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Application of Queueing theory to Salem railway ticket counters

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Abstract
In this paper the Queueing model (M/M/C: ∞/FIFO) is applied to the ticket counters of the Salem railway station. In this research the various characters of that queueing model have been analyzed and concluded.

Keywords: Queue, arrival rate, departure rate, traffic intensity

1. Introduction
A flow of customers from infinite or finite population towards the service facility forms a queue on account of lack of capability to serve them all at a time. Our main objectives of our research are

i. To apply the basic concepts of the selected queueing model to the Salem railway station, Tamil Nadu, India.
ii. To determine the various characteristics of the selected queueing model to the queueing system in the railway reservation centre at Salem Railway station.

2. Data Collection
We have visited the railway reservation counters at Salem railway Station on various days during 8.00am to 8pm and conducted a survey about the arrivals of customers to the reservation counters and about the service rendered to the customers. The data pertaining to the arrival and Departures of customers to the reservation counters of Salem railway station are given in this section.

The customers arrive at the railway reservation counters under poisson process and the service is done to the customers in the exponential rate. This queueing model involves three servers. The customers are given service under the FIFO (FIRST IN FIRST OUT) discipline. Thus the queueing model which will fit to our problem is (M/M/C: ∞/FIFO) which is also known as the multi server queueing model.

The details regarding the arrivals and departures of customers obtained in the Railway Reservation counters at Salem railway station are given in the following table (2.1) and Table (2.2)

Table 2.1

Time	Time-intervals	Number of Arrivals	Number of Departure
8.00-9.00	0-10	//	-
	10-20	/	/
	20-30	///	//
	30-40	//	///
	40-50	///	//
	50-60	//	///
9.00-10.00	0-10	///	////
	10-20	///// //	///// /
	20-30	/	///// /
	30-40	/////	/////
	40-50	///// /	/////
	50-60	///// //	///
10.00-11.00	0-10	////	//////
	10-20	///// /	/////
	20-30	////	///// /
	30-40	///// /	/////
	40-50	///// /	///// /
	50-60	///// ///	/////
11.00-12.00	0-10	///// /////	///// /////
	10-20	///// //	///// //
	20-30	///// /////	///// //
	30-40	///// ///	///// ///
	40-50	///// /	///
	50-60	///// ///// //	///// ///// /
12.00-13.00	0-10	///// /////	///// //
	10-20	///// ///	///// ///
	20-30	/////	////// ////
	30-40	///// /	///// /////
	40-50	///// /////	/////
	50-60	/////	/////
13.00-14.00	0-10	///	///// /
	10-20	///	///// //
	20-30	///	///// /
	30-40	//	/////
	40-50	/	///
	50-60	/	///
14.00-15.00	0-10	-	-
	10-20	/	//
	20-30	///	///
	30-40	///	/////
	40-50	/////	////
	50-60	///// /	///// /
15.00-16.00	0-10	////	//////
	10-20	////	///
	20-30	//	///
	30-40	///	///
	40-50	//	//
	50-60	/	///
16.00-17.00	0-10	///	//
	10-20	////	////
	20-30	/////	///
	30-40	/	//
	40-50	///	////
	50-60	/	//
17.00-18.00	0-10	///	///
	10-20	/////	////
	20-30	////	//
	30-40	//	/////
	40-50	///// /	///
	50-60	/	//
18.00-19.00	0-10	///// //	///
	10-20	///	/////
	20-30	////	///
	30-40	///	///// /
	40-50	////	////
	50-60	////	//

19.00-20.00	0-10	//	///
	10-20	//	/
	20-30	///	////
	30-40	/	///
	40-50	//	//
	50-60	/	///
	0-10	///	///
	10-20	////	/////
	20-30	///	///
	30-40	////	///// /
	40-50	////	//
	50-60	-	/

