Then $g^{-1}$ (G) is βwg-closed in Y. Since f is ßwg-irresolute, $f^{-1}$ ($g^{-1}$ (G)) = (g o f)$^{-1}$ (G) is βwg-closed in X. Therefore g o f: X → Z is contra Bwg-continuous function.

**Theorem 3.31:** If f: X → Y is a βwg-irresolute function and g: Y → Z is a contra continuous function then gof: X → Z is contra βwg-continuous.

**4. Approximately βwg- Continuous Maps**

Now, we define the following

**Definition 4.1:** A function f: (X, τ) → (Y, σ) is said to be approximately-βwg-continuous (briefly, ap- βwg-continuous) if βcl (F) ⊆ $f^{-1}$(U) whenever U is an open subset of Y and F is a βwg-closed subset of X such that F ⊆ $f^{-1}$ (U).

**Definition 4.2:** A function f: (X, τ) → (Y, σ) is said to be approximately-βwg - closed (briefly, ap-βwg-closed) function if f (F) ⊆ βint (V) whenever V is an open subset of Y and F is a βwg-closed subset of X such that F ⊆ $f^{-1}$ (V).

**Definition 4.3:** A function f: (X, τ) → (Y, σ) is said to be approximately-βwg-open (briefly, ap-βwg-open) if βcl (F) ⊆ f (U) whenever U is an open subset of X, F is a βwg -closed subset of Y and F ⊆ f (U).

**Definition 4.5:** A function f: X → Y is said to be contra βwg-closed (resp. contra βwg-open) if f (U) is ßwg-open (resp βwg-closed) in Y for each closed (resp. open) set U of X.

**Theorem 4.6:** Let f: X → Y be a function then
1. If f is contra precontinuous, then f is an ap-βwg-continuous.
2. If f is contra preclosed, then f is ap-βwg-closed.
3. If f is contra preopen, then f is ap-βwg-open.

**Proof: (i)** Let F ⊆ $f^{-1}$ (U) where U is a open subset in Y and F is a βwg-closed subset of X. Then βcl (F) ⊆ pcl ($f^{-1}$(U)). Since f is contra precontinuous, βcl (F) ⊆ pcl ($f^{-1}$(U)) = $f^{-1}$(U). This implies f is ap-βwg-continuous.

**(ii)** Let f (F) ⊆ V, where F is a closed subset of X and V is a βwg-open subset of Y. Therefore f (F) = βint(f(F)) ⊆ pint (V).Thus f is ap-βwg-closed.

**(iii)** Let F ⊆ f (U) where F is βwg-closed subset of Y and U is an open subset of X. Since f is contra preopen, f (U) is preclosed in Y for each open set U of X. Thus βcl (F) ⊆ pcl(f(U) = f(U). Therefore f is ap-ßwg-open.

**Theorem 4.7:** If a function f: X → Y is ap-βwg-continuous and preclosed function, then the image of each βwg-closed set in X is βwg-closed set in Y.

**Proof:** Let F be a βwg-closed subset of X. Let f (F) ⊆ V where V is an open subset of Y. Then F⊆ $f^{-1}$ (V) holds. Since f is ap-βwg-continuous, βcl (F) ⊆ $f^{-1}$(V).Thus f (βcl (F)) ⊆ V. Therefore, we have βcl (f (F)) ⊆ βcl(f(βcl(F)) = f(βcl(F)) ⊆ V. Hence f (F) is βwg-closed set in Y.

**Definition 4.8:** A function f: (X, τ) → (Y,σ) is called a contra βwg-irresolute if $f^{-1}$(V) is βwg-closed in X for each βwg-open set V in Y.

**Definition 4.9:** A space (X, τ) is said to be βwg-Lindelof if every cover of X by βwg-open sets has a countable sub cover.

**Theorem 4.10:** Let f: X →Y and g: Y → Z be two functions such that gof: X → Z.
1. If g is βwg-continuous and f is contra βwg-irresolute then gof is contra βwg-continuous.
2. If g is βwg-irresolute and f is contra βwg-irresolute, then gof is contra βwg-irresolute.

