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Rama Shanker
Department of Statistics College of Science, Eritrea Institute of Technology, Asmara, Eritrea

Kamlesh Kumar Shukla
Department of Statistics College of Science, Eritrea Institute of Technology, Asmara, Eritrea

Ravi Shanker
Department of Mathematics, G.L.A. College, N.P University, Daltonganj, Jharkhand, India

Correspondence
Ravi Shanker
Department of Mathematics, G.L.A. College, N.P University, Daltonganj, Jharkhand, India

A quasi Poisson-Akash distribution and its applications to ecology

Rama Shanker, Kamlesh Kumar Shukla and Ravi Shanker

Abstract
In this paper, a quasi-Poisson-Akash distribution (QPAD) of which Poisson-Akash distribution (PAD) of Shanker (2017 a) is a particular case, has been proposed by compounding Poisson distribution with quasi Akash distribution (QAD), introduced by Shanker (2015). The raw moments and central moments have been obtained. Expressions for coefficient of variation, skewness, kurtosis and index of dispersion have been given their behaviors have been studied for varying values of the parameters. The QPAD has been shown to be unimodal and always over-dispersed. The estimation of its parameters has been discussed using both the method of moments and the method of maximum likelihood. Finally, the goodness of fit of QPAD has been discussed with two real count datasets from ecology and the fit has been compared with that of Poisson distribution (PD), Poisson-Lindley distribution (PLD) and Poisson-Akash distribution (PAD).

Keywords: Poisson-Akash distribution, compounding, moments, log-concavity, over-dispersion, estimation of parameters, goodness of fit

1. Introduction

The probability mass function (pmf) of Poisson-Akash distribution (PAD) introduced by Shanker (2017a) [9] with parameter θ is given by

$$P_1(x;\theta)=\frac{\theta^3}{\theta^2+2}\cdot\frac{x^2+3x+\left(\theta^2+2\theta+3\right)}{(\theta+1)^{x+3}}\;;x=0,1,2,...,\theta>0 \tag{1.1}$$

Shanker (2017a) [9] have studied statistical and mathematical properties of PAD, estimation of parameter using both the method of moments and the method of maximum likelihood, and applications to model count data. Shanker (2017b, 2017c) [10, 11] have also obtained the size-biased and zero-truncated forms of PAD and discussed their statistical and mathematical properties along with the estimation of parameters and applications for modeling count data which structurally excludes zero counts.

The PAD has been obtained by compounding Poisson distribution with Akash distribution, introduced by Shanker (2015) [7] having probability density function (pdf)

$$f_1(x;\theta)=\frac{\theta^3}{\theta^2+2}\left(1+x^2\right)e^{-\theta x};x>0,\theta>0 \tag{1.2}$$

Shanker (2015) [7] has studied its statistical and mathematical properties including its shapes for varying values of parameter, coefficients of variation, skewness, kurtosis, index of dispersion, hazard rate function, mean residual life function, mean deviations, stochastic ordering, Renyi entropy measure, order statistics, Bonferroni and Lorenz indices, stress-strength reliability, estimation of parameter and application for modeling life time data from engineering and biomedical sciences.

The first four moments about origin and the variance of PAD (1.1) obtained by Shanker (2017a) [9] are given as

$$\mu_1' = \frac{\theta^2 + 6}{\theta\left(\theta^2 + 2\right)}$$

$$\mu_2' = \frac{\theta^3 + 2\theta^2 + 6\theta + 24}{\theta^2\left(\theta^2 + 2\right)}$$

$$\mu_3' = \frac{\theta^4 + 6\theta^3 + 12\theta^2 + 72\theta + 120}{\theta^3\left(\theta^2 + 2\right)}$$

$$\mu_4' = \frac{\theta^5 + 14\theta^4 + 42\theta^3 + 192\theta^2 + 720\theta + 720}{\theta^4\left(\theta^2 + 2\right)}$$

$$\mu_2 = \sigma^2 = \frac{\theta^5 + \theta^4 + 8\theta^3 + 16\theta^2 + 12\theta + 12}{\theta^2\left(\theta^2 + 2\right)^2}$$

Shanker (2017a) [9] has shown that PAD is better model than the Poisson-Lindley distribution (PLD) introduced by Sankaran (1970) [6] having pmf

$$P_2\left(x;\theta\right) = \frac{\theta^2\left(x + \theta + 2\right)}{\left(\theta + 1\right)^{x+3}}\ ; x = 0,1,2,...,\theta > 0 \tag{1.3}$$

Ghitany *et al.* (2008) [1] have obtained the zero-truncated versions of PLD and discussed its statistical properties, estimation of parameter and applications for modeling count data which structurally excludes zero counts. Note that PLD is a Poisson mixture of Lindley distribution introduced by Lindley (1958) [4] having pdf

$$f_2\left(x,\theta\right) = \frac{\theta^2}{\theta + 1}\left(1 + x\right)e^{-\theta x}\ ; x > 0, \theta > 0 \tag{1.4}$$

