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Self-similar magnetogasdynamic shock waves in an ideal gas with heat conduction and radiation heat-flux

Arvind Kumar Singh

Abstract
The propagation of a spherical shock wave in an ideal gas with heat conduction and radiation heat-flux, in the presence of a spacially decreasing azimuthal magnetic field, is investigated. The initial density of the gas is assumed to obey a power law. The heat conduction is expressed in terms of Fourier's law and the radiation is considered to be of the diffusion type for an optically thick grey gas model. The thermal conductivity K and the absorption coefficient α_R are assumed to vary with temperature and density. The shock wave is assumed to be driven by a piston moving with a variable velocity. Similarity solutions are obtained and the effects of variation of the heat transfer parameters and the variation of piston velocity (or initial density) and Alfven-Mach number are investigated.

Keywords: shock wave, self-similar flow, piston problem, ideal gas, magnetogasdynamics

Introduction

Parker [1] has pointed out that the hydrodynamic blast wave theory can usefully describe the large scale regime to which the flow due to a sudden expansion of the solar corona asymptotically converges. Using similarity assumptions, he has presented a number of numerical solutions for his idealized adiabatic 'solar wind' model. These solutions correspond to the flow driven by a spherical piston in power law motion whose surface is the contact discontinuity enveloping the fresh flare corona in the centre core. Lee and Chen [2] and Rosenau and Frankenthal [3] extended the work of Parker to the hydromagnetic cases. Director and Dabora [4] pointed out that the similarity solutions for time-dependent energy input cases as presented by Rogers [5] and Dabora [6] indicate that the blast waves resulting from linear time-dependent energy input are, in essence, 'piston driven' waves. They presented a numerical investigation of spherical blast waves with time-dependent energy deposition at the inner boundary which is a piston obeying a power law motion. Helliwell [7] studied the effects of radiative heat transfer upon the classical plane, cylindrically symmetric or spherically symmetric piston problem (Sedov) [8]. He has taken a power-law piston speed in view of that, in non-radiative hypersonic flow theory with slender bodies possessing power-law profiles, the flow in a shocked layer is given by the solution of analogous unsteady piston problems with the power-law piston speed (Mirelsh) [9].

Marshak [10] studied the effect of radiation on the shock propagation by introducing the radiation diffusion approximation. Using the same mode of radiation Elliott [11] discussed the conditions leading to self-similarity with a specified functional form of the mean free path of radiation and obtained a solution for self-similar spherical explosions. Gretler and Wehle [12] studied the propagation of blast waves with exponential heat release by taking internal heat conduction and thermal radiation in a detonating medium. Also, Abdel-Raouf and Gretler [13] obtained the non-self-similar solution for the blast waves with internal heat transfer effects. Ghoniem *et al*. [14] obtained a self-similar solution for spherical explosions taking into account the effects of both conduction and radiation in the two limit of Rosseland radiative diffusion and Plank radiative emission. In these works, where both the radiation and conduction effects are considered, the density of the medium ahead of the shock is taken to be uniform and the effects of presence of a magnetic field are omitted.

Since at high temperatures that prevail in the problems associated with shock waves a gas is inonized, electromagnetic effects may also be significant. A complete analysis of such a problem should therefore consist of the study of the gasdynamic flow and the electromagnetic and radiation with conduction fields simultaneously. Also, the results of the study of shock waves propagating in a non-uniform medium are more applicable to shocks formed in the stars (Sedov, Sakurai, Rogers, Summers) [8, 15, 16, 17].

The purpose of this study is therefore to obtain self-similar solutions for the propagation of a magnetogasdynamic shock wave in a non-uniform gas with heat conduction and radiation heat flux in presence of an azimuthal magnetic field, driven out by a cylindrical or spherical piston moving with time according to power law. The medium ahead and behind the shock front are assumed to be an inviscid one to behave as a thermally perfect gas. The piston velocity is assumed to vary as some power of time and the initial density of gas and the initial azimuthal magnetic field to vary as some powers of distance. The heat transfer fluxes are expressed in terms of Fourier's law for heat-conduction and a diffusion radiation mode for an optically grey gas, which is typical of large-scale explosions. The thermal conductivity and absorption coefficient of the gas are assumed to be proportional to appropriate powers of temperature and density (Ghoniem *et al*. [14]). Also, it is assumed that the gas is grey and opaque, and the shock is isothermal. The assumption that the shock is isothermal is a result of the mathematical approximation in which the heat flux is taken to be proportional to the temperature gradient, this excludes the possibility of temperature jump (Zel'dovich and Raizer [18], Rosenau and Frankenthal [19, 20], Bhowmick [21], Singh and Srivastava [22]). The counter pressure (the pressure ahead of the shock) is taken into account. The radiation pressure and radiation energy are neglected (Elliott [11], Wang [23], Abdel-Raouf and Gretler, Ghoniem *et al*.) [13, 14]. The assumption of an optically thick grey gas is physically consistent with the neglect of radiation pressure and radiation energy (Nicastro) [24]. The gas ahead of the shock is assumed to be at rest. Effects of viscosity and gravitation are not taken into account.

2. Fundamental Equations and Boundary Condition

The fundamental equations for one-dimensional unsteady flow of an electrically conducting and ideal gas with heat conduction and radiation heat flux taken into account in presence of an azimuthal magnetic field may, in Eulerian coordinates, be expressed as (Gretler and Wehle [12], Abdel-Raouf and Gretler [13], Ghoniem *et al*. [14], Christer and Helliwell [25], Summers [17]).

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \rho\frac{\partial u}{\partial r} + \frac{2\rho u}{r} = 0 \ , \qquad (2.1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{1}{\rho}\left[\frac{\partial p}{\partial r} + \mu h\frac{\partial h}{\partial r} + \frac{\mu h^2}{r}\right] = 0 \ , \qquad (2.2)$$

$$\frac{\partial h}{\partial t} + u\frac{\partial h}{\partial r} + h\frac{\partial u}{\partial r} + \frac{hu}{r} = 0 \ , \qquad (2.3)$$

$$\frac{\partial e}{\partial t} + u\frac{\partial e}{\partial r} - \frac{p}{\rho^2}\left[\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r}\right] + \frac{1}{\rho r^2}\frac{\partial}{\partial r}\left(r^2 q\right) = 0 \ , \qquad (2.4)$$

where r and t are independent space and time coordinates, ρ is the density, p the pressure, u the flow velocity, h the azimuthal magnetic field, e the internal energy per unit mass, q the heat flux, μ the magnetic permeability.