**Proof: (i)** Let V be closed set in Z. Then $g^{-1}(V)$ is βwg -closed in Y. Since f is contra βwg -irresolute, $f^{-1}(g^{-1}(V))$ is βwg-open in X. Hence gof is contra βwg-continuous.
**(ii)** Let V be βwg-closed in Z. Then $g^{-1}(V)$ is βwg-closed in Y. Since f is contra βwg-irresolute, $f^{-1}(g^{-1}(V))$ is βwg-open in X. Hence gof is contra βwg-irresolute.

**Theorem 4.11:** Let f: (X,τ) → (Y,σ) and g: (Y,) → **(**Z,η) be two functions such that g o f: (X,τ) → **(**Z,η).
1. If f is closed and g is ap-βwg-closed then gof is ap-βwg-closed.
2. If f is ap-βwg-closed and g is βwg-open and $g^{-1}$ preserves βwg-opensets then gof is ap-βwg-closed.
3. If f is ap-βwg-continuous and g is continuous then gof is ap-βwg-continuous.

**Proof: (i)** Suppose B is an arbitrary closed subset in X and A is a βwg-open subset of Z for which (g o f) (B) ⊆ A. Then f (B) is closed in Y because f is closed. Since g is ap-βwg-closed, g (f (B)) ⊆ βint (A). This implies gof is ap-βwg-closed.
**(ii)** Suppose B is an arbitrary closed subset of X and A is a βwg-open subset of Z for which (gof) (B) ⊆ A.Hence f (B) ⊆ $g^{-1}$(A).Then F (B) ⊆ βint ($g^{-1}$(A)) because $g^{-1}$(A) is βwg-open and f is ap-βwg-closed. Hence (gof) (B) = g (f (B)) ⊆ g (βint ($g^{-1}$(A))) ⊆ βint (g ($g^{-1}$(A))) ⊆ βint (A). This implies that gof is ap-βwg-closed.
**(iii)** Suppose F is arbitrary βwg-closed subset of X and U is open in Z for which F ⊆ $(gof)^{-1}$ (U). Then $g^{-1}$ (U) is open in Y, because g is continuous. Since f is ap-βwg-continuous then we have β Cl (F) ⊆ $f^{-1}$ ($g^{-1}$ (U)) = $(gof)^{-1}$(U). This shows that gof is ap-βwg-continuous.
Next, we define almost contra βwg-continuous functions in the followings

**Definition 4.12:** A function f: X →Y is called almost contra βwg-continuous if $f^{-1}$ (U) is βwg-closed set in X for every regular open set U in Y.

**Example 4.13:** Let X = Y ={a,b,c,d}, τ = { X, {a,b}, φ } and σ ={φ, {a}, {b,c}, {a,b,c},Y}. βwgC(X) = {X, φ, {a}, {b}, {c}, {d}, {a,c}, {a,d}, {b,c}, {b,d}, {c,d}, {a,c,d}, {b,c,d}} and regular open (Y) = { Y, φ,{a},{b,c}}. Now, define a function f: X →Y by f (a) = a, f (b) = b, f(c) = d and f (d) = c. Then f is almost contra βwg-continuous function.

**Theorem 4.14:** Every contra βwg-continuous function is almost contra βwg-continuous but not conversely.
**Proof:** Let G be a regular open set in Y. Since every regular open set is open then G is an open set in Y. Since f is contra βwg-continuous function then $f^{-1}$(G) is βwg -closed set in X. Therefore f is almost contra βwg-continuous.

**Example 4.15:** In above Example 4.13, f is almost contra βwg-continuous but not contra βwg-continuous. Since {a,b,c} is an open set in Y, $f^{-1}$({a,b,c}) = {a, b, d}is not a βwg-closed set in X.

