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# A self-gravitating non-ideal gas with radiation-heat flux

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**Abstract**
The propagation of a spherical shock wave in a non-ideal gas by taking into account the effects of radiation-heat flux. Radiation pressure and radiation energy are assumed to be negligible. The gas is supposed to be optically thick and self-gravitating and the initial density to be varying and obeying a power law. The shock is assumed to be transparent for radiation and the counter pressure is taken into account. Using the approximate method of Whitham, an ordinary differential equation is obtained which determine the shock velocity and the shock-Mach number. Effects of the radiation, the non-idealness of the gas and the inhomogeneity of the initial medium on the shock velocity and the shock-Mach number are obtained.

**Keywords:** self-gravitating, non-ideal gas, radiation-heat

## 1. Introduction

The problem of propagation of shock waves in a nonhomogeneous medium is of great interest in exploring the effects of explosion in stars and atmosphere of earth and in several other branches of engineering and science. Sakurai [1], Rogers [2], Zel'dovich and Raizer [3], Purohit [4], Rosenau and Frankenthal [5], Vishwakarma, Srivastava and Kumar [6], Vishwakarma and Yadav [7] and many others have discussed the propagation of shock waves in a conducting or non-conducting gas with varying density by self-similarity methods. Bhatnagar and Sachdev [8], and Singh and Yadav [9] have studied the propagation of spherical shock waves in a radiative, self-gravitating and non-conducting gas with decreasing density. They used Whitham's [10] rule to obtain the shock velocity and the flow-variables just behind the shock. In all of these works, the medium is assumed to be a gas obeying the equation of state of a perfect gas.

When the flow takes place at high temperatures, the assumption that the gas is ideal is no more valid. Anisimov and Spiner [11] have taken an equation of state for low density non-ideal gases in a simplified form, and investigated the effect of parameter for non-idealness on the problem of a point explosion. Ranga Rao and Purohit [12] and Ojha [13] have also studied the propagation of shock waves in gases with the above equation of state. In the present work, we analyse the propagation of a spherical shock wave in a non-ideal gas with the equation of state given by Anisimov and Spiner [11], taking into account the effects of radiation-heat flux. Radiation pressure and radiation energy are assumed to be negligible. The gas is supposed to be optically thick (Elliot [14]) and self-gravitating and the initial density to be varying and obeying a power law. The shock is assumed to be transparent for radiation (Wang [15], Helliwell [16]) and the counter pressure is taken into account. As the similarity solutions does not exist for a non-ideal gas with variable initial density (Vishwakarma and Pandey [17]), we study the present problem by using the approximate method of Whitham (Whitham's rule [10]). Although the Whitham's rule is approximate, it agrees well with exact solutions and with experimental results (Jumper [18]). Effects of the radiation, the non-idealness of the gas and the inhomogeneity of the initial medium on the shock velocity and the shock-Mach number are obtained.

## 2. Basic Equations and Boundary Conditions

The medium under consideration is a non-ideal gas whose equation of state is borrowed from the statistical physics (Landau and Lifshitz) [19], and simplified by Anisimov and Spiner [11] in the form

$$p = \Gamma\rho T(1+b\rho) , \qquad (2.1)$$

Where $b(<<1)$ is internal volume of the molecules, and $\Gamma$, $\rho$, p, and T are the gas constant, the density, the pressure and the temperature, respectively. Wu and Roberts [20] and Roberts and Wu [21] have used similar equation of state to study the shock wave theory of sonoluminescence.
The internal energy e per unit mass of the non-ideal gas is given by

$$e = \frac{p}{\rho(\gamma-1)(1+b\rho)} = \frac{p(1-b\rho)}{\rho(\gamma-1)} , \qquad (2.2)$$

Which implies that $c_p - c_v = \Gamma\left(1+\frac{b^2\rho^2}{1+2b\rho}\right) \cong \Gamma$ , $(2.3)$ neglecting the term $b^2\rho^2$. Here $c_p$ and $c_v$ are the specific heats of the gas at the constant pressure and constant volume, respectively, and $\gamma$ is the ratio of $c_p$ and $c_v$.
If we assume that the gas is inviscid, non-heat conducting, self-gravitating and radiating, the fundamental equations governing the spherically symmetric flow are (Verma, Vishwakarma and Sharan [22], Singh and Yadav [9])

