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New approach to almost periodicity in to pological dynamics

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Abstract
In this short paper we try to concerned with vector valued functions but for our first considerations, we restrict our attention to complex valued function of real variables Furthermore, we assume that all function considered in this section are continuous and make no further note of the matter. We want to make some basic observations before we proceed and for this we first define almost periodic function due to Bochner [1].

Keywords: Periodic, minimal, finite dimensional

Introduction

Basic Definitions

A functions f: $R \longrightarrow X$ is almost periodic if from every sequence $\{S_n\}$ there is a subsequence $\{\alpha_n\}$ (avoiding the necessity for double subscripts when taking subsequences) such that the sequence $\{f(t + \{f(t+\alpha_n)\}$ in linear space X converges uniformly on the real line R. The sequence $\{\alpha_n\}$ will be denoted byt α in the sequel and T α f=g means that g(t)=lim f(t+ α_n) and is written only when the limit exists, where T designates translation operator, The hull of almost periodic function is defined as

H (f)={g: there exists a sequence α with Tαf uniformly}

in order to talk about uniformly almost periodic family it is convenient to introduce Bochner's Translation function For an almost perodic function we express number

$V_f(\tau) = \sup.\{\| f(t+\tau) - f(t) \| \text{ for all } t \in R\}$ consequently.

A family $\mathcal{F}$ of almost periodic function on a linear space is said to be uniformly almost periodic family if it is bounded and if for a preassigned

number $\in > 0$. Then T (f, ∈) $= \cap\, T(f, \in)$ is relatively dense and includes on interval about 0.

Now we wish to enumerate the following important results on Bochner's translation function which will be frequently used in the sequel [2].

i) $V_f(\tau) \geq 0; V_f(\tau) = V_f(\tau)$.
ii) $V_f(0) = 0$
iii) $V_f(\tau + \sigma) + Vf(\sigma)$
iv) Vf is its own translation function
v) f is almost periodic if and only if V_f is almost periodic where the first three result hold trivially and (iv) follows from the fact: $V_f(t+s) - V_f(t) \leq V_f(s) \Longrightarrow \sup. \| V_f(t+s) - V_f(t) \| \leq V_f(s)$ and consequenlty (v) immediately follows.

Main Results

We will want to make some basic observation before we peoceed. Almost periodic functions are intended to be generalization of periodic functions in some sense. It is clear that the periodic functions satisfy the deficnition of an almost periodic funtion in the sense of Bochner. Indeed, if f has period τ, then from the given sequene $\{s_n\}$ we may select a subseqence $\{\alpha\}$ such that $\{\alpha_n(\text{mod}.\sigma)\}$ converges to σ_0.Then $\lim f(t+\alpha_n) = f(f+\alpha_0)$. Thus the almost periodicity of f for periodic f is a statement about the compactness of $A = \{f_\tau : f_\tau(t) = f(t+\tau)\}$ in the space of continuous function with the uniform norm. for general almost periodic functions we replace compactness by pre-compactness or by A having compact closure.

The concept of uniformly almost periodic family includes the notion of uniformly boundedness and equicontinuity. In fact all uniformly almost periodic families are uniformly bounded and equinuiformly continuous, that is $\| f \| \sup \| f(t) \| < M \; for \; all \; f \in \mathcal{F}$ where M depends only on the family, and there is a $\delta > 0 \; so \; that \; \| f(x) - f(y) \| < \in Ff$ whenever lx-yl<δ. Families with these two properties are pre-compact in the topology of uniform convergent on compacta of R [3]. This fact will be used in almost every argument in our later discussion on differential equation, so that it needs precision.
Let $\{f_n\}$ be a sequence of functions from R to C such that there is a number M for which $\|f_n\| \leq M$ for all n and for a preassigned number $\in > 0$ however small there corresponds $\delta > 0$ such that $1 f_n(x) - f_n(y) 1 \in$ for all n whenever $1x - y1 < \delta$.
Then there is a subsequence which converges uniformly on every compact subset of R.

Theorem 1: H (f) is compact in uniform norm if and only if f is almost periodic.

Proof: Let us assume that H(f) is compact. Then it is sequentially compact, that is, given a sequence $\{s_n\}$ there is a subsequence $\alpha = \{\alpha_n\}$ such that $T\alpha f$ exists uniformly on R. Hence f is almost periodic.
Conversely, let f be almost periodic and $\{\sigma_n\}$ is a sequence from H(f) then we pick s_n such that $\| f(t + s_n) - \sigma_n(t) \| < 1/n$.
We can find a subsequence of $\{s_n\}$ so that $T\alpha f$ exists uniformly. Let β be a subsequence of $\{n\}$ so that β and α are common subsequences, then $f(t + \alpha n) - \sigma_{\beta n}(t) \to 0$ so that $\sigma_{\beta n} \to T\alpha f$.
Since we are considering H (f) as a metric space we observe that the compactness is equivalent to the fact that the set of translates $\{f(t+\tau)\}$ for $\tau \in$ R is totally bounded
These remarks prove the following theorem which is adopted by Von Neumann as the definition.

Theorem 2: The function f is almost periodic if and only if for ever $\epsilon >$, there are numbers a_1, a_2......................, a_n and a function n(t) from R to {1,2..............n} such that

$$\| f(t + a_n \tau) \| < \in \text{ for all t and } \tau.$$

Theorem 3: If f is almost periodic, then for any $g \in H(f). H(g) = H(f)$.

Proof: If α is given with $T\alpha$ g=h uniformly $g \in H(f)$, then select a subsequence β so that $\| f(t + \beta_n) - g(t + \alpha_n) \| < 1/n$.
If follows that $T\beta f = h$ so that $h \in H(f)$. Thus H (g) $\subseteq$ H(f).
On the other hand, if $T\alpha f$=g then $\| f(t + \alpha_n) - g(t) \| \to 0$ so making the change of variable $\tau + \alpha_n \to s$, one has $\| f(s) - g(s - \alpha_n) \| \to 0$, i.e., $T_{-\alpha} g = f$.

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