International Journal of Statistics and Applied Mathematics 2018; 3(3): 226-230

International Journal of Statistics and Applied Mathematics

ISSN: 2456-1452
Maths 2018; 3(3): 226-230
© 2018 Stats & Maths
www.mathsjournal.com
Received: 18-03-2018
Accepted: 22-04-2018

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A view on rb-convergence in multiset topological spaces

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Abstract
This paper deals with the concepts of regular b-closed M-sets and regular b-continuous M-set functions are studied with necessary examples. Also the concepts of rb-convergence to a point, rb-accumulates to a point, rb-S-closed M-spaces are studied and some interesting properties are discussed. Finally, sequence of rb-convergence and rb-Hausdorff M-spaces are discussed.

Keywords: regular b-closed M-sets, regular b-continuous M-set functions, rb-convergence, rb-accumulates, rb-S-closed M-spaces
2010 AMS Subject Classification: 54A05, 54A10, 54A20

1. Introduction
The notion of M-topological space and the concept of open M-sets are introduced by Girish and sunil Jacob John [4]. Moreover, topologies on multisets can be associated to IC-bags or nk-bags introduced by Chakrabarthy [3] (Chakrabarty, 2000; Chakrabarty and Despi, 2007) with the help of rough set theory. The association of rough set theory and topologies on multisets through bags with interval counts (Chakrabarty and Despi, 2007) can be used to develop theoretical study of covering based rough sets with respect to universe as multisets.
Nagaveni and Narmadha [9, 10] introduced the notions of regular b-closed (rb-closed) sets in topological space. Regular open sets and b-open sets have been introduced and investigated by Stone [12] and Andrijevi′c [1]. Generalized α-closed sets have been introduced and investigated Maki, Devi and Balachandran [8].

2. Preliminaries
Definition 2.1 [4]: Let M ∈ [X]w and τ ⊆ P∗ (M). Then τ is called a Multiset topology of M if τ satisfies the following properties.
1. The M-set M and the empty M-set ϕ are in τ.
2. The M-set union of the elements of any sub collection of τ is in τ.
3. The M-set intersection of the elements of any finite sub collection of τ is in τ.

Definition 2.2 [4] Given a subM-set A of an M-topological space M in [X]w, the interior of A is denoted by Int (A) and it is is defined as follows: Int(A) = ∪ {G ⊆ M: G is an open M-set and G ⊆ A} and $C_{Int(A)}(x) = \max\{C_G(x) : G \subseteq A, G \in \tau\}$.

Definition 2.3 [4]: Given a subM-set A of an M-topological space M in [X]w, the closure of A is denoted by Cl (A) and is defined asfollows: Cl(A) = ∩ {K ⊆ M: K is a closed M-set and A ⊆ K} and $C_{Cl(A)}(x) = \min\{C_K(x) : A \subseteq K, K \in \tau^c\}$.

3. On rb-Closed M-Sets And rb-Continuous M-Set Functions
Definition 3.1: Let (M, τ) be an M-topological space in $[X]^W$. Any subM-set A of M is called a regular open M-set if A = int (cl (A)) with $C_A(x) = C_{int(cl(A))}(x)$, for all x ∈ X.

Definition 3.2: Let (M, τ) be an M-topological space in $[X]^W$. Any subM-set A of M is called a regular closed M-set if A = cl (int (A)) with $C_A(x) = C_{cl\,(int\,(A))}(x)$, for all x ∈ X.

Definition 3.3: Let (M, τ) be an M-topological space in $[X]^W$ and A be any subM-set of M. The M-set union of all regular open M-sets of (M, τ) contained in A is called the regular interior of A and is denoted by rint (A) i.e. rint(A) = ∪ {G ⊆ M: each G is a regular open M-set and G ⊆ A} with $C_{rint\,(A)}(x)$ = Max $\{C_G(x)$: G ⊆ A, each G is a regular open M-set}, for all x ∈ X.