The total heat flux q, which appears in the energy equation may be decomposed as

$$q = q_C + q_R, \qquad (2.5)$$

where $q_C =$ conduction heat flux, and $q_R =$ radiation heat flux.

According to Fourier's law of heat conduction

$$q_C = -K\frac{\partial T}{\partial r} \ , \qquad (2.6)$$

where K is the coefficient of thermal conductivity of the gas and T is the absolute temperature.

Assuming local thermodynamic equilibrium and using the radiative diffusion model for an optically thick grey gas (Pomraning [26]), the term q_R, which represents radiative heat flux, may be obtained from the differential approximation of the radiation-transport equation in the diffusion limit as

$$q_R = -\frac{4}{3}\left(\frac{\sigma}{\alpha_R}\right)\frac{\partial T^4}{\partial r} \ , \qquad (2.7)$$

Where σ is the Stefan-Boltzmann constant and α_R is the Rossel and mean absorption coefficient.

The electrical conductivity of the gas is assumed to be infinite. Therefore the diffusion term from the magnetic field equation is omitted, and the electrical resistivity is ignored. Also the effect of viscosity on the flow of the gas is assumed to be negligible.

The above system of equations should be supplemented with an equation of state. A ideal gas behaviour of the medium is assumed, so that

$$p = \Gamma \rho T \ , \qquad e = \frac{p}{\rho(\gamma - 1)} \ , \qquad (2.8)$$

Where Γ is the gas constant and γ the ratio of specific heats.

The thermal conductivity K and the absorption coefficient α_R are assumed to vary with temperature and density. These can be written in the form of power laws, namely (Ghoniem *et al*. [14])

$$K = K_0 \left(\frac{T}{T_0}\right)^{\beta_C} \left(\frac{\rho}{\rho_0}\right)^{\delta_C} \ , \quad \alpha_R = \alpha_{R_0} \left(\frac{T}{T_0}\right)^{\beta_R} \left(\frac{\rho}{\rho_0}\right)^{\delta_R} \ , \qquad (2.9)$$

Where subscript '0' denotes a reference state. The exponent in the above equations should satisfy the similarity requirements if a self-similar solution is sought.

A spherical shock is supposed to be propagating in the undisturbed ideal gas with variable density $\rho = Ar^w$, where A and w are constants. Also, the azimuthal magnetic field in the undisturbed gas is assumed to vary as $h = Br^{-k}$, (Rosenau [27]) where B and k are constants.

The flow variables immediately ahead of the shock front are

$$u = 0 \ , \ \rho = \rho_a = Ar_s^w \ , \ h = h_a = Br_s^{-k} \ , \qquad (2.10)$$

$$p = p_a = \frac{(1-k)}{2k} \frac{\mu B^2}{r_s^{2k}} \quad (0 < k < 1) \ , \ q = q_a = 0 \quad (\ [28]) \ ,$$

Where r_s is the shock radius, and the subscript 'a' denotes the conditions immediately ahead of the shock.

The shock is assumed to be isothermal (the formation of the isothermal shock is a result of the mathematical approximation in which the heat flux is taken to be proportional to the temperature gradient. This excludes the possibility of a temperature jump, see for example Zel'dovich and Raizer [18], Rosenau and Frankenthal [19, 20]) and hence, the conditions across it are

$$\rho_a V = \rho_n (V - u_n) \ , \ h_a V = h_n (V - u_n) \ , \ T_a = T_n \ ,$$

$$p_a + \frac{1}{2}\mu h_a^2 + \rho_a V^2 = p_n + \frac{1}{2}\mu h_n^2 + \rho_n (V - u_n)^2 \ , \qquad (2.11)$$

$$e_a + \frac{p_a}{\rho_a} + \frac{1}{2}V^2 + \frac{\mu h_a^2}{\rho_a} + \frac{q_n}{\rho_a V} = e_n + \frac{p_n}{\rho_n} + \frac{1}{2}(V - u_n)^2 + \frac{\mu h_n^2}{\rho_n} \ ,$$

where the subscript 'n' denotes conditions immediately behind the shock front, and $V = \frac{dr_s}{dt}$ denotes the velocity of the shock front. From equations (2.13), we get

$$u_n = (1-\beta)V \ , \ \rho_n = \frac{\rho_a}{\beta} \ , \ h_n = \frac{h_a}{\beta} \ ,$$

$$p_n = \left[\frac{1}{\gamma M^2} + (1-\beta) + \frac{M_A^{-2}}{2}\left(1 - \frac{1}{\beta^2}\right)\right]\rho_a V^2 \ , \qquad (2.12)$$

$$q_n = \left[\frac{1}{2}(\beta^2 - 1) + \frac{1}{\beta M_A^2}(1-\beta)\right]\rho_a V^3 \ ,$$

Where $M = \left(\frac{\rho_a V^2}{\gamma p_a}\right)^{\frac{1}{2}}$ is the shock-Mach number referred to the frozen speed of sound $\left(\frac{\gamma p_a}{\rho_a}\right)^{\frac{1}{2}}$ and $M_A = \left(\frac{\rho_a V^2}{\mu h_a^2}\right)^{\frac{1}{2}}$ is the Alfven-Mach number.

