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Product cordiality of path union of mix graphs

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Abstract
We obtain mixed path union P_m (G_1, G_2) and discuss it for product cordiality, where G_1 and G_2 are from C_3, C_4, bull graph. In this type of path union G_1 and G_2 are fused at alternate vertices of P_m. Note that the minimum length of a mixed path union is 2 if we have to notice that it is a mixed path union.

Keywords: labeling, cordial, product, bull graph, mixed path union, triangle, cycle C_4 Subject Classification: 05C78

1. Introduction
The graphs we consider are simple, finite, undirected and connected. For terminology and definitions we depend on Graph Theory by Harary [9], a dynamic survey of graph labeling by J. Gallian [8] and Douglas West. [11]. I. Cahit introduced the concept of cordial labeling [7]. There are variety of cordial labeling available in labeling of graphs. Sundaram, Ponraj, and Somasundaram [10] introduced the notion of product cordial labeling. A product cordial labeling of a graph G with vertex set V is a function f from V to {0, 1} such that if each edge uv is assigned the label f (u) f (v), the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a product cordial labeling is called a product cordial graph. We use v_f (0, 1) = (a, b) to denote the number of vertices with label 1 are a in number and the number of vertices with label 0 are b in number. Similar notion on edges follows for e_f (0, 1) = (x, y).
A lot of work is done in this type of labeling so far. One interested in survey may refer Dynamic survey in Graph labeling by J. Gillian. We mention some part of it. Sundaram, Ponraj, and Somasundaram have shown that trees; unicyclic graphs of odd order; triangular snakes; dragons; helms; Pm∪Pn; Cm∪Pn; Pm∪K1,n; Wm∪Fn (Fn is the fan Pn + K1); K1,m∪K1,n; Wm∪ K1,n; Wm ∪Pn; Wm ∪Cn; the total graph of Pn (the total graph of Pn has vertex set V (Pn) ∪E (Pn) with two vertices adjacent whenever they are neighbors in Pn); Cn if and only if n is odd; $C_n^{(t)}$, the one-point union of t copies of C_n, provided t is even or both t and n are even; K2+mK1 if and only if m is odd; $C_m \cup P_n$ if and only if m+n is odd; $K_{m,n} \cup$Ps if s > mn; Cn+2∪K1,n; Kn∪Kn, (n-1)/2 when n is odd; Kn∪Kn-1,n/2 when n is even; and P2 n if and only if n is odd. Are product cordial graphs. They also prove that $K_{m,n}$ (m, n > 2), $P_m \times P_n$ (m,n > 2) and wheels are not product cordial and if a (p, q)-graph is product cordial graph, then q = 6 (p-1) (p + 1)/4 + 1. In this paper we show that P_m (G_1, G_2) where G_1 and G_2 from {C_3, C_4, bull graph} are product cordial graphs.

2. Preliminaries
2.1 Fusion of vertex
Let G be a (p, q) graph. Let u≠v be two vertices of G. We replace them with single vertex w and all edges incident with u and that with v are made incident with w. If a loop is formed is deleted. The new graph has p-1vertices and at least q-1 edges. If u϶G_1 and v϶G_2, where G_1 is (p_1, q_1) and G_2 is (p_2, q_2) graph. Take a new vertex w and all the edges incident to u and v are joined to w and vertices u and v are deleted. The new graph has p_1+p_2-1 vertices and $q_1 + q_2$ edges. Sometimes this is referred as u is identified with the concept is well elaborated in John Clark, Holton [6]

A bull graph bull (G) was initially defined for a C_3-bull. It has a copy of G with an pendent edge each fused with any two adjacent vertices of G. For G is a (p, q) graph, bull (G) has p+2 vertices and q+2 edges. In this we refer bull graph to C3- bull [11].

Path union of G, i.e. P_m (G) is obtained by taking a path p_m and take m copies of graph G. Then fuse a copy each of G at every vertex of path at given fixed point on G. It has mp vertices and mq + m-1 edges. Where G is a (p, q) graph.

Mixed path union $P_m (G_1, G_2)$ is obtained by taking a path $P_m = (v_1, e_1, v_2, e_2,.. v_m)$ and at v_i of it a copy of G_1 is fused if $i \equiv 1$ (mod 2) and a copy of G_2 is fused if $i \equiv 0$(mod 2) at a fixed vertex of G_1 and that of G_2 where $G_1 = (p_1, q_1)$ and G_2 is (p_2, q_2) graph. It is important to note that $P_m (G_1, G_2) \neq P_m (G_2, G_1)$ (in the sense of isomorphism). The minimum length of $P_m (G_1, G_2)$ is m = 2. It has $xp_1 + xp_2$ vertices and $xq_1 + xq_2 + 2x-1$ edges when m = 2x and $(x+1)\ p_1 + xp_2$ vertices and $xq_1 + xq_2 + 2x+q_2$ edges when m = 2x+1.