$$\rho_t + u\rho_r + \rho u_t + \frac{2\rho u}{r} = 0 , \qquad (2.4)$$

$$u_t + uu_r + \frac{1}{\rho}p_r + \frac{Gm}{r^2} = 0 , \qquad (2.5)$$

$$e_t + ue_r + p\left[\left(\frac{1}{\rho}\right)_t + u\left(\frac{1}{\rho}\right)_r\right] + \frac{1}{\rho r^2}\left(r^2F\right)_r = 0 , \qquad (2.6)$$

$$m_r - 4\pi\rho r^2 = 0 , \qquad (2.7)$$

Where m, G, u, F, r, and t denote the mass of the gas within the radius r, the gravitational constant, the fluid velocity, the radiation heat-flux, the distance from the point of symmetry and the time, respectively. Here, the radiation pressure and the radiation energy are assumed to be negligible.
Assuming local thermodynamic equilibrium and a diffusion model for an optically thick grey gas (Pomraning [23]), the differential approximation of the radiation transport equation can be written in the following form

$$F = -\frac{c\mu}{3}\left(\sigma T^4\right)_r , \qquad (2.8)$$

where $\frac{1}{4}\sigma c$ is the Stefan-Baltzmann constant, c the velocity of light and $\mu$ the Rosseland mean free path for radiation, which is a function of density and temperature.
Following Wang [15], we take
$\mu = \mu_0\rho^\alpha T^\beta$ , $(2.9)$ where $\mu_0$, $\alpha$ and $\beta$ are constants.
We assume that the medium in which an explosive shock wave caused by an energy release, is propagating, is inhomogeneous and the density vary inversely as some power of distance (Rosenau [24], Vishwakarma and Yadav [7], Pai [25]),
i.e. $\rho_0 = \rho_c r^{-w}$ , $(2.10)$ where $\rho_c$ and w are constants.
Since the fluid originally is in hydrostatic equilibrium, we have

$$m_0 = \frac{4\pi\rho_c}{3-w} r^{3-w} , \qquad (2.11)$$

$$p_0 = \frac{2\pi G\rho_c^2}{(3-w)(w-1)} r^{2(1-w)} , \qquad (2.12)$$

$$F_0 = -\frac{4c\sigma\mu_0}{3\Gamma^{\beta+4}} \rho_c^{\alpha-\beta-4} p_c^{\beta+4} R^{2\beta-w(\alpha+\beta+4)+7} \frac{\left(2-w+2\delta r^{-w}\right)}{\left(1+\delta r^{-w}\right)^{\beta+5}} , \qquad (2.13)$$

Where $\delta = b\rho_c$ is the parameter of non-idealness of the gas.
Equation (2.12) may be written as

$$p_0 = p_c r^{2(1-w)} , \qquad (2.14)$$

where $$p_c = \frac{2\pi G \rho_c^2}{(3-w)(w-1)} , \quad 1 < w < 3. \qquad (2.15)$$

The frozen speed of sound (the speed of sound when b = 0) $a_f$ and the speed of sound in the non-ideal gas $a_0$, in the undisturbed state, are given by

$$a_f^2 = \frac{\gamma p_0}{\rho_0} , \qquad (2.16)$$

and $$a_0^2 = \frac{\gamma p_0}{\rho_0}\left(\frac{1+2b\rho_0}{1+b\rho_0}\right) . \qquad (2.17)$$

Therefore $$a_f = \left(\frac{\gamma p_c}{\rho_c}\right)^{\frac{1}{2}} r^{\frac{2-w}{2}} , \qquad (2.18)$$

and $$a_0 = a_f\left(\frac{1+2\delta r^{-w}}{1+\delta r^{-w}}\right)^{\frac{1}{2}} . \qquad (2.19)$$

The conservation conditions across the shock front may be written as

$$\rho_2(U-u_2) = \rho_1 U = m_s\,(\text{say}) , \qquad (2.20a)$$

$$p_2 - p_1 = m_s u_2 , \qquad (2.20b)$$

$$e_2 + \frac{p_2}{\rho_2} + \frac{1}{2}(U-u_2)^2 - \frac{F_2}{m_s} = e_1 + \frac{p_1}{\rho_1} + \frac{1}{2}U^2 - \frac{F_1}{m_s} , \qquad (2.20c)$$

$$m_2 = m_1 , \qquad (2.20d)$$

Where subscripts “1” and “2” denote the states immediately ahead and behind the shock front and U is the shock velocity. The transition region within the shock wave is taken to be transparent to the radiative-flux (Wang [15], Helliwell [16]), so that