Table 2.2

Time intervals Per Hour	No. of arrivals Per Hour	No. of Departure Per Hour
8.00-9.00	12	11
9.00-10.00	29	29
10.00-11.00	34	33
11.00-12.00	53	46
12.00-13.00	43	44
13.00-14.00	13	32
14.00-15.00	16	19
15.00-16.00	17	17
16.00-17.00	21	19
17.00-18.00	25	23
18.00-19.00	11	15
19.00-20.00	18	20

3. Evaluation of the Characteristics

From the above table,

1. Average number of arrivals to the system per hour (λ) = $\frac{292}{12}$ per hour = 24.33 per hour = 24 per hour (appr.)
2. Average number of departure from the system (μ) per hour = $\frac{308}{12}$ per hour = 25.67 per hour μ = 26 (appr.)
3. Since C = 4, μ =$\frac{26}{4}$, i.e. μ = 6.5000 per queue.
4. To find the traffic intensity,$\rho = \frac{\lambda}{C\mu}$, Here λ = 24, μ = 6.5000 and C = 3, $\rho = \frac{\lambda}{C\mu} = \frac{24}{26}$ = 0.92
5. P_n= probability that there are n- customers in the system both Waiting and in service.

$$P_n = \begin{cases} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n P_0; \ 1 \leq n \leq C \\ \\ \frac{1}{C^{n-1}C!}\left(\frac{\lambda}{\mu}\right)^n P_0; \ n \geq C \end{cases} \quad \ldots (3.3.1)$$

$$\text{Where } P_0 = \left[\sum_{n=0}^{C-1} \frac{1}{n!}\left(\frac{\lambda}{\mu}\right)^n + \frac{1}{C!}\left(\frac{\lambda}{\mu}\right)^C \frac{C\mu}{C\mu-\lambda}\right]^{-1} \quad \ldots (3.3.2)$$

$$\text{To find } P_0 = \left[1 + \frac{1}{1!}\left(\frac{24}{6.5000}\right)^1 + \frac{1}{C!}\left(\frac{24}{6.5000}\right)^2 + \frac{1}{3!}\left(\frac{24}{6.5000}\right)^3 + \frac{1}{4!}\left(\frac{24}{6.5000}\right) \frac{24}{26-24}\right]^{-1}$$

$$= [1 + 3.6923 + 6.8165 + 8.3896 + 100.6746]^{-1}$$

$$= [120.5730]^{-1}$$

$$P_0 = 0.0083$$

5. The values of P_n are calculated for n = 1, 2, 3........

Using the formulae (1.3.1) and (1.3.2) these values are given in the Table,

Table 3.1

N	0	1	2	3	4	5	6	7	8	9
P_n	0.0083	0.0306	0.0566	0.0696	0.0643	0.0593	0.0548	0.0506	0.0467	0.0431

N	10	11	12	13	14	15	16	17	18	19
P_n	0.0398	0.0210	0.0327	0.0296	0.0273	0.0252	0.0232	0.0214	0.0199	0.0183

N	20	21	22	23	24	25	26
P_n	0.0168	0.0155	0.0143	0.0132	0.0122	0.0112	0.0104

From the table 3.3.1 we can easility see that

$\sum_{n=0}^{C-1} P_n + \sum_{n=C}^{\infty} P_n = 1$

4. The Results of Characteristics of Our System

$[(\text{M/M/C}):(\infty/\text{FIFO})]$

(i) P (n≥ C) $=$ Probability that an arrival has to wait

$= \frac{C\mu(\frac{\lambda}{\mu})^C}{C!(C\mu-\lambda)} P_0 = \frac{4(6.500)\,(\frac{24}{6.5000})^3 0.0083}{4!(26-24)} = 0.8356$

(ii) Probability that an arrival enters the service without wait

$= 1 - \text{P}(\text{n} \geq \text{C}) = 1 - 0.8356 = 0.1644$

(iii) Average queue length, Lq

$\text{Lq} = \frac{\lambda\mu\,(\frac{\lambda}{\mu})^C\,P_0}{(C-1)!(C\mu-\lambda)^2} = \frac{24(6.5000)\,(\frac{24}{6.5000})^4 0.0083}{24}$