3. Main Results

Theorem 3.1

$G = P_m(G_1,G_2)$ is product cordial where G_1 is C_3 and G_2 is C_4 for all m as even number and odd m = 2x+1 for even x only. Proof: We take a path $P_m = (v_1, e_1, v_2, e_2... v_m)$. At v_i we fuse C_3 if $i\equiv 1$ (mod 2) and C_4 if $i \equiv 0$ (mod 2) .The copy of C_3 fused at v_i is given by $(u_{i,1}, e_{i,1}, u_{i,2}, e_{i,2},u_{i,3}, e_{i,3},u_{i,1})$ and that copy of C_4 fused at v_i is given by $(v_{i,1}, e_{i,1}, v_{i,2}, e_{i,2},v_{i,3}, e_{i,3},v_{i,4},ei_{,4},v_{i,1})$.Note that $u_{i,j}$ is same as vi, i = 1, 3,.. and $v_{i,j}$ is same as v_i ; i = 2, 4,.. Thus when m = 2x we have |V (G)| =7x. and|E (G) | = 9m-1

Define a function f: V (G)-{0, 1} as follows.

Case m = 2x.

F $(u_{i,j})$ = 0 for i = 1,3,5,..,x;

F $(v_{i,j})$= 0 for i = 2, 4,..,x;

F $(u_{i,j})$ = 1 for i = x+1,x+3,x+5,..,2x;

F $(v_{i,j})$= 1 for i = x+2,x+ 4,..,2x.

The label number distribution is given by $v_f (0, 1) = (7x, 7x)$ and $e_f (0, 1) = (7y+x, 7y+x-1)$ where $y = \frac{x}{2}$ for x is even number. When x is odd number then $v_f (0,1) = (7x+3, 7x+4)$;take $y = \frac{x-1}{2}$, $e_f(0,1)=(7y+x+3,7y+x+4)$.

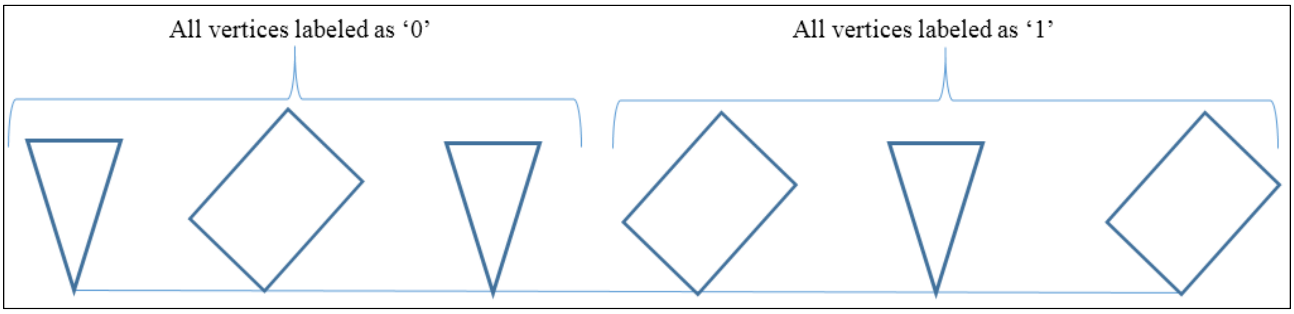

Fig 1: mixed path union on C_3 and C_4 with m = 6

Case m = 2x+1 (only even x)

To obtain a labeled copy of $P_m (G_1, G_2)$ from one end we label $P_{2x} (G_1, G_2)$ part of $P_{2x+1}(G_1, G_2)$ as above. The copy of C_3 fused at last vertex of P_{2x+1} is labeled as follows:

F $(u_{i,\ 1})$ = 1;

F $(u_{i,\ 2})$ = 1;

F $(u_{i,\ 3})$ = 0.

The label number distribution is given by $v_f(0,1) = (7x+1, 7x+2)$ and $e_f(0, 1)=(7y+x+2, 7y+x+1)$ where $y = \frac{x}{2}$ for x is even number.

#

Theorem 3.2 $G = P_m (G_1, G_2)$ is product cordial where G_1 is C_4 and G_2 is C_3. For all m even number and m is odd number 2x+1 then x is odd number only.