In this paper, a two-parameter quasi Poisson-Akash distribution (QPAD) which includes one parameter Poisson-Akash distribution (PAD) of Shanker (2017a) [9] as particular case, has been proposed. Its moments and moments based measures including coefficient of variation; skewness, kurtosis and index of dispersion have been studied. Its statistical properties including over-dispersion, increasing hazard rate and unimodality and generating function have been discussed. The method of moments and the method of maximum likelihood estimation have been discussed for estimating the parameters of the distribution. The goodness of fit of the proposed distribution has been established through two examples of observed datasets from ecology and the fit has been observed to be quite satisfactory over Poisson distribution, Poisson-Lindley distribution (PLD), and Poisson-Akash distribution (PAD).

2. A Quasi Poisson-Akash Distribution

Shanker (2016) [8] has introduced a quasi Akash distribution (QAD) with parameters θ and α having pdf

$$f_3\left(x;\theta,\alpha\right) = \frac{\theta^2}{\alpha\theta + 2}\left(\alpha + \theta x^2\right)e^{-\theta x}; x > 0, \theta > 0,\ \alpha\theta + 2 > 0 \tag{2.1}$$

It can be easily verified that Akash distribution (1.2) is a special case of QAD (2.1) for $\alpha = \theta$. Shanker (2016) [8] has studied various statistical and mathematical properties of QAD including its shapes for varying values of parameters, raw moments, central moments, hazard rate function, mean residual life function, stochastic ordering, mean deviations, order statistics, Reny entropy measure, Bonferroni and Lorenz curves, stress-strength reliability. Shanker (2016) [8] have also discussed the estimation of parameters using both the method of moments and the method of maximum likelihood along with the applications of QAD for modeling real lifetime data from medical science and engineering.

Suppose the parameter λ of Poisson distribution follows QAD (2.1). Then, a Poisson mixture of QAD can be obtained as

$$P_3(x;\theta,\alpha) = \int_0^\infty \frac{e^{-\lambda}\lambda^x}{x!}\frac{\theta^2}{\alpha\theta+2}\left(\alpha+\theta\lambda^2\right)e^{-\theta\lambda}\,d\lambda \tag{2.2}$$

$$= \frac{\theta^2}{(\alpha\theta+2)\,x!}\int_0^\infty e^{-(\theta+1)\lambda}\left[\alpha\lambda^x+\theta\lambda^{x+2}\right]d\lambda$$

$$= \frac{\theta^2}{(\alpha\theta+2)\,x!}\left[\frac{\alpha\,\Gamma(x+1)}{(\theta+1)^{x+1}}+\frac{\theta\,\Gamma(x+3)}{(\theta+1)^{x+3}}\right]$$

$$= \frac{\theta^2}{(\alpha\theta+2)}\frac{\left(x^2+3x\right)\theta+\left(\alpha\theta^2+2\alpha\theta+2\theta+\alpha\right)}{(\theta+1)^{x+3}};\ x=0,1,2,\ldots \tag{2.3}$$

We would name this probability distribution quasi Poisson-Akash distribution (QPAD). It can be easily verified that QPAD (2.3) reduces to PAD (1.1) at $\alpha=\theta$. The behavior of QPAD for varying values of its parameters θ and α have been explained through graphs and presented in figure 1.