The quantity $\beta(0 < \beta < 1)$ is obtained by the relation

$$\beta^2 - \beta\left[\frac{1}{\gamma M^2} + \frac{M_A^{-2}}{2}\right] - \frac{M_A^{-2}}{2} = 0 \ . \qquad (2.13)$$

3. Similarity Solutions

The inner boundary of the flow-field behind the shock is assumed to be on an expanding surface (piston). In the framework of self-similarity (Sedov [8]) the velocity $U_p = \frac{dr_p}{dt}$ of the piston is assumed to follow a power law which reads (Steiner and Hirschler [29])

$$U_p = \frac{dr_p}{dt} = U_0\left(\frac{t}{t_0}\right)^n \ , \qquad (3.1)$$

Where r_p is the radius of the piston, t_0 denotes a reference time, U_0 is the piston velocity at $t = t_0$. The consideration of heat flux imposes a restriction on n, that is $n \neq 0$ (see relations (3.13)). Also, spherical geometries do not permit $n \leq -1$ for physical reasons. Thus, using equation (3.5), either $n > 0 \ (-2 < w + 2k < 0)$ or $-1 < n < 0 \ (0 < w + 2k < \infty)$. For $n > 0$ the piston is continuously accelerated. A shock is formed which reaches the strong shock limit at large times. For $-1 < n < 0$ the piston velocity jumps, almost instantaneously, from zero to infinity leading to the formation of a shock of high strength in the initial phase. Concerning the shock boundary conditions self-similarity requires that the velocity of the shock $V = \frac{dr_s}{dt}$ is proportional to the velocity of the piston

$$V = \frac{dr_s}{dt} = CU_0\left(\frac{t}{t_0}\right)^n \ , \qquad (3.2)$$

Where C is a constant. The time and space coordinates can be transformed into a dimensionless self-similarity variable as follows

$$\lambda = \frac{r}{r_s} = \left[\frac{(n+1)t_0^n}{U_0 C}\right]\left(\frac{r}{t^{n+1}}\right) \ . \qquad (3.3)$$

Evidently, $\lambda = \lambda_p = \frac{r_p}{r_s}$ at the piston and $\lambda = 1$ at the shock. We express the flow velocity u, density ρ, pressure p, azimuthal magnetic field h and the total heat flux q as (Abdel-Raouf and Gretler [13], Ghoniem *et al.* [14], Vishwakarma and Yadav [30])

$$u = V\phi(\lambda) \ , \ \rho = \rho_a \Lambda(\lambda) \ , \ p = \rho_a V^2 \psi(\lambda) \ ,$$
$$\mu^{1/2} h = \rho_a^{1/2} V H(\lambda) \ , \ q = \rho_a V^3 \eta(\lambda) \ , \qquad (3.4)$$

Where $\phi, \ \Lambda, \ \psi, \ H$ and η are functions of λ only.

For the existence of similarity solutions M and M_A should be constants, therefore

$$w + 2k + \frac{2n}{n+1} = 0. \qquad (3.5)$$

Thus,

$$M^2 = \frac{2k}{\gamma(1-k)} M_A^2 \ , \qquad (3.6)$$

Where $0 < k < 1$.

The conservation equations (2.1) to (2.4) can be transformed into a system of ordinary differential equations with respect to λ

$$(\phi - \lambda)\frac{d\Lambda}{d\lambda} + \Lambda\frac{d\phi}{d\lambda} + \frac{2\Lambda\phi}{\lambda} + w\Lambda = 0 \ , \qquad (3.7)$$

$$(\phi - \lambda)\frac{d\phi}{d\lambda} + \frac{n\phi}{n+1} + \frac{1}{\Lambda}\frac{d\psi}{d\lambda} + \frac{H}{\Lambda}\frac{dH}{d\lambda} + \frac{H^2}{\lambda\Lambda} = 0 \ , \qquad (3.8)$$

$$(\phi - \lambda)\frac{dH}{d\lambda} + \frac{nH}{n+1} + H\frac{d\phi}{d\lambda} + \frac{\phi H}{\lambda} + \frac{wH}{2} = 0 \ , \qquad (3.9)$$

$$(\phi - \lambda)\Lambda\frac{d\psi}{d\lambda} - \gamma\psi(\phi - \lambda)\frac{d\Lambda}{d\lambda} + (\gamma - 1)\Lambda\frac{d\eta}{d\lambda} +$$

$$\frac{2n\psi\Lambda}{n+1}+\frac{2(\gamma-1)\Lambda\eta}{\lambda}-w(\gamma-1)\Lambda\psi=0\ . \qquad (3.10)$$

By using equations (2.6), (2.7) and (2.9) in (2.5), we get

$$q=-\frac{K_0}{T_0^{\beta_C}\rho_0^{\delta_C}}T^{\beta_C}\rho^{\delta_C}\frac{\partial T}{\partial r}-\frac{16}{3}\frac{\sigma T_0^{\beta_R}\rho_0^{\delta_R}}{\alpha_{R_0}}T^{3-\beta_R}\rho^{-\delta_R}\frac{\partial T}{\partial r}\ . \qquad (3.11)$$

Using the equations (2.8) and (3.4) in (3.11), we get

$$\eta=-(n+1)\frac{d}{d\lambda}\left(\frac{\psi}{\Lambda}\right)\left[\frac{K_0A^{\delta_C-1}t_0^{w(\delta_C-1)-1}\psi^{\beta_C}V^{2\beta_C-1+\left(1+\frac{1}{n}\right)\left[w(\delta_C-1)-1\right]}}{T_0^{\beta_C}\rho_0^{\delta_C}\Gamma^{\beta_C+1}(CU_0)^{\frac{1}{n}\left[w(\delta_C-1)-1\right]}(n+1)^{w(\delta_C-1)}\Lambda^{\beta_C-\delta_C}}+\right.$$

$$\left.\frac{16\sigma T_0^{\beta_R}\rho_0^{\delta_R}A^{-1-\delta_R}t_0^{-1-w(\delta_R+1)}\psi^{3-\beta_R}V^{5-2\beta_R-\left(1+\frac{1}{n}\right)\left[w(\delta_R+1)+1\right]}}{3\alpha_{R_0}\Gamma^{4-\beta_R}(CU_0)^{-\frac{1}{n}\left[w(\delta_R+1)+1\right]}(n+1)^{-w(\delta_R+1)}\Lambda^{3-\beta_R+\delta_R}}\right]\ . \qquad (3.12)$$