Proof: We take a path $P_m = (v_1, e_1, v_2, e_2... v_m)$. At v_i we fuse C_3 if $i\equiv 0$ (mod 2) and C_4 if $i \equiv 1$ (mod 2). The copy of C_3 fused at v_i is given by $(u_{i,1}, e_{i,1}, u_{i,2}, e_{i,2}, u_{i,3}, e_{i,3}, u_{i,1})$ and that copy of C_4 fused at v_i is given by $(v_{i,1}, e_{i,1}, v_{i,2}, e_{i,2}, v_{i,3}, e_{i,3}, v_{i,4}, ei_{,4}, v_{i,1})$. Note that $u_{i,j}$ is same as vi, i = 2, 4, 6,.. And $v_{i,\ j}$ is same as v_i; i = 1, 3.

Thus when m = 2x we have |V (G)| =7x. And| E (G) | = 9m-1

Define a function f: V (G)-{0, 1} as follows.

Case m = 2x.

F $(u_{i,j})$ = 0 for i = 2, 4,..x;

F $(v_{i,j})$= 0 fori = 1,3,5,..,x;

F $(u_{i,j})$ = 1 for i = x+2,x+ 4,..,2x;

F $(v_{i,j})$= 1 for i = x+1,x+3,x+5,..,2x;

The label number distribution is given by $v_f(0, 1) = (7x, 7x)$ and $e_f(0, 1)=(7y+x, 7y+x-1)$ where $y = \frac{x}{2}$ for x is even number. When x is odd number then $v_f(0,1) = (7y+4, 7y + 3)$; $y = \frac{x-1}{2}$, $e_f(0, 1)=(7y+x), 7y+x-1)$.

Case m = 2x+1 (only odd x)
To obtain a labeled copy of P_m (G_1, G_2) from one end we label P_{2x} (G_1, G_2) part of P_{2x+1} (G_1, G_2) as above. The copy of C_4 fused at last vertex of P_{2x+1} is labeled as follows: (x is odd number)
F ($u_{i,1}$) = 1;
F ($u_{i,2}$) = 1;
F ($u_{i,3}$) = 1,
F ($u_{i,4}$) = 0.

When x is odd number then $v_f(0,1) = (7y+5, 7y+6)$; $y = \frac{x-1}{2}$, $e_f(0,1)=(7y+x+2),7y+x+2)$. #

Theorem 3.3 G = P_m (G_1, G_2) is product cordial where G_1 is C_3 and G_2 is bull graph for m= 2x, x is even number and m = 2x+1 when x is even number.
Proof: We take a path $P_m = (v_1, e_1, v_2, e_2... v_m)$. At v_i we fuse C_3 if i≡ 1 (mod 2) and bull if i ≡ 0 (mod 2). The copy of C_3 fused at v_i with vertex $u_{i,1}$ of C_3 is given by ($u_{i,1}$, $e_{i,1}$, $u_{i,2}$, $e_{i,2}$,$u_{i,3}$, $e_{i,3}$,$u_{i,1}$) and that copy of bull fused at v_i at $v_{i,1}$of bull is given by {($v_{i,1}$, $e_{i,1}$, $v_{i,2}$, $e_{i,2}$,$v_{i,3}$, $e_{i,3}$,$v_{i,1}$) and pendent vertices $v_{i,4}$, $v_{i,5}$ with corresponding pendent edges ($v_{i,2}v_{i,4}$) and ($v_{i,3}$ $v_{i,5}$). Note that $u_{i,j}$ is same as v_i, i = 1, 3, 5,... and $v_{i,j}$ is same as v_i ; i = 2, 4, 6,.. Thus when m = 2x we have |V (G)| =8x. And |E (G)| = 10x-1.and |V (G)|= 8x+ 3 |E (G)| = 10x+3 if m = 2x+1.
Define a function f: V (G)-{0, 1} as follows.
Case m = 2x.
F ($u_{i,j}$) = 0 fori = 1,3,5,..,x;
F ($v_{i,j}$) = 0 for i = 2, 4,..,x;
F ($v_{i,4}$) = 0, f ($v_{i,5}$) = 0 for i≤ x
F ($u_{i,j}$) = 1 for i = x+1, x+ 3,..,2x;
F ($v_{i,j}$)= 1 for i = x+2, x+4, x+6,..,2x;
F ($v_{i,4}$) = 1, f ($v_{i,5}$) = 1for x ≤ i≤ 2x