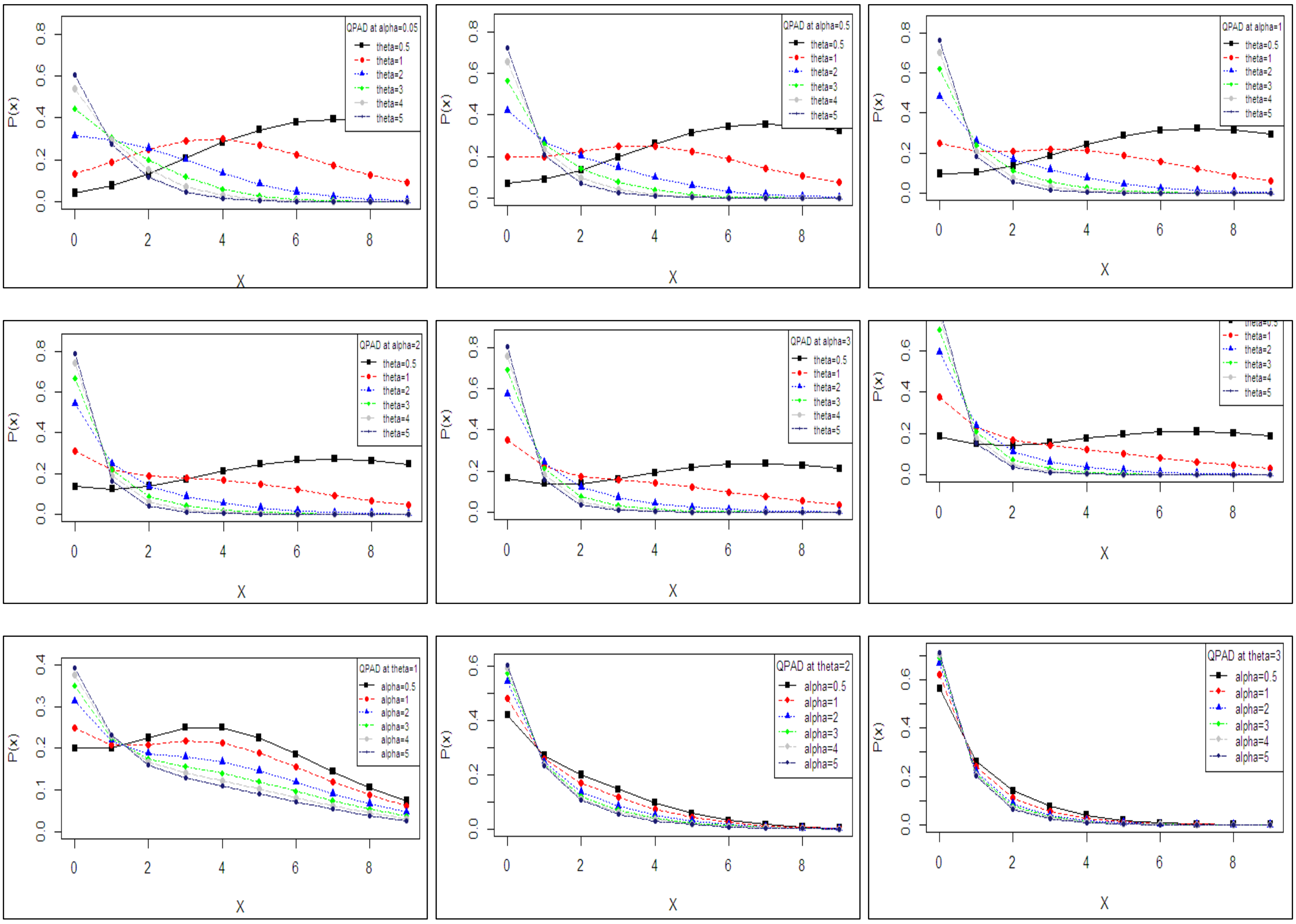

Fig 1: Graphs of pmf of QPAD for varying values of parameters θ and α

3. Moments and Moments Based Statistical Constants

The r th factorial moment about origin of QPAD (2.3) can be obtained as

$$\mu_{(r)}' = E\left[E\left(X^{(r)}\mid\lambda\right)\right], \text{ where } \quad X^{(r)} = X(X-1)(X-2)\ldots(X-r+1).$$

Using (2.2), the r th factorial moment about origin of QPAD (2.3) can be obtained as

$$\mu_{(r)}' = \frac{\theta^2}{\alpha\theta^2+2}\int_0^\infty \left[\sum_{x=0}^\infty x^{(r)} \frac{e^{-\lambda}\lambda^x}{x!}\right]\left(\alpha+\theta\lambda^2\right)e^{-\theta\lambda}\,d\lambda$$

$$= \frac{\theta^2}{\alpha\theta^2+2}\int_0^\infty \lambda^r \left[\sum_{x=r}^\infty \frac{e^{-\lambda}\lambda^{x-r}}{(x-r)!}\right]\left(\alpha+\theta\lambda^2\right)e^{-\theta\lambda}\,d\lambda$$

Taking $x+r$ in place of x within the bracket, we get

$$\mu_{(r)}' = \frac{\theta^2}{\alpha\theta^2+2}\int_0^\infty \lambda^r \left[\sum_{x=0}^\infty \frac{e^{-\lambda}\lambda^{x}}{x!}\right]\left(\alpha+\theta\lambda^2\right)e^{-\theta\lambda}\,d\lambda$$

$$= \frac{\theta^2}{\alpha\theta^2+2}\int_0^\infty \lambda^r \left(\alpha+\theta\lambda^2\right)e^{-\theta\lambda}\,d\lambda$$

Using gamma integral and a little algebraic simplification, we get finally, a general expression for the r th factorial moment about origin of QPAD (2.3) as

$$\mu_{(r)}' = \frac{r!\left[\alpha\theta+(r+1)(r+2)\right]}{\theta^r(\alpha\theta+2)}\;;r=1,2,3,.... \qquad (3.1)$$