Definition 3.4: Let (M, τ) be an M-topological space in $[X]^W$ and A be any subM-set of M. The M-set intersection of all regular closed M-sets of (M, τ) containing A is called the regular closure of A and it is denoted by rcl (A). i.e. rcl(A) = ∩ { K ⊆ M: each K is a regular closed M-set and A ⊆ K } with $C_{rcl(A)}(x)$ = Min $\{C_k(x)$: A ⊆ K, each K is a regular closed M-set}, for all x ∈ X.

Definition 3.5: Let (M, τ) be an M-topological space in $[X]^W$. Any subM-set A of M is called a b-open M-set if A ⊆ int(cl(A)) ∪ cl(int(A)) with $C_A(x) \le$ Max { $C_{int(cl(A)}(x)$, $C_{cl(int(A))}(x)$ }, for all x ∈ X.

Definition 3.6: Let (M, τ) be an M-topological space in $[X]^W$. Any subM-set A of M is called a regular b-closed (rb-closed) M-set if rcl(A) ⊆ U with $C_{rcl(A)}(x) \le C_U(x)$, whenever A ⊆ U with $C_A(x) \le C_U(x)$ and U is a b-open M-set in (M, τ), for all x ∈ X. The complement of a regular b-closed M-set is called a regular b-open (rb-open) M-set.

Definition 3.7: Let (M, τ) be an M-topological space in $[X]^W$. For any subM-set A of M, the rb-interior of A is defined as the M-set union of all rb-open M-sets of (M, τ) contained in A and is denoted by rb-int(A) i.e. rb-int(A) = ∪ { G ⊆ M: each G is a rb-open M-set and G ⊆ A } with $C_{rb\text{-}int(A)}(x)$ = Max { $C_G(x)$: G ⊆ A, each G is a rb-open M-set }, for all x ∈ X.

Definition 2.8: Let (M, τ) be an M-topological space in $[X]^W$. For any subM-set A of M, the rb-closure of A is defined as the M-set intersection of all rb-closed M-sets of (M, τ) containing A and is denoted by rb-cl (A). i.e. rb-cl(A) = ∩ { K ⊆ M: each K is a rb-closed M-set and A ⊆ K } with $C_{rcl(A)}(x)$ = Min $\{C_k(x)$: A ⊆ K, each K is a rb-closed M-set }, for all x ∈ X.

Definition 3.9: Let (M, τ) be an M-topological space in $[X]^W$. Any subM-set A of M is called an α-open M-set if A ⊆ int (cl (int (A))) with $C_A(x) \le C_{int(cl(int(A)))}(x)$, for all x ∈ X. The complement of an α-open M-set is called an α-closed M-set.

Definition 3.10: Let (M, τ) be an M-topological space in $[X]^W$ and A be any sub M-set of M. The M-set intersection of all α-closed M-sets containing A is called the α-closure of A and is denoted by αcl (A). Similarly, the M-set union of all α-open M-sets of (M, τ) contained in A is called the α-interior of A and is denoted by αint (A). i.e. αcl(A) = ∩ { K ⊆ M: each K is an α-closed M-set and A ⊆ K } with $C_{\alpha cl(A)}(x)$ = Min $\{C_k(x)$: A ⊆ K, each K is an α-closed M-set }, for all x ∈ X. Similarly, αint (A) = ∪ { K ⊆ M: each K is an α-open M-set and A ⊇ K } with $C_{\alpha int(A)}(x)$ = Max $\{C_k(x)$: A ⊇ K, each K is an α-open M-set }, for all x ∈ X.

Definition 3.11: Let (M, τ) be an M-topological space in $[X]^W$. Any subM-set A of M is called a generalized α-closed (gα-closed) M-set if αcl(A) ⊆ U with $C_{\alpha cl(A)}(x) \le C_U(x)$, whenever A ⊆ U with $C_A(x) \le C_U(x)$, and U is an α-open M-set in (M, τ), for all x ∈ X.

Proposition 3.1.1: Every regular closed M-set is a rb-closed M-set.

Remark 3.1.2: the converse of the above Proposition 2.1.1 need not be true as shown in the Example 2.1.12.