Equation (3.12) shows that similarity solution of the present problem exists when

$$\beta_C=1+\frac{1}{2}\left[\frac{1}{n}-w\left(1+\frac{1}{n}\right)(\delta_C-1)\right]\ ,\ \text{and}\ \beta_R=2-\frac{1}{2}\left[\frac{1}{n}+w\left(1+\frac{1}{n}\right)(\delta_R+1)\right]. \qquad (3.13)$$

Therefore equation (3.12) becomes

$$\eta=-X\left[\frac{1}{\Lambda}\frac{d\psi}{d\lambda}-\frac{\psi}{\Lambda^2}\frac{d\Lambda}{d\lambda}\right]\ , \qquad (3.14)$$

Where Γ_C and Γ_R are the conductive and radiative non-dimensional heat transfer parameters, respectively. The parameters Γ_C and Γ_R depend on the thermal conductivity K and the mean free path of radiation $\frac{1}{\alpha_R}$, respectively, and also on the exponents n and w, and they are given by

$$\Gamma_C=\frac{K_0A^{\delta_C-1}\left(\sqrt{\Gamma T_0}\right)^{w(\delta_C-1)}}{T_0\Gamma^2\rho_0^{\delta_C}}\left\{t_0\left[\frac{\sqrt{\Gamma T_0}}{CU_0}\right]^{\frac{1}{n}}\right\}^{w(\delta_C-1)-1}\ ,$$

$$\text{and}\quad \Gamma_R=\frac{16\sigma A^{-\delta_R-1}\rho_0^{\delta_R}T_0^2\left(\sqrt{\Gamma T_0}\right)^{-w(\delta_R+1)}}{3\alpha_{R_0}\Gamma^2}\left\{t_0\left[\frac{\sqrt{\Gamma T_0}}{CU_0}\right]^{\frac{1}{n}}\right\}^{-w(\delta_R+1)-1}\ .$$

Using the self-similarity transformations (3.4) and the equation (3.2), equations (2.12) can be rewritten as

$$\phi(1)=1-\beta\ ,\ \Lambda(1)=\frac{1}{\beta}\ ,\ H(1)=\frac{M_A^{-1}}{\beta}\ ,$$

$$\psi(1)=\left[\frac{1}{\gamma M^2}+(1-\beta)+\frac{M_A^{-2}}{2}\left(1-\frac{1}{\beta^2}\right)\right]\ ,$$

$$\eta(1)=\left[\frac{1}{2}\left(\beta^2-1\right)+\frac{1}{\beta M_A^2}(1-\beta)\right]\ . \qquad (3.15)$$

By solving equations (3.7)-(3.10) and equation (3.14) for $\frac{d\phi}{d\lambda}, \frac{dH}{d\lambda}, \frac{d\psi}{d\lambda}, \frac{d\eta}{d\lambda}, \frac{d\Lambda}{d\lambda},$ we have

$$\frac{d\phi}{d\lambda} = -\frac{(\phi-\lambda)}{\Lambda}\frac{d\Lambda}{d\lambda} - \frac{2\phi}{\lambda} - w \ , \qquad (3.16)$$

$$\frac{dH}{d\lambda} = \frac{H}{\Lambda}\frac{d\Lambda}{d\lambda} - \frac{nH}{(n+1)(\phi-\lambda)} + \frac{H\phi}{\lambda(\phi-\lambda)} + \frac{wH}{2(\phi-\lambda)} \ , \qquad (3.17)$$

$$\frac{d\psi}{d\lambda} = \left[(\phi-\lambda)^2 - \frac{H^2}{\Lambda}\right]\frac{d\Lambda}{d\lambda} + \frac{2\phi\Lambda(\phi-\lambda)}{\lambda} - \frac{n\phi\Lambda}{n+1} + \frac{H^2\left[\lambda(2n+1)-2(n+1)\phi\right]}{(n+1)\lambda(\phi-\lambda)} + w\left[\Lambda(\phi-\lambda) - \frac{H^2}{2(\phi-\lambda)}\right] , \qquad (3.18)$$

$$\frac{d\eta}{d\lambda} = \frac{(\phi-\lambda)}{\Lambda(\gamma-1)}\left[H^2 + \gamma\psi - (\phi-\lambda)^2\Lambda\right]\frac{d\Lambda}{d\lambda} + \frac{(\phi-\lambda)}{(\gamma-1)}\left[\frac{n\phi\Lambda}{n+1} - \frac{2\phi(\phi-\lambda)\Lambda}{\lambda} - \frac{H^2\left[\lambda(2n+1)-2\phi(n+1)\right]}{(n+1)\lambda(\phi-\lambda)}\right] - w\left[\frac{\Lambda(\phi-\lambda)^2}{\gamma-1} - \frac{H^2}{2(\gamma-1)} - \psi\right] - \frac{2n\psi}{(n+1)(\gamma-1)} - \frac{2\eta}{\lambda} \ , \qquad (3.19)$$

$$\frac{d\Lambda}{d\lambda} = \frac{\Lambda^2}{\left[\psi - \left\{(\phi-\lambda)^2\Lambda - H^2\right\}\right]}\left[\frac{\eta}{X} + \frac{2\phi(\phi-\lambda)}{\lambda} - \frac{n\phi}{n+1} + \frac{H^2\left[\lambda(2n+1)-2\phi(n+1)\right]}{(n+1)\lambda(\phi-\lambda)\Lambda} + w\left\{(\phi-\lambda) - \frac{H^2}{2(\phi-\lambda)\Lambda}\right\}\right] . \qquad (3.20)$$

Also, the total energy of the disturbance is given by

$$E = 4\pi\int_{r_p}^{r_s}\rho\left(e + \frac{u^2}{2} + \frac{uh^2}{2\rho}\right)r^2 dr \ . \qquad (3.21)$$

Using (3.4) and (2.8), equation (3.21) becomes

$$E = 4\pi A(CU_0)^{\frac{2}{n+1}} t_0^{-\frac{2n}{n+1}} r_s^{w+3+\frac{2n}{n+1}} (n+1)^{\frac{2n}{n+1}} \int_{\lambda_p}^{1}\left(\frac{\psi}{\gamma-1} + \frac{\wedge\phi^2}{2} + \frac{H^2}{2}\right)\lambda^2 d\lambda \ . \qquad (3.22)$$

Hence the total energy of the shock wave is non-constant and varies as $r_s^{w+3+\frac{2n}{n+1}}$.