**Theorem 4.16:** Let f: (X,τ) → (Y,σ) be for a function. Then, the following statements are equivalent:
1. Fi is almost contra βwg-continuous.
2. F-1 (F) ∈βwg O(X, τ) for every F ∈ RC(Y, σ).
3. F-1 (int (cl (G)) ∈ βwgC (X, τ) for every open subset G of Y.
4. F-1 (int (cl (F)) ∈ βwgO (X, τ) for every closed subset F of Y.

**Proof: (i)** ⇒ **(ii)** Let F ∈ RC (Y, σ).Then Y-F ∈ RO(Y, σ) by assumption.
Hence $f^{-1}$((Y-F) = X-$f^{-1}$(F) ∈ βwgC(X, τ). This implies that $f^{-1}$(F) ∈ βwgO(X, τ).
**(ii)** ⇒ **(i)** Let V ∈ RO(Y, σ). Then by assumption (Y-V) ∈ RC(Y,σ).
Hence $f^{-1}$((Y-F) =X - $f^{-1}$ (F) ∈ ßwgO(X, τ). This implies that $f^{-1}$(F) ∈ βwgC(X, τ).
**(i)** ⇒ **(iii)** Let G be a open subset of Y. Since int (cl (G)) is regular open then by (i), $f^{-1}$ (int (cl (G)) ∈ βwg-C(X, τ).
**(iii)** ⇒ **(i)** Let V∈RO(Y, σ). Then V is open in Y. By (ii), $f^{-1}$(int (cl (G)) ∈ βwgC (X, τ).
This implies that $f^{-1}$(V) ∈ ßwg C(X, τ)
**(ii)** ⇔ **(iv)** is similar as **(i)** ⇔ **(iii).**

**Theorem 4.17:** If f: X →Y is an almost contra βwg-continuous function and A is a open subset of X, then the restriction f /A : A→Y is almost contra βwg-continuous.

**Proof:** Let F∈RC(Y). Since f is almost contra βwg -continuous, $f^{-1}$(F) ∈ βwg -O(X).Since A is open, it follows that $(f/A)^{-1}$(F) = A ∩ $f^{-1}$(F) ∈ βwgO(A).Therefore f/A is an almost contra-βwg -continuous.

**Definition 4.18:** A functioin f: (X, τ) → (Y, σ) is called regular set connected if $f^{-1}$(U) is clopen in X for every regular open set U in Y.

**Theorem 4.19**: If a function f: (X, τ) → (Y, σ) is almost contra βwg-continuous and almost continuous then f is regular set connected.

**Proof:** Let U be a regular open set in Y. Since f is almost contra βwg-continuous and almost continuous then $f^{-1}(U)$ is βwg-closed and open. Hence $f^{-1}(U)$ is clopen. Therefore, f is regular set connected.

**Theorem 4.20:** Let f: X →Y and g: Y → Z be two functions. Then the following properties hold.

1. If f is almost contra βwg-continuous and g is regular set connected, then gof: X → Z is almost contra βwg-continuous and almost βwg-continuous.
2. If f is almost contra βwg-continuous and g is perfectly continuous, then gof: X → Z is βwg-continuous and contra βwg-continuous.

**Proof: (i**) Let U be regular open in Z. Since g is regular set connected, $g^{-1}$ (U) is clopen in Y. Since f is almost contra βwg-continuous, $f^{-1}(g^{-1}(U)) = (gof)^{-1}$ (U) is βwg-open and βwg-closed. Therefore (g o f) is almost contra βwg-continuous and almost βwg-continuous.

**(ii)** Let G be open in Z. Since g is perfectly continuous, $g^{-1}(G)$ is clopen in Y. Since f is almost contra βwg-continuous, $f^{-1}(g^{-1}(G)) = (gof)^{-1}(G)$ is βwg-open and βwg-closed. Hence gof is contra βwg-continuous and βwg-continuous function.

## 5. Conclusion

In this research article, we have focused on contra βwg-continuity and its characteristics and contra βwg-irresolute in topological spaces. Further with help these functions almost contra βwg-continuous functions, contra βwg-closed functions were studied.