It can be easily verified that at $\alpha=\theta$, the expression (3.1) reduces to the corresponding expression of PAD (1.1).
Substituting $r=1,2,3,\text{and }4$ in (3.1), the first four factorial moments about origin can be obtained and using the relationship between factorial moments about origin and moments about origin, the first four moment about origin of the QPAD (2.3) are obtained as

$$\mu_1' = \frac{\alpha\theta+6}{\theta(\alpha\theta+2)}$$

$$\mu_2' = \frac{\alpha\left(\theta^2+2\theta\right)+6(\theta+4)}{\theta^2(\alpha\theta+2)}$$

$$\mu_3' = \frac{\alpha\left(\theta^3+6\theta^2+6\theta\right)+6\left(\theta^2+12\theta+20\right)}{\theta^3\left(\theta^2+2\right)}$$

$$\mu_4' = \frac{\alpha\left(\theta^4+14\theta^3+36\theta^2+24\theta\right)+6\left(\theta^3+28\theta^2+120\theta+120\right)}{\theta^4(\alpha\theta+2)}$$

Using the relationship between moments about mean and the moments about origin, the moments about mean of the QPAD (2.3) are obtained as

$$\mu_2=\sigma^2=\frac{\alpha^2\theta^2(\theta+1)+8\alpha\theta(\theta+2)+12(\theta+1)}{\theta^2(\alpha\theta+2)^2}$$

$$\mu_3=\frac{\alpha^3\theta^3\left(\theta^2+3\theta+2\right)+2\alpha^2\theta^2\left(5\theta^2+27\theta+30\right)+4\alpha\theta\left(7\theta^2+33\theta+18\right)+24\left(\theta^2+3\theta+2\right)}{\theta^3(\alpha\theta+2)^3}$$

$$\mu_4=\frac{\begin{bmatrix}\alpha^4\theta^4\left(\theta^3+10\theta^2+18\theta+9\right)+4\alpha^3\theta^3\left(3\theta^3+47\theta^2+132\theta+96\right)\\ +8\alpha^2\theta^2\left(6\theta^3+103\theta^2+258\theta+153\right)+16\alpha\theta\left(5\theta^3+85\theta^2+180\theta+108\right)\\ +48\left(\theta^3+16\theta^2+30\theta+15\right)\end{bmatrix}}{\theta^4(\alpha\theta+2)^4}$$

The coefficient of variation $(C.V)$, coefficient of Skewness $\left(\sqrt{\beta_1}\right)$, coefficient of Kurtosis (β_2), and index of dispersion (γ) of the QPAD (2.3) are thus given by

$$C.V = \frac{\sigma}{\mu_1'} = \frac{\sqrt{\alpha^2\theta^2(\theta+1)+8\alpha\theta(\theta+2)+12(\theta+1)}}{\alpha\theta+6}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\mu_2^{3/2}} = \frac{\left[\begin{array}{l}\alpha^3\theta^3\left(\theta^2+3\theta+2\right)+2\alpha^2\theta^2\left(5\theta^2+27\theta+30\right)+4\alpha\theta\left(7\theta^2+33\theta+18\right)\\ +24\left(\theta^2+3\theta+2\right)\end{array}\right]}{\left[\alpha^2\theta^2(\theta+1)+8\alpha\theta(\theta+2)+12(\theta+1)\right]^{3/2}}$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{\left[\begin{array}{l}\alpha^4\theta^4\left(\theta^3+10\theta^2+18\theta+9\right)+4\alpha^3\theta^3\left(3\theta^3+47\theta^2+132\theta+96\right)\\ +8\alpha^2\theta^2\left(6\theta^3+103\theta^2+258\theta+153\right)+16\alpha\theta\left(5\theta^3+85\theta^2+180\theta+108\right)\\ +48\left(\theta^3+16\theta^2+30\theta+15\right)\end{array}\right]}{\left[\alpha^2\theta^2(\theta+1)+8\alpha\theta(\theta+2)+12(\theta+1)\right]^2}$$

$$\gamma = \frac{\sigma^2}{\mu_1'} = \frac{\alpha^2\theta^2(\theta+1)+8\alpha\theta(\theta+2)+12(\theta+1)}{\theta(\alpha\theta+2)(\alpha\theta+6)}.$$

It can be easily verified that at $\alpha = \theta$, expressions of these statistical constants of QPAD reduce to the corresponding expressions for PAD.

The nature and behavior of coefficient of variation (C.V), coefficient of skewness (C.S), coefficient of kurtosis (C.K) and index of dispersion (I.D) of QPAD for varying values of parameters θ and α have been explained through graphs and presented in figure 2.