The piston path coincides at $\lambda_p = \frac{r_p}{r_s}$ with a particle path. Using equations (3.1) and (3.4) the relation

$$\phi(\lambda_p) = \lambda_p = \frac{1}{C} = \frac{U_p}{V} \qquad (3.23)$$

Can be derived. In addition to shock conditions (3.15), the kinematic condition (3.23) at the piston surface must be satisfied. For exhibiting the numerical solutions it is convenient to write the field variables in the non-dimensional form as

$$\frac{u}{u_n} = \frac{\phi(\lambda)}{\phi(1)} \ , \ \frac{\rho}{\rho_n} = \frac{\Lambda(\lambda)}{\Lambda(1)} \ , \ \frac{h}{h_n} = \frac{H(\lambda)}{H(1)} \ , \ \frac{p}{p_n} = \frac{\psi(\lambda)}{\psi(1)} \ , \ \frac{q}{q_n} = \frac{\eta(\lambda)}{\eta(1)} \ . \qquad (3.24)$$

4. Result and Discussion

Distribution of flow variables behind the shock front are obtained by numerical integration of the equations (3.16) to (3.20) with the boundary conditions (3.15). For the purpose of numerical integration, values of the constant parameters are taken to be (Ghoniem *et al.* [14], Rosenau [27], Singh and Srivastava [22]) $\gamma = \frac{5}{3}$; $M_A^{-2} = 0.05,\ 0.1$; $\delta_C = 1$, $\delta_R = 2$; $\Gamma_C = 0.01,\ 1,\ 10$; $\Gamma_R = 0.001$, 0.1, 10; $k = \frac{1}{2}$; $n = 2,\ 3$. The value $n = 2,\ 3$ correspond to $w = -\frac{7}{3},\ -\frac{5}{2}$ respectively i.e. to ambient media with decreasing density.

Figures 1-10 show the variation of the flow variables $\frac{u}{u_n}, \frac{\rho}{\rho_n}, \frac{h}{h_n}, \frac{p}{p_n}, \frac{q}{q_n}$ with λ at various values of the parameters $M_A^{-2}, \Gamma_C, \Gamma_R, n$. It is shown that, as we move inward from the shock front towards the inner contact surface (piston), the reduced density $\frac{\rho}{\rho_n}$ and the reduced pressure $\frac{p}{p_n}$ increase, and the reduced velocity $\frac{u}{u_n}$ increases and after attaining a maximum near the piston starts to decrease (figures 1, 2, 4, 6, 7, 9). The reduced magnetic field $\frac{h}{h_n}$ increases behind the shock front and after attaining the maximum, decrease abruptly near the piston (figures 3 and 8). The reduced total heat flux $\frac{q}{q_n}$ also shows, somewhat, the same behaviour in some cases as shown by $\frac{h}{h_n}$, and in the remaining cases it decreases throughout behind the shock front (figures 5 and 10).

As can be seen from the equation (3.20) for Λ, there is a singularity at the piston where $\phi = \lambda$, because this equation becomes singular there. The singularity is non-removable and derivative of the density tends to positive infinity, as shown in figures 2 and 7, in most of the cases. This singularity can be physically interpreted as follows (Steiner and Hirschler [29]): the path of the accelerated piston converges with the path of the particle immediately ahead condensing the gas to infinity. This can also be interpreted from the adiabatic integral as follows:

The effects of an increase in M_A^{-2}, i.e. in the ambient magnetic field are (from figures 1 to 10)

(i) To increase the velocity $\frac{u}{u_n}$, and to decrease the density $\frac{\rho}{\rho_n}$ and the pressure $\frac{p}{p_n}$ at any point in the flow-field behind the shock, in general;

(ii) To increase the distance of maxima point in the profiles of the magnetic field $\frac{h}{h_n}$ from the shock front;

(iii) To remove the tendency of formation of a maxima point in the profiles of total heat flux $\frac{q}{q_n}$ at higher values of $\Gamma_C\,(=10)$ (see figure 5), and to increase the distance of maxima point of $\frac{q}{q_n}$ from the shock front at smaller values of $\Gamma_C\,(=0.01, 1)$ (see figure 10);

(iv) To increase the distance of the piston (the inner surface) from the shock front (see tables 1 and 2). This shows that there is a decrease in the strength of shock due to presence of magnetic field.

It is found that the effects of an increase in the value of radiative heat transfer parameter Γ_R are (from figures 1 to 5)

(i) to decrease the velocity $\frac{u}{u_n}$, density $\frac{\rho}{\rho_n}$, magnetic field $\frac{h}{h_n}$, pressure $\frac{p}{p_n}$ and total heat flux $\frac{q}{q_n}$;

(ii) to remove the tendency of the total heat flux $\frac{q}{q_n}$ of attaining a maximum in the flow behind the shock.

(iii) In general, somewhat, flattening of the profiles of the flow variables.

The distance between the piston and the shock front is almost unaffected (see table 1).

Effects of an increase in the value of conduction heat transfer parameter Γ_C are (from figures 6 to 10)

(i) To decrease the velocity $\frac{u}{u_n}$, density $\frac{\rho}{\rho_n}$, magnetic field $\frac{h}{h_n}$, and total heat flux $\frac{q}{q_n}$; and to increase the pressure $\frac{p}{p_n}$. These effects are significant when $M_A^{-2} = 0.1$, i.e. when the initial magnetic field is strong.

(ii) To increase the distance of the piston from the shock when $M_A^{-2} = 0.1$, and this increase is, somewhat, significant when $w = -\frac{5}{2}$ i.e. when the initial density is decreasing faster.

(iii) In general flattening of the profiles of $\frac{u}{u_n}$, $\frac{\rho}{\rho_n}$, $\frac{h}{h_n}$ and $\frac{q}{q_n}$.

Above results show that the effects of conduction heat transfer are significant in the presence of stronger ambient magnetic field.