Example 3.1.3 Let X = {a, b, c}, W = 2. Let M = {1/a, 2/b, 1/c}. Let τ = { M, ϕ, { 1/a }, { 2/b }, { 1/a, 2/b } }. Then τ^c = {ϕ, M, {2/b, 1/c}, {1/a, 1/c}, {1/c}}. Clearly, τ is an M-topology and the ordered pair (M, τ) is an M-topological space. Also, the collection of the regular closed M-sets of (M, τ) is { M, ϕ, { 2/b, 1/c }, { 1/a, 1/c } } and the collection of the b-open M-sets of (M, τ) is { ϕ, M, { 1/a }, { 2/b }, { 1/b }, { 1/a, 2/b }, { 2/b, 1/c }, { 1/a, 1/c }, { 1/b, 1/a }, { 1/a, 1/b, 1/c } }.
Let A = {1/c} be a sub M-set of M and A ⊂ U = {1/a, 1/c} is the b-open M-set in (M, τ). Then rcl (A) = {1/c} ⊂ U. Therefore, rcl(A) ⊂ U with $C_{rcl(A)}(x) < C_U(x)$, for all x ∈ X. whenever A ⊆ U and U is b-open M-set with $C_A(x) \le C_U(x)$, for all x ∈ X. Hence, A is a rb-closed M-set. But int (A) = ϕ and cl (int (A)) = ϕ. Thus, A ≠ cl (int (A)) with $C_A(c) = 1 \ne 0 = C_{cl(int(A))}(c)$, $C_A(a) = C_{cl(int(A))}(a) = 0$ and $C_A(c) = C_{cl(int(A))}(c) = 0$, for all x ∈ X. Hence, A is not a regular closed M-set.

Proposition 3.1.4: Every regular closed M-set is a gα-closed M-set.

Remark 3.1.5: The converse of the Proposition 2.1.2 need not be true as shown in the Example 2.1.13

Example 3.1.6: Let X = {a, b, c}, W = 2. Let M = {1/a, 2/b, 1/c}. Let τ = { ϕ, M, { 1/a }, { 1/a, 2/b }, { 2/b } }. Then τ^c = {M, ϕ, {2/b, 1/c}, {1/c}, {1/a, 1/c}}. Clearly, τ is an M-topology and the ordered pair (M, τ) is an M-topological space. Now the collection of the α-open M-sets of (M, τ) is { M, ϕ, { 1/a }, { 1/a, 2/b }, { 2/b } } and α-closed M-sets of (M, τ) is { M, ϕ, { 2/b, 1/c }, { 1/c }, { 1/a, 1/c } }.
Let A = {1/b, 1/c} be a sub M-set of M. Then A ⊂ U = M is the α-open M-set in (M, τ). Then αcl (A) = {2/b, 1/c} ⊂ U. Therefore, αcl (A) ⊂ U with $C_{\alpha cl\,(A)}(x) < C_U(x)$, for all x ∈ X whenever A ⊂ U and U is an α-open M-set. Hence, A is a gα-closed M-set. But int (A) = ϕ and cl (int (A)) = ϕ. Thus A ≠ cl(int(A)) with $C_A(c) = 1 \ne 0 = C_{cl(int(A))}(c)$, $C_A(a) = C_{cl(int(A))}(a) = 0$ and $C_A(b) = C_{cl(int(A))}(b) = 0$. Hence, A is not a regular closed M-set.

Proposition 3.1.7: Every rb-closed M-set is a gα-closed M-set.

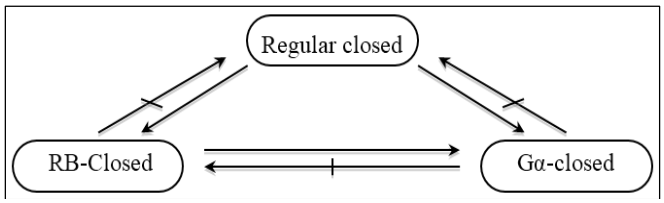
Remark 3.1.8: The converse of the Proposition 2.1.3 need not be true as shown in the Example 2.1.14