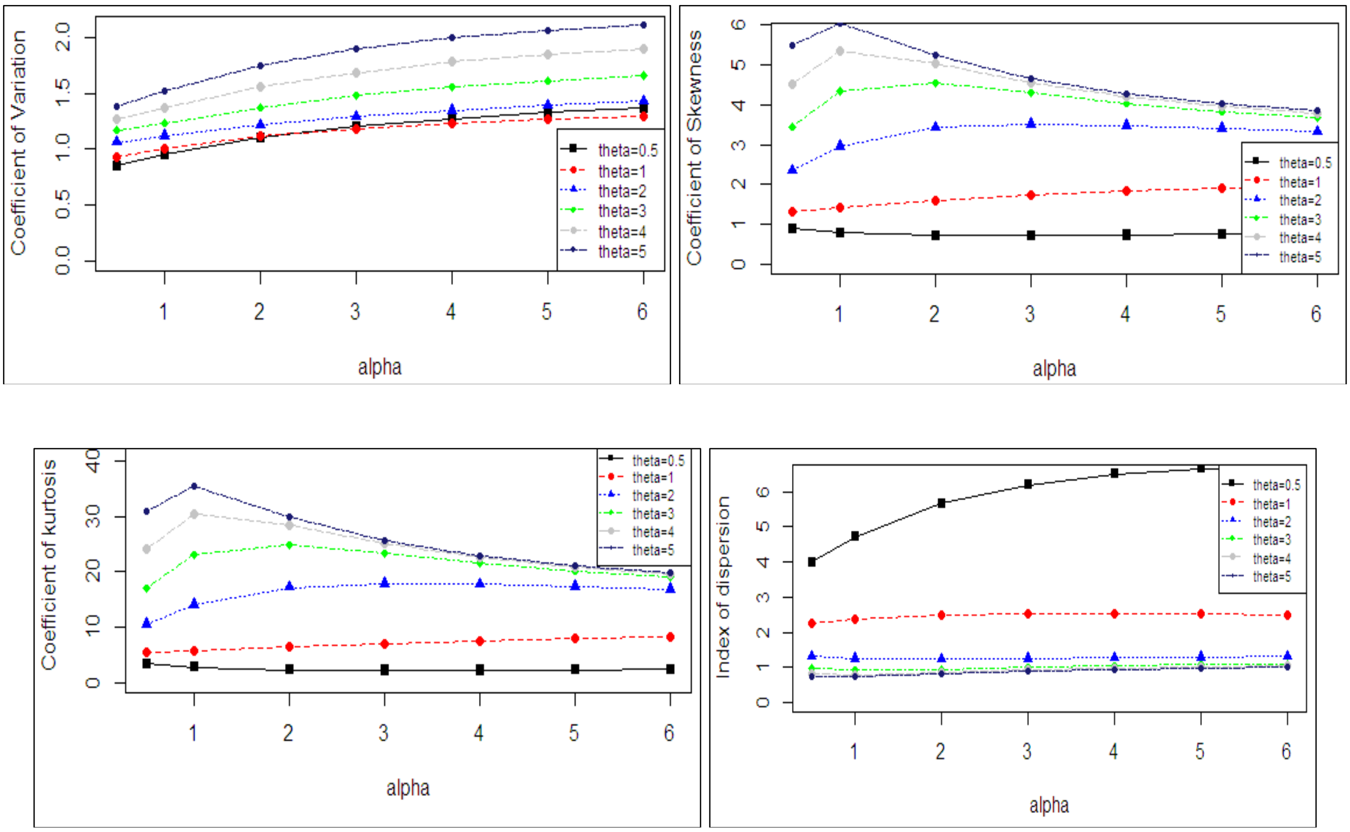

Fig 2: Graphs of coefficient of variation (C.V), coefficient of skewness (C.S), coefficient of kurtosis (C.K) and index of dispersion (I.D) of QPAD for varying values of parameters θ and α

4. Statistical Properties

4.1 Over-dispersion

The QPAD (2.3) is always over-dispersed $\left(\sigma^2 > \mu\right)$. We have

$$\sigma^2 = \frac{\alpha^2\theta^2(\theta+1)+8\alpha\theta(\theta+2)+12(\theta+1)}{\theta^2(\alpha\theta+2)^2}$$

$$= \frac{\alpha\theta+6}{\theta(\alpha\theta+2)}\left[\frac{\alpha^2\theta^2(\theta+1)+8\alpha\theta(\theta+2)+12(\theta+1)}{\theta(\alpha\theta+2)(\alpha\theta+6)}\right]$$

$$= \mu\left[1+\frac{\alpha^2\theta^2+16\alpha+12}{\theta(\alpha\theta+2)(\alpha\theta+6)}\right] > \mu$$

This shows that QPAD (2.3) is always over dispersed and thus it can be used for over-dispersed data (variance greater than the mean).