The effects of an increase in the piston velocity index n, i.e. the effects of a decrease in the ambient density variation index w are (from figures 1-10 and tables 1-2).

(i) To increase the velocity $\frac{u}{u_n}$, density $\frac{\rho}{\rho_n}$, magnetic field $\frac{h}{h_n}$ and total heat flux q/q_n, and to decrease the pressure p/p_n, at lower values of $\Gamma_C\,(=0.01,\ 1)$. At $\Gamma_C = 10$, the effects are small.

(ii) To decrease the distance of the piston from the shock front at lower values of Γ_C. At $\Gamma_C = 10$, the effects are very small (see tables 1 and 2). Actually, the piston speed index n, ambient density variation index w and ambient magnetic field variation index k are connected by the relation $w = -2k - \frac{2n}{n+1}$. For $k = \frac{1}{2}$, an increase in n from 2 to 3 results in a comparatively small decrease in w from $-7/3$ to $-5/2$. Therefore, the increase in shock velocity due to decrease in w is overcome by increase in piston velocity due to increase in n, and this fact causes the decrease in the distance of piston from the shock front.

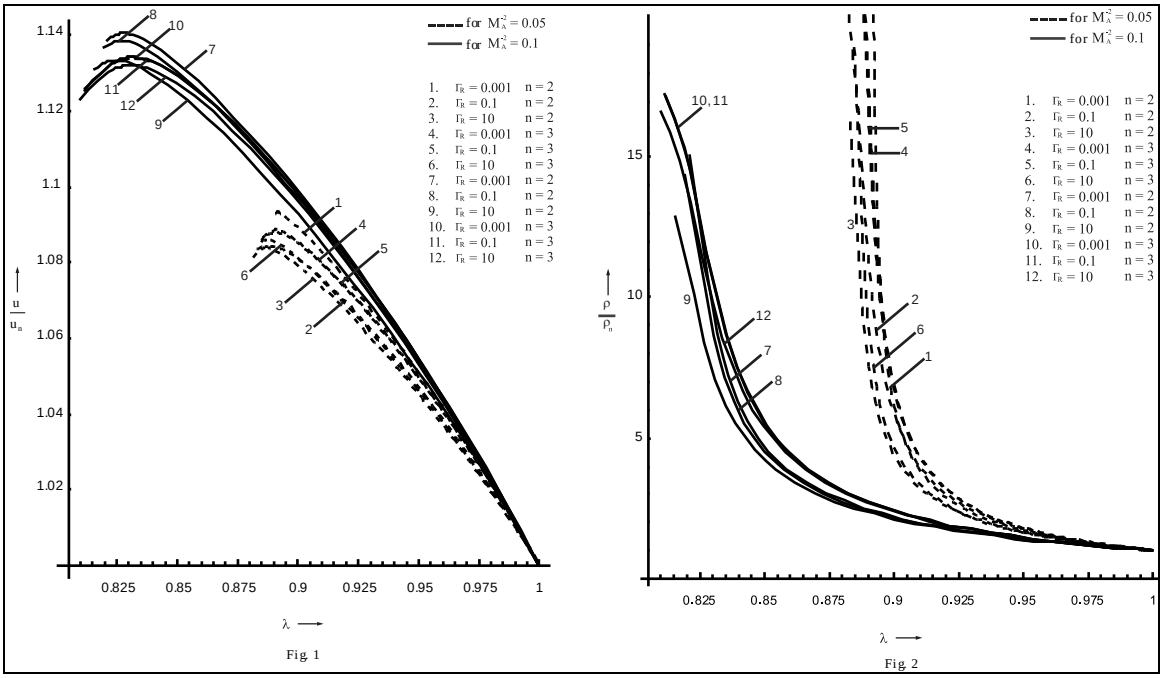
Table 1: Piston position λ_p at different values of Γ_R

Γ_C	M_A^{-2}	n	w	Γ_R	λ_p
10	0.05	2	$-\frac{7}{3}$	0.001	0.8908
				0.1	0.8847
				10	0.8830
		3	$-\frac{5}{3}$	0.001	0.8854
				0.1	0.8848
				10	0.8814
	0.1	2	$-\frac{7}{3}$	0.001	0.8208
				0.1	0.8193
				10	0.8153
		3	$-\frac{5}{3}$	0.001	0.8113
				0.1	0.8116
				10	0.8096

Table 2: Piston position λ_p at different values of Γ_C

Γ_R	M_A^{-2}	n	w	Γ_C	λ_p
0.01	0.05	2	$-\frac{7}{3}$	0.01	0.8865
				1	0.8859
				10	0.8847
		3	$-\frac{5}{3}$	0.01	0.8955
				1	0.8946
				10	0.8848
	0.1	2	$-\frac{7}{3}$	0.01	0.8317
				1	0.8285
				10	0.8193
		3	$-\frac{5}{3}$	0.01	0.8506
				1	0.8458
				10	0.8114

Fig.1-5 Variation of the reduced velocity, density, magnetic field, pressure, total heat flux in the region behind the shock front with $\Gamma_C = 10$ and Fig.6-10 Variation of the reduced velocity, density, magnetic field, pressure, total heat flux in the region behind the shock front with $\Gamma_R = 0.1$