Example 3.1.9: Let X = {a, b, c}, W = 2. Let M = {1/a, 2/b, 1/c}. Let τ = { ϕ, M, { 1/a }, { 1/a, 2/b }, { 2/b } }. Then τ^c = { M, ϕ, { 2/b, 1/c }, { 1/c }, { 1/a, 1/c } }. Now τ is an M-topology and the ordered pair (M, τ) is an M-topological space. Now the collection of the regular-closed M-sets of (M, τ) is { ϕ, M, { 1/c, 2/b }, { 1/c, 1/a } }, the collection of the α-closed M-sets of (M, τ) is { M, ϕ, { 2/b, 1/c }, { 1/c }, { 1/a,

1/c } }, the collection of the b-open M-sets of (M, τ) is { ϕ, M, { 1/a }, { 2/b }, { 1/b }, { 1/a, 2/b }, { 2/b, 1/c }, { 1/a, 1/c }, { 1/b, 1/a }, { 1/a, 1/b, 1/c } } and the collection of the α-open M-sets of (M, τ) is { M, ϕ, { 2/a }, { 1/b }, { 2/a, 1/b } }. Let A = {1/a, 1/b, 1/c} be a sub M-set of M and A ⊂ U = M is the α-open M-set in (M, τ). Then αcl (A) = M = U. Therefore, αcl (A) = U with $C_{\alpha cl\,(A)}(x) = C_U(x)$ whenever A ⊂ U and U is α-open M-set with $C_A(x) < C_U(x)$, for all x ∈ X. Hence, A is a gα-closed M-set. But A = U = {1/a, 1/b, 1/c} is the b-open M-set. Then rcl (A) = M ⊈ U with $C_{rcl\,(A)}(x) \nleq C_U(x)$, for all x ∈ X, whenever A = U and U is b-open M-set. Hence, A is not a rb-closed M-set.

Remark 3.1.10: From the above discussions, the following implications hold: In the following diagram by A⟶B means A implies B

A⟵┼ B does not imply A.



3.2 Regular b-continuous M-set functions

Definition 3.2.1 Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function f: (M, τ) ⟶ (N, σ) is said to be a continuous M-set function if f^{-1} (A) is closed (resp. open) M-set in (M, τ), for every closed (resp. open) M-set A in (N, σ).

Definition 3.2.2 Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function f: (M, τ) ⟶ (N, σ) is said to be a regular continuous M-set function if f^{-1} (A) is regular closed (resp. regular open) M-set in (M, τ), for every closed (resp. open) M-set A in (N, σ).

Definition 3.2.3 Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function f: (M, τ) ⟶ (N, σ) is said to be a regular b-continuous (rb-continuous) M-set function if f^{-1} (A) is rb-closed (res. rb-open) M-set in (M, τ), for every closed (res. open) M-set A in (N, σ).

Definition 3.2.4 Let (M, τ) and (N, σ) be any two M-topological spaces. Any M-set function f: (M, τ) ⟶ (N, σ) is said to be a generalized α-continuous (gα-continuous) M-set function if f^{-1} (A) is gα-closed (res. gα-open) M-set in (M, τ), for every closed (res. open) M-set A in (N, σ).

Proposition 3.2.5 Let (M, τ) and (N, σ) be any two M-topological spaces in $[X]^W$. For any M-set function f: (M, τ) → (N, σ), the following statements are equivalent:

i. f is a rb-continuous M-set function.
ii. f(rb-cl(A)) ⊆ cl(f(A)), for each A ⊆ M with $C_{f(rb\text{-}cl(A))}(x) \le C_{clf(A)}(x)$, for all x ∈ X
iii. rb-cl((B)) ⊆ f^{-1}(cl(B)), for each B ⊆ N with $C_{rb\text{-}cl(f^{-1}(B))}(x) \le C_{f^{-1}(cl(B))}(x)$, for all x ∈ X.
iv. f^{-1}(int(B)) ⊆ rb-int(f^{-1}(B)), for each B ⊆ N with
v. $C_{(f^{-1}(int(B)))}(x) \le C_{rb\text{-}int(f^{-1}(B))}(x)$, for all x ∈ X.

Proposition 3.2.6: Every regular continuous M-set function is a continuous M-set function.