4.2 Increasing Hazard Rate and Unimodality

The QPAD (2.3) has an increasing hazard rate (IHR) and unimodal. Since

$$\frac{P_3(x+1;\theta,\alpha)}{P_3(x;\theta,\alpha)} = \frac{1}{\theta+1}\left[1+\frac{2(x+2)\theta}{\left(x^2+3x\right)\theta+\left(\alpha\theta^2+2\alpha\theta+2\theta+\alpha\right)}\right]$$

Is decreasing function in x, $P_3(x;\theta,\alpha)$ is log-concave. Therefore, QPAD has an increasing hazard rate and unimodal. The interrelationship between log-concavity, unimodality and increasing hazard rate (IHR) of discrete distributions has been discussed in Grandell (1997).

4.3 Generating Functions

The probability generating function of QPAD can be obtained as

$$P_X(t) = \frac{\theta^2}{(\alpha\theta+2)(\theta+1)^3}\left[\theta\sum_{x=0}^{\infty}x^2\left(\frac{t}{\theta+1}\right)^x+3\theta\sum_{x=0}^{\infty}x\left(\frac{t}{\theta+1}\right)^x+\left(\alpha\theta^2+2\alpha\theta+2\theta+\alpha\right)\sum_{x=0}^{\infty}\left(\frac{t}{\theta+1}\right)^x\right]$$

$$= \frac{\theta^2}{(\alpha\theta+2)(\theta+1)^2}\left[\frac{\alpha\theta^2+2\alpha\theta+2\theta+\alpha}{\theta+1-t}+\frac{2\theta t\left\{2(\theta+1)-t\right\}}{(\theta+1-t)^3}\right]$$

The moment generating function of QPAD is thus given by

$$M_X(t) = \frac{\theta^2}{(\alpha\theta+2)(\theta+1)^2}\left[\frac{\alpha\theta^2+2\alpha\theta+2\theta+\alpha}{\theta+1-e^t}+\frac{2\theta e^t\left\{2(\theta+1)-e^t\right\}}{\left(\theta+1-e^t\right)^3}\right]$$

5. Estimation

5.1 Method of Moment Estimate (MOME): Since QPAD (2.3) has two parameters to be estimated, first two moments about origin are required to have MOME of parameters. We have

$$\frac{\mu_2'-\mu_1'}{\left(\mu_1'\right)^2} = \frac{2(\alpha\theta+12)(\alpha\theta+2)}{(\alpha\theta+6)^2} = k\ (\text{Say})$$

Assuming $\alpha\theta = b$, following quadratic equation in b can be obtained

$$(2-k)b^2+(28-12k)b+(48-36k) = 0$$

Solving the above quadratic equation for b and substituting $\alpha\,\theta = b$ in the expression for mean, MOME $\left(\tilde{\theta},\tilde{\alpha}\right)$ of (θ,α) are obtained as

$$\tilde{\theta} = \frac{b+6}{(b+2)\,\bar{x}} \text{ And } \tilde{\alpha} = \frac{b}{\tilde{\theta}} = \frac{b\,(b+2)\,\bar{x}}{b+6}.$$

5.2 Maximum Likelihood Estimate (MLE): Let $(x_1, x_2, \ldots, x_n)$ be a random sample of size n from the QPAD (2.3) and let f_x be the observed frequency in the sample corresponding to $X = x$ $(x = 1, 2, 3, \ldots, k)$ such that $\sum_{x=1}^{k} f_x = n$, where k is the largest observed value having non-zero frequency. The likelihood function L of the QPAD (2.3) is given by

$$L = \left(\frac{\theta^2}{\alpha\,\theta+2}\right)^n \frac{1}{(\theta+1)^{\sum_{x=1}^{k}(x+3)f_x}} \prod_{x=1}^{k}\left[\left(x^2+3x\right)\theta+\left(\alpha\,\theta^2+2\alpha\,\theta+2\theta+\alpha\right)\right]^{f_x}$$

The log likelihood function is thus obtained as

$$\log L = n\log\left(\frac{\theta^2}{\alpha\,\theta+2}\right) - \sum_{x=1}^{k}(x+3)f_x\log(\theta+1) + \sum_{x=1}^{k} f_x\log\left[\left(x^2+3x\right)\theta+\left(\alpha\,\theta^2+2\alpha\,\theta+2\theta+\alpha\right)\right]$$

The maximum likelihood estimates $\left(\hat{\theta},\hat{\alpha}\right)$ of (θ,α) of QPAD (2.3) is the solutions of the following log likelihood equations

$$\frac{\partial \log L}{\partial\theta} = \frac{2n}{\theta} - \frac{n\,\alpha}{\alpha\,\theta+2} - \frac{n\left(\bar{x}+3\right)}{\theta+1} + \sum_{x=1}^{k}\frac{\left[\left(x^2+3x\right)+2\left(\alpha\,\theta+\alpha+1\right)\right]f_x}{\left[\left(x^2+3x\right)\theta+\left(\alpha\,\theta^2+2\alpha\,\theta+2\theta+\alpha\right)\right]} = 0$$

$$\frac{\partial \log L}{\partial\alpha} = -\frac{n\,\theta}{\alpha\,\theta+2} + \sum_{x=1}^{k}\frac{\left(\theta^2+2\theta+1\right)f_x}{\left[\left(x^2+3x\right)\theta+\left(\alpha\,\theta^2+2\alpha\,\theta+2\theta+\alpha\right)\right]} = 0$$

Where $\bar{x}$ is the sample mean.