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## 7. References

1. Andrijevic D. Semi-preopen sets, Mat. Vesnik. 1986; 38(1):24-32.
2. Cladas M, Navalagi G. On weak forms of preopen and preclosed functions, Archivum
3. Mathematicum (BRNO), 40, 2004, 119-128.
4. Dontchev J. On generalizing semi-preopen sets, Mem. Fac. Sci. Kochi. Univ. Ser. A. Math. 1995; 16:35-48.
5. Dontchev J. Contra continuous functions and strongly S-closed spaces, Int. Math. Sci. 1996; 19(2):303-310.
6. Dontchev J, Noiri T. Contra semi-continuous functions, Math. Pannon. 1999; 10(2); 159-168.
7. Ekici E. Almost contra pre-continuous functions, Bull. Malaysian Math. Sci. Soc. 2004; 27:53-65.
8. Gnanambal Y. On generalized preregular closed sets in topological spaces, Indian J Pure. Appl. Math. 1997; 28(3):351-360.
9. Jafari S, Noiri T. Contra α-continuous functions between topological spaces, Iran. Int. J Sci. 2001; 2(2):153-167.
10. Jafari S, Noiri T. On contra pre-continuous functions, Bull. Malays. Math Sci. Soc. 2002; 25(2):115-128.
11. Jayakumar P, Mariappa K, Sekar S. On generalized gp*-closed set in topological spaces, Int. Journal of Math. Analysis. 2013; 33(7):1635-1645.
12. Levine N. Semi-open sets and semi-continuity in topological spaces, Amer. Math. Monthly. 1963; 70:36-41.
13. Maki H, Umehara J, Noiri T. Every topological space is pre-$T_{1/2}$ space, Mem. Fac. Sci. Kochi Univ. Ser. A. Math. 1996; 17:33-42.
14. Mashhour AS, Abd El-Monsef ME, El-Deeb SN. On pre-continuous and weak pre-continuous mapings, Proc. Math. and Phys. Soc. Egypt. 1982; 53:47-53.
15. Navalagi GB, Bhavikatti KM. Beta Weakly Generalized Closed sets in Topology, Journal Of Computer And Mathematical Sciences, May (Accepted), 2018, 9(5).
16. Govinappa Navalagi, Kantappa Bhavikatti M. On βwg-Continuous and βwg- Irresolute Functions in Topological Spaces, IJMTT, (May). 2018; 57(1):9-20.
17. Njastad O. On some classes of nearly open sets, Pacific J Math. 1965; 15:961-970.
18. Noiri T. Super-continuity and some strong forms of continuity, Indian J Pure Appl. Math. 1984; 15:241-250.
19. Patil PG, Rayanagoudar TD, Mahesh Bhat K. On some new functions of g*p-continuity, Int. J Contemp. Math. Sciences. 2011; 6:991-998.
20. Pauline Mary Helen M, Kulandhai Therese, (gsp) ∗ -Closed Sets In Topological Spaces, Int. Journal of Mathematics Trends and Technology. 2014; 6:75-86.
21. Rajakumari J, Sekar C. Contra Alpha generalised Star pre-Continuous Functions in Topological Spaces, Journal of Global Research in Mathematical Archive. 2016; 3(8):1-7.
22. Sekar S, Jayakumar P. Contra gp*-continuous Functions, IOSR Journal of
23. Mathematics. 2014; 10(4):55-60.
24. Singal MK, Singal AR. Almost continuous mappings, Yokohama. Math. 1968; 3:63-73.
25. Stone M. Application of the theory of Boolean rings to general topology. Trans. Amer. Math. Soc. 1937; 41:374-381.
26. Veera kumar MKRS. G*-preclosed sets, Indian J Math. 2002; 44(2):51-60.
27. Veera kumar MKRS. Pre-semi closed sets, Acta ciencia Indica (Maths) Meerut, (M). 2002; XXVIII(1):165-181.
28. Veerakumar MKRS. Contra pre-semicontinuous functions, Bull. Malays. Math. Sci. Soc. 2005; 28 (2):67-71.