Remark 3.2.7: The converse of the above Proposition need not be true as shown in the Example 2.2.5

Example 3.2.8: Let X = {a, b, c}, W_1 = 2, Y = {x, y, z}, and W_2 = 1. Let M = {1/a, 2/b, 1/c} and N = {1/x, 1/y, 1/z} be two M-sets. Let τ = { M, ϕ, { 1/a }, { 2/b }, { 1/a, 2/b } } and σ = { N, ϕ, { 1/x, 1/y } } be the M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. Now the collection of the closed M-sets of (N, σ) is { ϕ, N, { 1/z } } and the collection of the regular closed M-sets of (M, τ) is { M, ϕ, { 2/b, 1/c }, { 1/a, 1/c } } and the collection of the closed M-sets of (M, τ) is { ϕ, M, { 2/b, 1/c}, {1/a, 1/c}, {1/c}}.
Let M-set function f: (M, τ) ⟶ (N, σ) be defined by f = { (1/a, 1/x)/1, (2/b, 1/y)/2, (1/c, 1/z)/1 }. For the closed M-set A = {1/z} in (N, σ), f^{-1} (A) = {1/c} is an closed M-set in (M, τ). Trivially f^{-1} (ϕ) = ϕ and f^{-1}(N) = M are closed M-sets in (M, τ). But {1/c} is not a regular closed M-set in (M, τ). Hence, every continuous M-set function need not be a regular continuous M-set function.

Proposition 3.2.9: Every regular continuous M-set function is a rb-continuous M-set function.

Remark 3.2.10: The converse of the Proposition 2.2.3 need not be true as shown in following Example.

Example 3.2.11: Let X = {a, b, c}, W_1 = 2, Y = {x, y, z}, and W_2 = 1. Let M = {1/a, 2/b, 1/c} and N = {1/x, 1/y, 2/z} be two M-sets. Let τ = { M, ϕ, { 1/a }, { 2/b }, { 1/a, 2/b } } and σ = { N, ϕ, { 1/x, 1/y } } be two M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. Now the collection of the regular closed M-sets of (M, τ) is { M, ϕ, { 2/b, 1/c }, { 1/a, 1/c } } and the collection of the rb-closed M-sets of (M, τ) is { M, ϕ, { 1/c, 2/b }, { 1/a, 1/c }, { 1/c } } and the collection of the closed M-sets of (N, σ) is { ϕ, N, { 2/z } }.
Let M-set function f: (M, τ) ⟶ (N, σ) be defined by f = { (1/a, 1/x)/1, (2/b, 1/y)/2, (1/c, 2/z)/2 }. For the closed M-set A = {2/z} in (N, σ), f^{-1} (A) = {1/c} is an rb-closed M-set in (M, τ). Trivially f^{-1} (ϕ) = ϕ and f^{-1} (N) = M are rb-closed M-sets in (M, τ). But {1/c} is not a regular closed M-set in (M, τ). Hence, every rb-continuous M-set function need not be a regular continuous M-set function.

Proposition 3.2.12: Every rb-continuous M-set function is a gα-continuous M-set function.

Remark 3.2.13: The converse of the Proposition 2.2.4 need not be true as shown in the Example 2.2.7

Example 3.2.14: Let X = {a, b, c}, W_1 = 2, Y = {x, y, z} and W_2 = 2. Let M = {1/a, 2/b, 1/c} and N = {2/x, 1/y, 1/z} be two M-sets. Let τ = { ϕ, M, { 1/a }, { 2/b }, { 1/a, 2/b } } and σ = { N, ϕ, {2/x}} be the M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. Now the collection of the closed M-sets of (N, σ) is { ϕ, N, { 1/y, 1/z } } and the collection of the rb-closed M-sets of (M, τ) is { M, ϕ, { 1/c }, { 2/b, 1/c }, { 1/c, 1/a } } and the collection of the gα-closed M-sets of (M, τ) is { M, ϕ, { 1/c }, { 1/b, 1/c }, { 1/c, 2/b }, { 1/c, 1/a }, { 1/a, 1/b, 1/c } }.
Let M-set function f: (M, τ) ⟶ (N, σ) be defined by f = { (1/a, 2/x)/2, (1/b, 1/y)/2, (1/c, 1/z)/1 }. For the closed M-set A = {1/y, 1/z} in (N, σ), f^{-1} (A) = {1/b, 1/c} is a gα-closed M-set in (M, τ). Trivially f^{-1} (ϕ) = ϕ and f^{-1} (N) = M are gα-

closed M-sets in (M, τ), but {1/b, 1/c} is not a rb-closed in (M, τ). Hence every gα-continuous M-set function need not be a rb-continuous M-set function.