These two log likelihood equations do not seem to be solved directly. However, the Fisher's scoring method can be applied to solve these equations. We have

$$\frac{\partial^2 \log L}{\partial\theta^2} = -\frac{2n}{\theta^2} + \frac{n\,\alpha^2}{(\alpha\,\theta+2)^2} + \frac{n\left(\bar{x}+3\right)}{(\theta+1)^2} + \sum_{x=1}^{k}\frac{\left[2\alpha\left\{\left(x^2+3x\right)+2\left(\alpha\,\theta+\alpha+1\right)\right\} - \left\{\left(x^2+3x\right)+2\left(\alpha\,\theta+\alpha+1\right)\right\}^2\right]f_x}{\left[\left(x^2+3x\right)\theta+\left(\alpha\,\theta^2+2\alpha\,\theta+2\theta+\alpha\right)\right]^2}$$

$$\frac{\partial^2 \log L}{\partial\alpha^2} = \frac{n\,\theta^2}{(\alpha\,\theta+2)^2} - \sum_{x=1}^{k}\frac{\left(\theta^2+2\theta+1\right)^2 f_x}{\left[\left(x^2+3x\right)\theta+\left(\alpha\,\theta^2+2\alpha\,\theta+2\theta+\alpha\right)\right]^2}$$

$$\frac{\partial^2 \log L}{\partial\theta\,\partial\alpha} = -\frac{2n}{(\alpha\,\theta+2)^2} + \sum_{x=1}^{k}\frac{\left[\begin{array}{l}2(\theta+1)\left\{\left(x^2+3x\right)\theta+\left(\alpha\,\theta^2+2\alpha\,\theta+2\theta+\alpha\right)\right\}\\ -\left(\theta^2+2\theta+1\right)^2\left\{\left(x^2+3x\right)+2\left(\alpha\,\theta+\alpha+1\right)\right\}\end{array}\right]f_x}{\left[\left(x^2+3x\right)\theta+\left(\alpha\,\theta^2+2\alpha\,\theta+2\theta+\alpha\right)\right]^2} = \frac{\partial^2 \log L}{\partial\alpha\,\partial\theta}$$

For the maximum likelihood estimates $\left(\hat{\theta},\hat{\alpha}\right)$ of (θ,α) of QPAD (2.3), following equations can be solved

$$\begin{bmatrix} \dfrac{\partial^2 \log L}{\partial \theta^2} & \dfrac{\partial^2 \log L}{\partial \theta \, \partial \alpha} \\ \dfrac{\partial^2 \log L}{\partial \theta \, \partial \alpha} & \dfrac{\partial^2 \log L}{\partial \alpha^2} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}} \begin{bmatrix} \hat{\theta}-\theta_0 \\ \hat{\alpha}-\alpha_0 \end{bmatrix} = \begin{bmatrix} \dfrac{\partial \log L}{\partial \theta} \\ \dfrac{\partial \log L}{\partial \alpha} \end{bmatrix}_{\substack{\hat{\theta}=\theta_0 \\ \hat{\alpha}=\alpha_0}}$$

Where θ_0 and α_0 are the initial values of θ and α, respectively. These equations are solved iteratively till sufficiently close values of $\hat{\theta}$ and $\hat{\alpha}$ are obtained. Note that the starting initial values of the parameters are the values of parameters given by MOME.

6. Applications to Ecology

The QPAD has been fitted to a number of data sets to test its goodness of fit over Poisson distribution (PD), Poisson-Lindley distribution (PLD) and Poisson-Akash distribution (PAD). The maximum likelihood estimate (MLE) has been used to fit the QPAD. Two examples of observed count datasets from ecology, for which the PD, PLD, PAD and QPAD has been fitted, are presented. The first dataset is the number of Student's historic data on Haemocytometer counts of yeast cells available in Gosset (1908) [2] and the second data set is the number of European corn-borer of Mc. Guire *et al.* (1957) [5]. The fitted plots of distributions for datasets in tables 1 and 2 are presented in figure 3. It is clear from the goodness of fit of QPAD and from the fitted plots of distributions that QPAD gives much closer fit than PD, PLD, and PAD and hence it can be considered as an important distribution in ecology.