The label number distribution is given by $v_f(0,1) = (8y,8y)$ and $e_f(0,1)=(8y+x+1,8y+x-1)$ where $y = \frac{x}{2}$ for x is even number.
When x is odd number then there is no vertex prime labeling.
Case m =2x+1
To obtain a labeled copy of P_m (G_1, G_2) from one end we label P_{2x} (G_1, G_2) part of P_{2x+1} (G_1, G_2) as above. The copy of C_3 fused at last vertex of P_{2x+1} is labeled as follows:
F ($u_{i,1}$) = 1;
F ($u_{i,2}$) = 1;
F ($u_{i,3}$) = 0;

The label number distribution is given by $v_f(0,1) = (8y+2,8y+1)$ and $e_f(0,1)=(8y+x+2,8y+x+1)$ where $y = \frac{x}{2}$ for x is even number.
For odd number x there is no product cordial labeling. #

Theorem 3.4 G = P_m (G_1, G_2) is product cordial where G_1 is bull graph and G_2 is C_3 cycle for m = 2x, x is even and for all m.
Proof: We take a path $P_m = (v_1, e_1, v_2, e_2... v_m)$. At v_i we fuse C_3 if i≡ 0 (mod 2) and bull if i ≡ 1 (mod 2). The copy of C_3 fused at v_i with vertex $u_{i,1}$ of C_3 is given by ($u_{i,1}$, $e_{i,1}$, $u_{i,2}$, $e_{i,2}$, $u_{i,3}$, $e_{i,3}$, $u_{i,1}$) and that copy of bull fused at v_i at $v_{i,1}$of bull is given by {($v_{i,1}$, $e_{i,1}$, $v_{i,2}$, $e_{i,2}$, $v_{i,3}$, $e_{i,3}$, $v_{i,1}$) and pendent vertices $v_{i,4}$, $v_{i,5}$ with corresponding pendent edges ($v_{i,2}v_{i,4}$) and ($v_{i,3}$ $v_{i,5}$). Note that $u_{i,j}$ is same as v_i, i = 2, 4, 6,.. and $v_{i,j}$ is same as v_i ; i = 1, 3, 5,.. Thus when m = 2x we have |V (G)| =8x. And |E (G)| = 10x-1.and |V (G)|= 8x+ 3 |E (G)| = 10x+3 if m = 2x+1.
Case m = 2x.
F ($u_{i,j}$) = 0 for i = 2,4,6,..,x;
F ($v_{i,j}$) = 0 for i = 1, 3,..x;
F ($v_{i,4}$) = 0, f ($v_{i,5}$) = 0; for i≤ x
F ($u_{i,j}$) = 1 for i = x+2,x+ 4,.., 2x;
F ($v_{i,j}$)= 1 for i = x+1,x+3,x+5,.., 2x;
F ($v_{i,4}$) = 1, f ($v_{i,5}$) = 1 for x ≤ i≤ 2x

The label number distribution is given by $v_f(0,1) = (8y,8y)$ and $e_f(0,1)=(8y+x+1,8y+x-1)$ where $y = \frac{x}{2}$ for x is even number.
When x is odd number then there is no vertex prime labeling.
Case m = 2x+1

To obtain a labeled copy of P_m (G_1, G_2) from one end we label P_{2x} (G_1, G_2) part of P_{2x+1} (G_1, G_2) as above. The copy of bull fused at last vertex of P_{2x+1} is labeled as follows:
F ($u_{i,1}$) = 1;
F ($u_{i,2}$) = 1;
F ($u_{i,3}$) = 1;
F ($u_{i,4}$) = 0;
F ($u_{i,5}$) = 0;

The label number distribution is given by v_f (0, 1) = (8y+2, 8y+3) and e_f (0, 1) = (8y+x+2, 8y+x+3) where $y = \frac{x}{2}$ for x is even number.

When x is odd number given by 2y+1, y= 0, 1…We have v_f (0, 1) = (8y+7, 8y+6) and e_f (0, 1) = (10y+8, 10y+7) where $y = \frac{x-1}{2}$. #

4. Conclusions

A mixed path union P_m (G_1, G_2) has at odd vertex of P_m a copy of G_1 is fused and at even vertex of path P_m the copy of G_2 is fused. We take G1 and G2 from {C3, C4, bull graph}. We have proved following results:

1) P_m (G_1, G_2) is product cordial where G_1 is C_3 and G_2 is C_4 for all m as even number and odd m = 2x+1 for even x only
2) P_m (G_1, G_2) is product cordial where G_1 is C_4 and G_2 is C_3. For all m even number and m is odd number 2x+1 then x is odd number only.
3) P_m (G_1, G_2) is product cordial where G_1 is C_3 and G_2 is bull graph for m= 2x, x is even number and m = 2x+1 when x is even number.
4) P_m (G_1, G_2) is product cordial where G_1 is bull graph and G_2 is C_3 cycle for m = 2x, x is even and for all m.

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