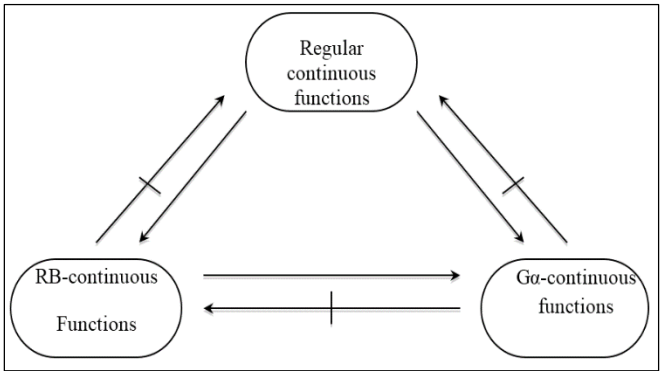
Proposition 3.2.15: Every regular continuous M-set function is gα-continuous M-set function.

Remark 3.2.16: The converse of the Proposition 2.2.5 need not be true as shown in the Example 2.2.6

Example 3.2.17: Let X = {a, b, c}, $W_1 = 2$, Y = {x, y, z} and $W_2 = 2$. Let M = {1/a, 2/b, 1/c} and N = {2/x, 1/y, 1/z} be two M-sets. Let τ = { ϕ, M, { 1/a }, { 2/b }, { 1/a, 2/b } } and σ = { N, ϕ, { 2/x, 1/y } } be the M-topologies on M and N respectively. Then (M, τ) and (N, σ) are M-topological spaces. Now the collection of the closed M-sets of (N, σ) is { ϕ, N, { 1/z } } and the collection of the regular closed M-sets of (M, τ) is { M, ϕ, { 2/b, 1/c }, { 1/c, 1/a } } and the collection of the gα-closed M-sets of (M, τ) is { M, ϕ, { 2/b, 1/c }, { 1/c }, { 1/b, 1/c }, { 1/c, 1/a }, { 1/a, 1/b, 1/c } }.
Let M-set function f: (M, τ) ⟶ (N, σ) be defined by f = { (1/a, 2/x)/2, (2/b, 1/y)/2, (1/c, 1/z)/1 }. For the closed M-set A = {1/z} in (N, σ), f^{-1} (A) = {1/c} is an gα-closed M-set in (M, τ). Trivially f^{-1} (ϕ) = ϕ and f^{-1} (N) = M are gα-closed M-sets in (M, τ), but {1/c} is not a regular closed in (M, τ). Hence, every gα-continuous M-set function need not be a regular continuous M-set function.

Remark 3.2.18: From the above discussions, the following implications hold: In the following diagram by A B means A implies B

A ⟵┼ B does not imply A.



4. On rb-Convergence in Multiset Topological Spaces

In this chapter, the concepts of rb-convergence to a point, rb-accumulates to a point, rb-S-closed M-spaces are studied and some interesting properties are discussed. Finally, sequence of rb-convergence and rb-Hausdorff M-spaces are discussed.

Definition 4.1: A filter is a non-empty collection F of subM-sets of an M- topological space (M, τ) such that

1. ϕ ∉ F;
2. If A ∈ F and B ⊇ A with C_B (x) ≥ C_A(x), for all x ∈ X, then B ∈ F and
3. If A ∈ F and B ∈ F, then A ∩ B ∈ F with $C_{A \cap B}$ (x) ≤ C_F (x), for all x ∈ X.

Definition 4.2: Let (M, τ) be an M-topological space. A filter base of a filter F is a sub collection B of M such that for every F ∈ F there exists B ∈ B with B ⊆ F with C_B (x) ≤ C_F (x), for all x ∈ X. Such a subcollection B satisfies the following conditions:

1. ϕ ∉ B
2. For every B_1, B_2 ∈ B there exists B_3 ∈ B with $B_3 \subseteq B_1 \cap B_2$ with CB_3 (x) ≤ CB_1 (x) ∩ CB_2 (x), for all x ∈ X.