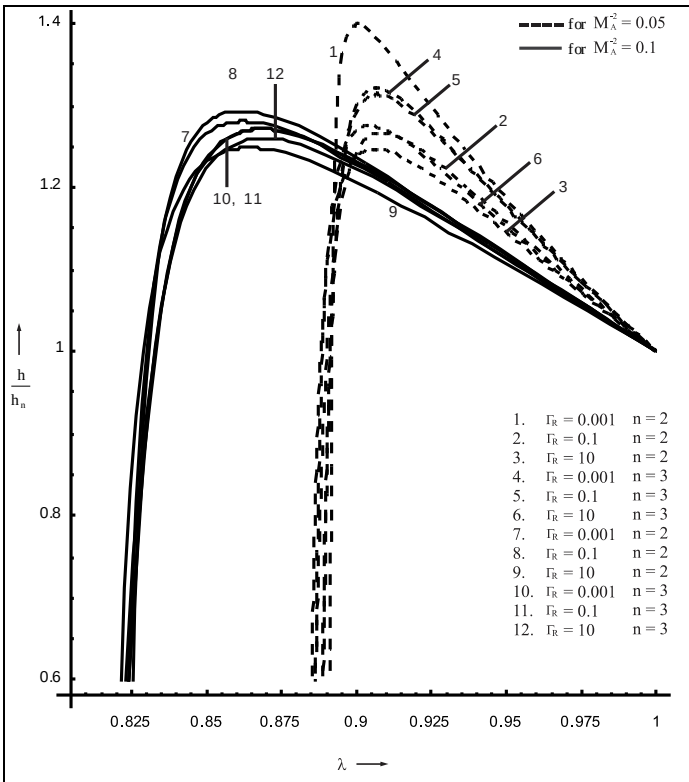


Fig. 3

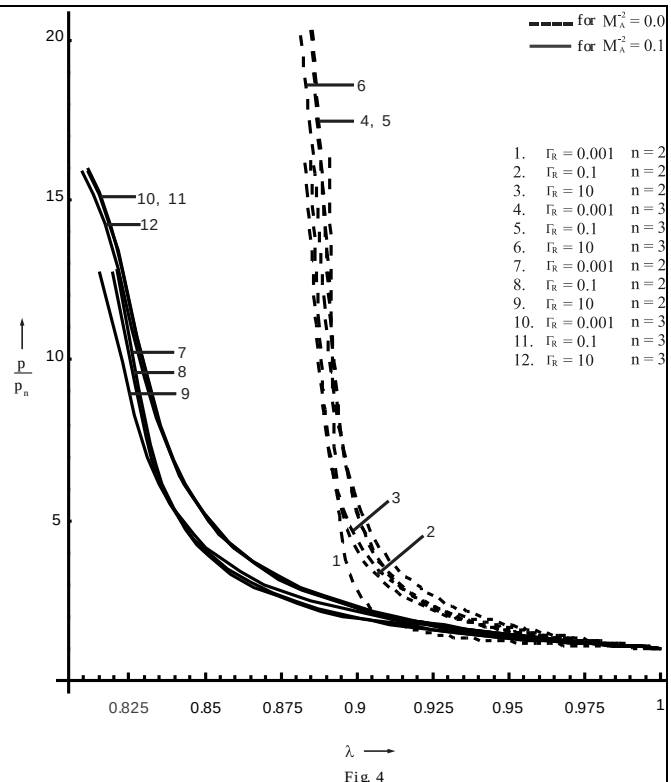


Fig. 4

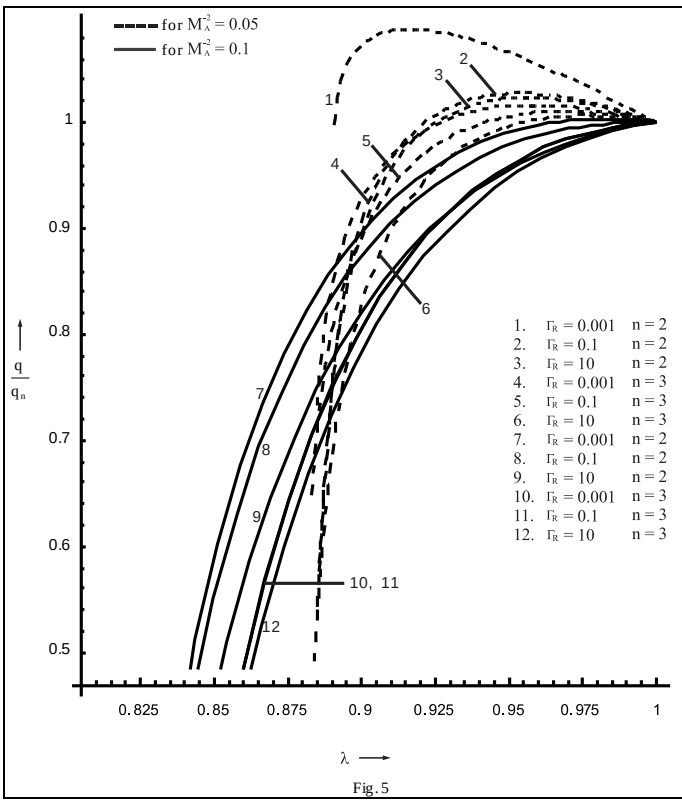


Fig.5

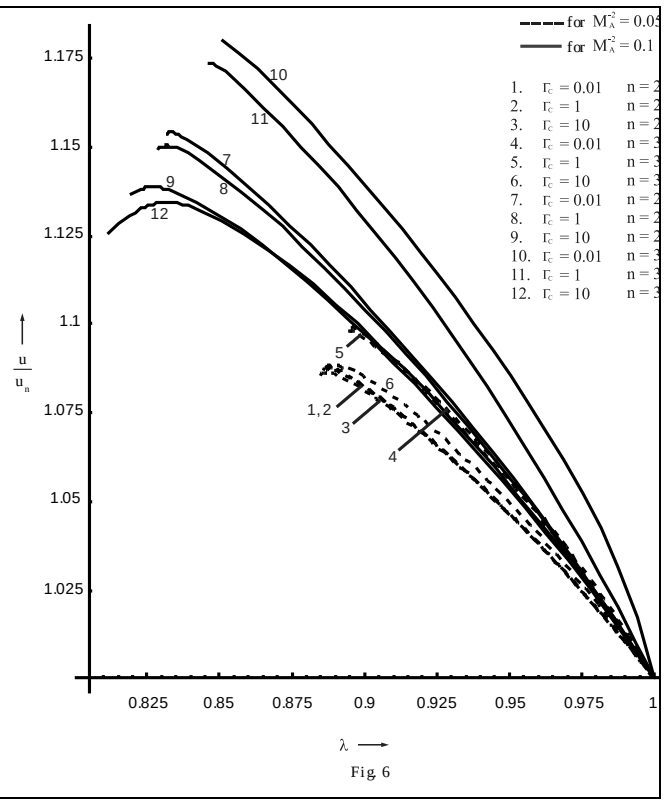


Fig 6

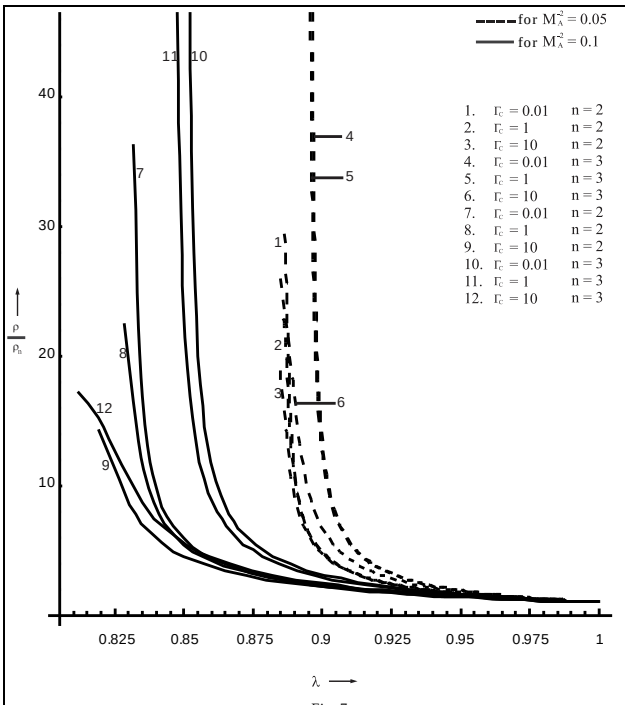