$$F_2 = F_1 \qquad (2.20e)$$

and the jump conditions across the front reduce to those of a non-radiative shock passing into a non-ideal gas at rest.
The jump conditions may, therefore, be written as

$$u_2 = \frac{2f(R,\ M)a_f M}{\gamma+1} , \qquad (2.21)$$

$$\rho_2 = \frac{\rho_1(\gamma+1)}{\gamma+1-2f(R,\ M)} , \qquad (2.22)$$

$$p_2 = \frac{p_1}{\gamma+1}\left[2\gamma M^2\left(1-\delta R^{-w}\right)-\gamma+1\right] , \qquad (2.23)$$

where $$f(R,\ M) = \left(1-\delta R^{-w} - M^{-2}\right) \qquad (2.24)$$

and $$M^2 = \frac{U^2}{a_{f1}^2} .$$

Here R is the shock radius and M the shock-Mach number referred to sound speed $a_{f1}$.

**3. Solution of the Problem**

Now, we apply Whitham's rule to get an expression for shock velocity. The equation of motion along the positive characteristic $\frac{dr}{dt} = u + a$ is

$$dp + \rho a du + \left[\frac{\rho a G m}{r^2} + \frac{2\rho u a^2}{r} + \frac{(\gamma - 1)(1 + b\rho)}{r^2}\left(r^2 F\right)_r\right]\frac{dr}{u + a} = 0 \ , \qquad (3.1)$$

Where $\quad a^2 = \frac{\gamma p}{\rho}\left(\frac{1 + 2b\rho}{1 + b\rho}\right)$ .

For diverging shocks, Whitham's rule is to apply the characteristic equation (valid along the positive characteristic) to the flow quantities just behind the shock front. Therefore, substituting the values of $u_2, \ p_2, \ \rho_2, \ F_2$ and $m_2$ from the equations (2.21), (2.22), (2.23) and (2.20d, e) in the equation (3.1), we get after some simplifications

$$\begin{aligned} & RM_R\left[2\gamma Mg + \eta\gamma\left(f + 2M^{-2}\right)\right] + \left\{2\gamma g M^2 - \gamma + 1\right\}(1 - w) + \gamma\delta w M^2 R^{-w} \\ & \quad + \left(\frac{2 - w}{2}\right)\gamma\eta f M + \gamma\delta w \eta M R^{-w} + \left[(w - 1)(\gamma + 1)^2 \eta + 2\gamma M f \eta^2 (\gamma + 1 - 2f)\right. \\ & \quad - \frac{2N(\gamma - 1)(\gamma + 1)^2 \gamma^{-\frac{1}{2}} \Gamma^{-\beta - \frac{5}{2}} \rho_c^{\alpha - \beta - \frac{7}{2}} p_c^{\beta + \frac{5}{2}} \xi R^{2(2-\beta) - w\left(\alpha + \beta + \frac{3}{2}\right)}}{\left(1 + \delta R^{-w}\right)^{\beta + 5}} \times \\ & \quad \left.\left\{-2w\delta R^{-w} + \left(\frac{8 - 5w}{2}\right)\left(2 - w + 2\delta R^{-w}\right) + \frac{(\beta + 5)(2 - w) w\delta R^{-w}}{\left(1 + \delta R^{-w}\right)}\right\}\right] \times \\ & \qquad \left[2fM + (\gamma + 1 - 2f)\eta\right]^{-1} = 0 \ , \qquad (3.2) \end{aligned}$$

Where $\quad g = \left(1 - \delta R^{-w}\right)$ ,

$$\eta(R, M) = \left[\frac{\left(2\gamma g M^2 - \gamma + 1\right)\left\{(\gamma + 1)\left(1 + 2\delta R^{-w}\right) - 2f\right\}}{(\gamma + 1 - 2f)\left\{(\gamma + 1)\left(1 + \delta R^{-w}\right) - 2f\right\}}\right]^{\frac{1}{2}} ,$$

$$\xi(R, M) = \left[\frac{(\gamma + 1)\left(1 + \delta R^{-w}\right) - 2f}{(\gamma + 1 - 2f)}\right] ,$$

and $N = \frac{c\sigma\mu_0}{3\Gamma^{\frac{3}{2}}}$ is the radiation parameter.