Definition 4.3: Let F be a filter on B. Then F is a maximal filter if every filter including F coincides with F

Definition 4.4: Let (M, τ) be an M-topological space. A filter base B = { { A_α }$_{\alpha \varepsilon J}$, where J is an indexed set and A_α ∈ M } rb-converges to a point x_o $\in^m$ M if for each rb-open M-set V containing m/x_o there exists an A_α ∈ B such that A_α ⊆ rb-cl(V) with CA_α (x) ≤ $C_{rb\text{-}cl(V)}$ (x), for all x ∈ X.

Definition 4.5: A filter base B = { { A_α }$_{\alpha \in J}$, where J is indexed set and A_α ∈ M } rb-accumulates to a point x_o $\in^m$ M if for each rb-open M-set V containing m/x_o and A_α ∈ B, A_α ∩ rbcl (V) ≠ ϕ with Min { $C_{A\alpha}$(x), $C_{rb\text{-}cl\,(V)}$ (x) } ≠ 0, for all x ∈ X.

Definition 4.6: Let (M, τ) be an M-topological space. A collection $\mathcal{A}$ of sub M-sets of a space M is said to cover M, or to be a covering of M, if the union of the elements of $\mathcal{A}$ is equal to M. It is called an open covering of M if its elements are open sub M-sets of M.

Definition 4.7: Let (M, τ) be an M-topological space.An M-topological space (M, □) is said to be rb-S-closed if for every cover { V_α\α ∈ I, I is an indexed set } of (M, τ) by rb-open M-sets of M, there exists a finite subM-set I_0 of I such that ∪ { rb-cl (V_α)\α ∈ I_0 } = M with Max { $C_{rb\text{-}cl(V\alpha)}$ (x)\ α ∈ I_0 } = C_M (x), for all x ∈ X.

Definition 4.8: Let (M, τ) be an M-topological space. A subM-set K of an M-topological space (M, □) is said to be rb-closed relative to M if for any cover {U_i: i ∈ I, I is an indexed set } of K by rb-open M-sets, there exists a finite subM-set I_0 of I such that K ⊆ ∪ { rb-cl (U_i)\ i ∈ I_0 } with C_K (x) ≤ Max { $C_{rb\text{-}cl(Ui)}$(x)\ i ∈ I_0 }, for all x ∈ X.

Remark 4.9: The family of all rb-open (res, rb-closed) M-sets of (M, τ) is denoted by RBO(X) (res, RBC(X)). The family of all rb-open M-sets of (M, τ) containing a point x $\in^m$ M is denoted by RBO (M, m/x).

Remark 4.10: A filter base B is rb-convergent to a point x $\in^m$ M if and only if B contains the collection {rbcl (U): U ∈ RBO (M, m/x)}.

Proposition 4.11: For an M-topological space (M, τ), the following statements are equivalent:

1. M is an rb-S-closed M-space.
2. For each family of rb-closed M-sets { F_α }$_{\alpha \in I}$, where I is an indexed set, such that ∩ (F_α) = ϕ with Min { $C_{F\alpha}$ (x), α ∈ I } = 0, for all x ∈ X, there exists a finite subfamily $\{F_{\alpha_i}\}_{i=1}^{n}$ such that $\bigcap_{i=1}^{n}$ rb-int ($F_{\alpha i}$) = ϕ with Min { $C_{rb\text{-}int(F\alpha i)}$(x), i = 1, 2,…n } = 0, for all x ∈ X.
3. Each filter base B = { A_α }$_{\alpha \in I}$ with C_B (x) = $C_{\{ A\alpha \}\alpha \in I}$(x), for all x ∈ X, rb-accumulates to some point x_o $\in^m$ M.
4. Each maximum filter base B Rb-converges.

Proposition 4.12: A subM-set A of a M-topological space (M, τ) is rb-closed relative to M if and only if every filter base B on M with A ∈ B rb-converges to a point in A.