Lq = 10.0272 …….. (3.4.1)

(v) Average number of customers is the system $[E(n)]$

Ls = Lq+$\frac{\lambda}{\mu}$ = 10.0272+3.6923

Ls = 13.7195

(iv) Aerage waiting time of an arrival $[Wq]$

$\text{Wq} = \frac{1}{\lambda}$

$\text{Lq} = \frac{10.0272}{24}$

E (ω) $=$ 0.4176 or 25 minutes …. (3.4.3)

(vi) Average waiting time an arrival spends in the system $[Ws]$

Ws = Wq+$\frac{1}{\mu}$

= 0.4178+ $\frac{1}{6.5000}$

= 0.4178 + 0.1538

Ws = 0.5716 or 34 minutes … (3.4.4)

4.1 Verification of the Little‘s formulae

The relationships between the characteristics are given by means of the following formulae known as little formulae there are

(i) Ls = λ Ws

(ii) Lq = λ Wq

(iii) Ws = Wq + $\frac{1}{\mu}$

Our findings also satisfy the Little's formulae.
(i) To verify Ls = λ Ws from equation (3.4.2)
In this section 3.4 we have obtained that Lq = 13.085 from
Equation (3.4.4) we have found that λ Ws = 24× 0.5716
= 13.7184
= Ls (appr.)

(ii) To prove Lq = Wq
From the equation (3.4.1) we see that Lq = 10.0272
Also from equation (3.4.3) λ Wq = 24 × 0.4178
= 10.0272
= Lq.

5. Observations, Summary

5.1 Observations

In our study we have made the following observations the following results.

i. Traffic intensity, ρ = 0.9231
ii. Probability that there are no customers in the system, $P_0 = 0.0083$
iii. Probability that an arrival has to wait, p (n≥c) = 0.8356
iv. Probability that an arrival enters the service without wait 1-p (n≥c) = 0.1644
v. Average queue length at any time Lq = 10.0272 customers
vi. Average number of customer in the system at any time Ls = 13.7195 customers
vii. Average waiting time of an arrival in the queue, Wq = 0.4178 or hour 25.0680 minutes.
viii. Average waiting time of an arrival spends in the systemWs =0.5716 or 35 minutes (appr)
ix. Average time of service =6 minutes (appr)
x. Average number of arriving rate, λ = 24 per hour.
xi. Average number of service rate, μ = 6.5000 per hour.

5.2 Summary

After conducting the survey and the analysis of data obtained from the reservation centre of salem railway station the following predictions are made regarding the characteristics of the queueing made 1 (M/M/C: ∞/FIFO):

i. An average of 24 customers are arriving to the reservation counters in an hour.
ii. An average of 26 customers are being served in the counters in an hour.
iii. The traffic intensity p= $\frac{\lambda}{c\mu}$ =3.6923<1 and therefore the queueing system followed in the salem railway reservation centre attains stability.
iv. The probability that all the servers bring idle is nearly 2 percent.
v. The probability that an arrival has to wait before getting served is 86 percent.
vi. The probability that an arrival enters the service straight away without having to wait is 14 percent.
vii. At any time an average of 10 customers are found waiting in the queue.
viii. At any time an average of 13 customers are found waiting in the system.
ix. On arrival, a customer has to wait in the queue for 25 minutes before getting served.
x. On arrival, a customer has to wait in the system for 34minutes before getting served.
xi. A reservation clerk takes an average of 7 minutes to reserve a ticket.

6. Conclusion

In our study we have found that it is optional that there are four reservation clerks employed in the salem railway reservation. It means that if we add one more counter to the existing set up, the chance for a reservation clerk to be idle would increase enormously. And also, if one counter is removed from the existing set up, the number of customers waiting in the queue would increase rapidly, thus it would end up in chaos.

Also we have analyzed that the traffic intensity of the queueing system prevailing in salem railway station always remains less then unity and therefore the system attains stability.

7. References

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