Fig. 7

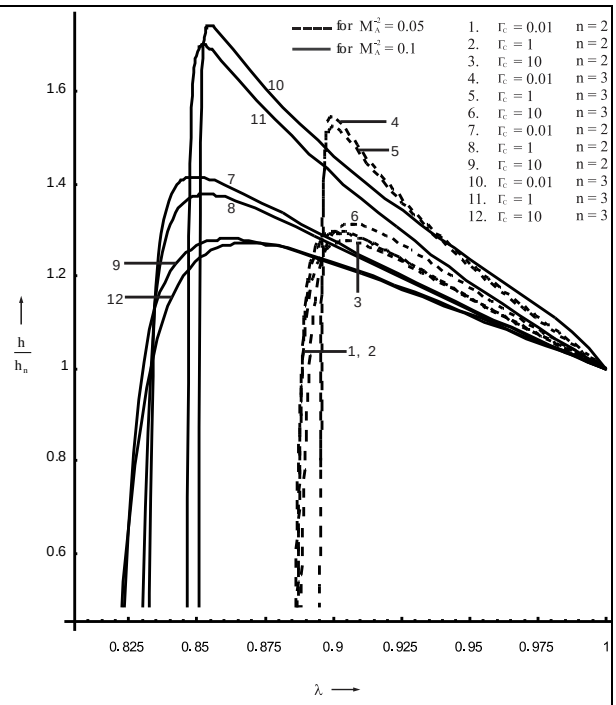


Fig. 8

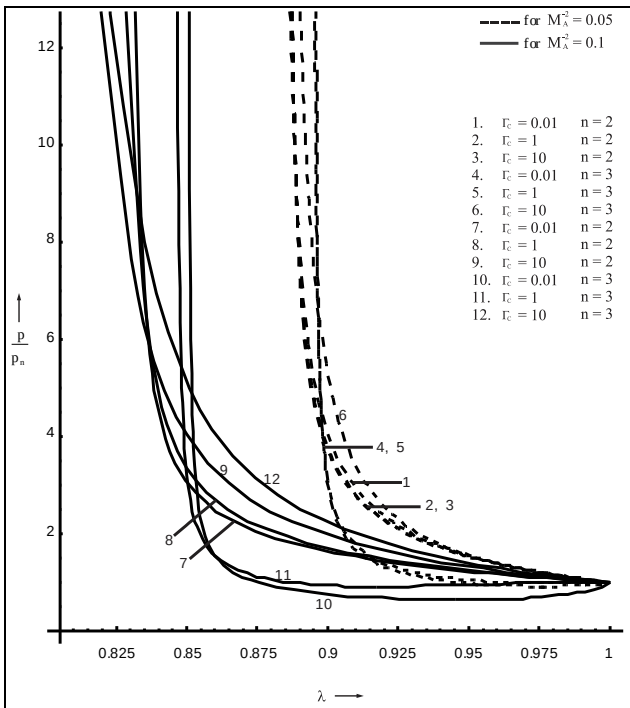


Fig. 9

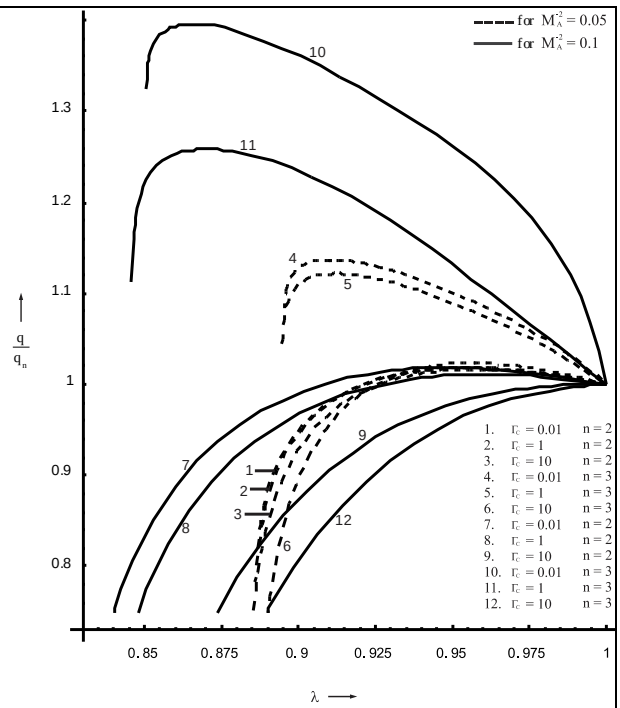


Fig. 10

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