The fitted plot of distributions for datasets in table 1 and 2 are shown in figure 3. It is obvious from the goodness of fit in tables 1 and 2 and the fitted plot of distributions in figure 3 that QPAD gives better fit.

Table 1: Observed and expected number of Haemocytometer yeast cell counts per square observed by Gosset (1908) [2]

Number of yeast cells per square	Observed frequency	Expected frequency			
		PD	PLD	PAD	QPAD
0	213	202.1	234.0	236.8	213.4
1	128	138.0	99.4	95.6	124.0
2	37	47.1	40.5	39.9	44.6
3	18	10.7 }	16.0 }	16.6 }	13.2 }
4	3	1.8 }	6.2 }	6.7 }	3.5 }
5	1	0.2 }	2.4 }	2.7 }	0.8 }
6	0	0.1 }	1.5 }	1.7 }	0.5 }
Total		400.0	400.0	400.0	400.0
ML Estimates		$\hat{\theta}=0.6825$	$\hat{\theta}=1.9502$	$\hat{\theta}=2.2603$	$\hat{\theta}=4.6817$ $\hat{\alpha}=-0.0380$
χ^2		10.08	11.04	14.68	2.46
D.F		2	2	2	1
p-value		0.0065	0.0040	0.0006	0.1168

Table 2: Observed and expected number of European corn-borer of Mc. Guire *et al.* (1957) [5]

Number of corn-borer per plant	Observed frequency	Expected frequency			
		PD	PLD	PAD	QPAD
0	188	169.4	194.0	196.3	186.9
1	83	109.8	79.5	76.5	87.3
2	36	35.6	31.3	30.8	33.5
3	14	7.8 }	12.0 }	12.4 }	11.4 }
4	2	1.2 }	4.5 }	4.9 }	3.4 }
5	1	0.2 }	2.7 }	3.1 }	1.5 }
Total	324	324.0	324.0	324.0	324.0
ML Estimates		$\hat{\theta}=0.6481$	$\hat{\theta}=2.0432$	$\hat{\theta}=2.3451$	$\hat{\theta}=3.7463$ $\hat{\alpha}=0.2137$
χ^2		15.19	1.29	2.33	0.42
d.f.		2	2	2	1
p-value		0.0005	0.5247	0.3119	0.5169

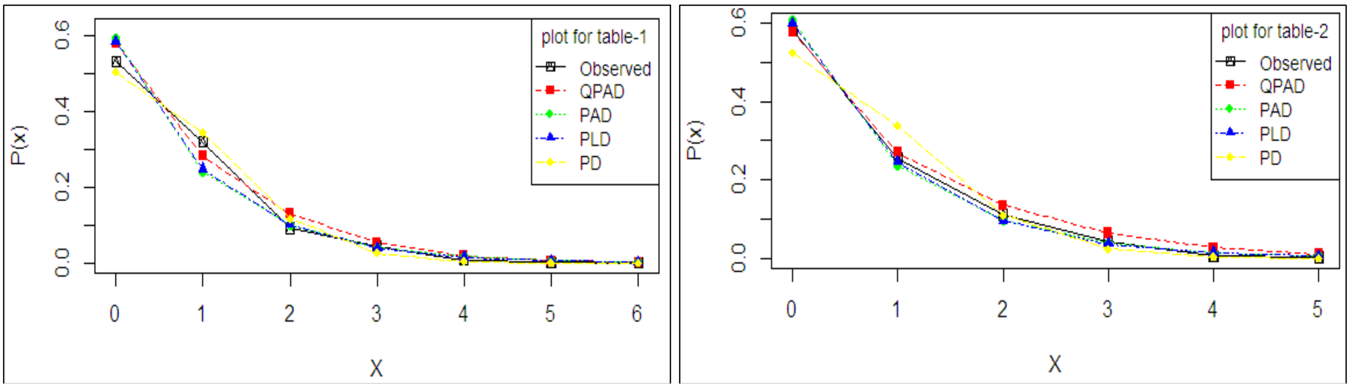

Fig 3: Fitted plot of distributions for datasets in tables 1 and 2

7. Concluding Remarks

In this paper a Quasi Poisson-Akash distribution (QPAD) has been obtained by compounding Poisson distribution with a quasi Akash distribution (QAD) introduced by Shanker (2016) [8]. The expression for the r th factorial moment has been derived and hence the moments about origin and the moments about mean have been given. The expressions for coefficient of variation, skewness and kurtosis have been obtained and their natures have been studied graphically. The method of moments and the method of maximum likelihood estimation have been discussed for estimating the parameter of the proposed distribution. The distribution has been fitted using maximum likelihood estimate to two datasets from ecology to test its goodness of fit over Poisson distribution (PD), Poisson-Lindley distribution (PLD) and Poisson-Akash distribution (PAD) and found that QPAD gives much closer fit than PD, PLD, and PAD in the considered datasets.

8. References

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