Definition 4.13: Let (M, τ) be an M-topological space. A sequence in a M-set M is a function f: ℕ → M. Then m/x_n = f(n) for all n ∈ ℕ, where ℕ is positive integer and denoted the sequence as $(m/x_n)_{n \in \mathbb{N}}$.

Definition 4.14: Let (M, τ) be an M-topological space, let x $\in^m$ M and N ⊆ M. Then N is said to be a rb-neigh bor hood of m/x if there is an rb-open M-set V in τ such that x $\in^m$ V and $C_V(y) \leq C_N(y)$, for all y ≠ x.
I.e. a rb-neighborhood of m/x in M means any rb-open M-set containing m/x. Here m/x is said to be an interior point of N.

Definition 4.15: Let (M, τ) be an M-topological space. A sequence m/x_1, m/x_2, m/x_3… of points in M is said to be rb-converges to a point x $\in^m$ M if for each rb-neighborhood U of m/x, there is a positive integer ℕ such that $x_n \in^m$ U, for all n ≥ ℕ. It is denoted by $(m/x_n) \to m/x$.

Definition 4.16: An M-topological space (M, τ) is said to be a rb-Hausdorff M-space, if each pair of distinct points m/x and m/y in M, there exist rb-open M-sets U and V containing m/x and m/y respectively such that U ∩ V = ϕ with $C_{U \cap V}(x) = 0$, for all x ∈ X.

Definition 4.17: Let (M, τ) be an M-topological space. A rb-countable basis at a point m/x is a countable M-set { U_n | n ∈ ℕ } of rb-neighbourhood of m/x, such that for any rb-neighbourhood V of m/x there is an n ∈ ℕ such that $U_n \subseteq V$ with $C_{Un}(x) \leq C_V(x)$, for all x ∈ X.
An M-topological space (M, τ) is said to be first rb-countable if every point has a rb-countable basis.

Proposition 4.3 Let (M, τ) be an M-topological space. Let M be a rb-Hausdorff space and $x_n \in^m$ M a rb-convergent sequence. Then the limit $\lim_{n\to\infty} m/x_n$ is unique.

5. Acknowledgement
The authors would like to express their sincere gratitude to the referees for their valuable suggestions and comments for the betterment of this paper.

6. References

1. Andrijevic D. On b-open sets, mat. Vesnik. 1996; 48:59-64.
2. Beceren Y. Almost α -irresolute functions, bull. Calcutta Math. Soc. 2000; 92:213-218.
3. Chakrabarty, Kankana. Ioan Despi. n^k - bags. Int. J Intell. Syst. 2007s; 22(2):223-236.
4. Girish KP. Sunil Jacob John. On Multiset Topologies, 2007s.
5. Computer and Mathematics with Applications, 2012, 37-52
6. James Munkres R. Topology, Prentice Hall of India Private Limited, New Delhi.
7. Jeyanthi V, Janaki C. On πgr-continuous functions, International Jornal of Engineering Research and Applications, Jan-Feb. 2013; 3(1):1861-1870.
8. John Kelly L. General Topology, Spring India Private Limited.
9. Maki H, Devi R, Balachandran K. Associated topologies of generalized α-closed sets and α-generalized closed sets, Mem, Fac. Sci, Kochi Univ. Ser. A Math. 1994; 15:51-63.
10. Nagaveni N, Narmadha A. A note on regular b-open sets in topological spaces, Proc, UGC and DST-Curie sponsored International Conference on Mathematics and its Applications-A New Wave, ICMANW, Dec. 2011, 93-100, 21-22.
11. Nagaveni N, Narmadha A. On regular b-closed sets in topological spaces, Heber International Conference on Applications of Mathematics and Statistics, HICAMS, 2012, 5-7, 81-87.
12. Narmatha A, Nagaveni N, Noiri T. On Regular b-open sets in Topological spaces, Int. Jornal of Math. Analysis. 2013; 7(19):937-948.
13. Stone MH. Applications of the theory of Boolean rings to general topology", Trans. Math, Soc. 1937; 41:375-381.
14. Thamson T. S-closed spaces, Proc. Amer. Math, Soc. 1976; 60:335-338.
15. Thomas Wright. Convergence of sequences and Nets in Metric and Topological space, July 1, 2013, 1-6.