For the removal of dimensional constants $\rho_c$ and $p_c$, we assume $\alpha = 1$ and $\beta = -\frac{5}{2}$; and after some simplifications, we get

$$M_R + \frac{1}{R}\left[\frac{K_2}{K_1} + \frac{K_4}{K_1 K_3}\right] - \frac{K_5}{K_1 K_3}\frac{1}{R^2} = 0 \ , \qquad (3.3)$$

Where

$$K_1(R, \ M) = 2\gamma g M + \gamma\eta\left(f + 2M^{-2}\right) \ ,$$

$$K_2(R, \ M) = \left\{2\gamma g M^2 - \gamma + 1\right\}(1 - w) + \gamma w\delta M^2 R^{-w} + \frac{2 - w}{2}\gamma\eta f M + \gamma\eta w M\delta R^{-w},$$

$$K_3(R, \ M) = 2fM + (\gamma + 1 - 2f) \ ,$$

$$K_4(R, \ M) = \eta(w - 1)(\gamma + 1)^2 + 2\gamma M f\eta^2(\gamma + 1 - 2f) \ ,$$

$$K_5(R,\ M) = \frac{2N(\gamma-1)(\gamma+1)^2 \gamma^{-\frac{1}{2}}\xi}{\left(1+\delta R^{-w}\right)^{\frac{5}{2}}}\left[-2w\delta R^{-w} + \left(\frac{8-5w}{2}\right)\left(2-w+2\delta R^{-w}\right) + \frac{5w(2-w)\delta R^{-w}}{2\left(1+\delta R^{-w}\right)}\right].$$

The expressions for the effective shock-Mach number $M_e$ and the shock velocity U in terms of the shock-Mach number M and the shock radius R can be written as

$$M_e = M\left[\frac{1+\delta R^{-w}}{1+2\delta R^{-w}}\right]^{\frac{1}{2}}, \qquad (3.4)$$

and $$U\Big/\left(\frac{\gamma p_c}{\rho_c}\right)^{\frac{1}{2}} = MR^{\frac{2-w}{2}}. \qquad (3.5)$$

Equations (3.3), (3.4) and (3.5) give the shock-Mach number, the effective shock-Mach number and the shock velocity as functions of R.

## 4. Results and Discussion

Values of the shock-Mach number M, the effective shock-Mach number $M_e$ and the shock velocity $U\Big/\left(\frac{\gamma p_c}{\rho_c}\right)^{\frac{1}{2}}$ are calculated from equations (3.3), (3.4) and (3.5) by numerical integration. For numerical integration we have taken $\gamma = 1.4;\ w = 1.1,\ 1.2;$ $N = 0,\ 1,\ 5;\ \delta = 0,\ 0.025.$ (Rosenau [24], Ghoniem *et al.* [26], Ranga Rao and Purohit [12]) and the initial conditions as $M = 4$ at $R = 0.05$ (c.f. Vishwakarma and Yadav [27], Singh and Yadav [9]). The particular values $\delta = 0$ and $N = 0$ are, respectively, associated with the case of a perfact gas and the radiation free case.

The variation of the shock-Mach number M with the shock radius R. It is found that M decreases as R increases, and this decrease is very rapid in the beginning, becomes slower there after and continues until M is reduced to unity. The effective shock-Mach number $M_e$ display similar behaviour as M. It means that the shock decays very fast after its formation, and reduces into a sound wave. In fact, as R increases, the reduced sound speed $\left\{a_1\Big/\left(\frac{\gamma p_c}{\rho_c}\right)^{\frac{1}{2}}\right\} = \left(\frac{1+2\delta R^{-w}}{1+\delta R^{-w}}\right)^{\frac{1}{2}} R^{\frac{(2-w)}{2}}$ increases faster than the reduced shock velocity $U\Big/\left(\frac{\gamma p_c}{\rho_c}\right)^{\frac{1}{2}}$ and this results in a decrease in the shock strength $M_e$.

The effects of an increase in the radiation parameter N are

(i) to increase the shock-Mach number M and the effective shock-Mach number $M_e$, and

(ii) to increase the shock velocity $U\Big/\left(\frac{\gamma p_c}{\rho_c}\right)^{\frac{1}{2}}$

Thus the effects of radiation-heat flux on the shock wave are to increase its velocity and strength.

The effects of an increase in the value of the inhomogeneity index w are to increase the shock-Mach number and the effective shock-Mach number in the case of perfect gas ($\delta = 0$), and to decrease these quantities in the case of non-ideal gas ($\delta = 0.025$)

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